

## ADAPTIVE FUZZY CONTROL SYSTEM FOR MULTI-DIMENSIONAL DYNAMIC OBJECT UNDER THE CONDITIONS OF UNCERTAINTY OF INFORMATION

*PhD.dots., U.O. Xujanazarov,  
PhD.dots., G.R. Alimova*

**Аннотация.** Предложена математическая модель динамических объектов в пространстве состояния в условиях нечеткой исходной информации. Синтезирован адаптивно-нечеткий алгоритм управления технологическими объектами на основе динамической модели с робастными свойствами. Для придания свойства робастности алгоритму управления предложено использование дискретного алгоритма скоростного градиента в параметрической форме, что позволит обеспечить минимум сложности и учесть ограничения на управляющий сигнал и скорость его изменения. Предложенный алгоритм реализован для управления процессами в ректификационной установке, который позволил уменьшение переходного процесса на 20% и отклонение вектора переменных состояний из эталонного на 15%.

**Abstract.** A mathematical model of dynamic objects in the state space under conditions of fuzzy initial information is proposed. An adaptive-fuzzy control algorithm for technological objects is synthesized based on a dynamic model with robust properties. To impart robustness to the control algorithm, it is proposed to use a discrete velocity gradient algorithm in a parametric form, which will provide a minimum of complexity and take into account the constraints on the control signal and its rate of change. The proposed algorithm was implemented to control the processes in the rectification plant, which allowed a decrease in the transient process by 20% and the deviation of the vector of state variables from the reference one by 15%.

**Keywords:** mathematic model, dynamic object, fuzzy initial information, robustness algorithm, control signal.

Modern technological objects are complex weakly formalized systems operating under conditions of great uncertainty, incomplete knowledge and unclear descriptions of both the system itself and the disturbances acting on it. To manage such objects, it is not enough to use classical methods of control theory, and there is a need to develop new methods and approaches using the achievements of modern information technologies. One of these approaches, based on the theory of fuzzy sets and fuzzy logic, is the basis for the creation of an intellectualized control system for technological objects operating under conditions of information uncertainty.

In this case, in order to increase the efficiency of managing technological objects using modern management methods, it is necessary to solve the following tasks:

- • assessment of quality indicators of uncertainty
- • reduction (or compensation) of a priori uncertainty of knowledge about the process due to the use of operational information from measuring instruments and construction of an adaptation loop
- • the formation of such a control law that would guarantee stability and specified indicators of accuracy and quality of the control system (CS) in conditions of uncompensated (a posteriori) uncertainty.

One of the possible ways to solve these problems is the use of methods of adaptive, robust and fuzzy control [1, 2].

An important point in this case is the construction of a mathematical model for representing knowledge about dynamic objects in conditions of fuzzy information, which should make it possible to determine the uncertainty indicators, give a description of random processes, invariant to their distribution law.

- • fuzzy equation of state

$$d\bar{x}/dt = \bar{A} \otimes \bar{x} \oplus \bar{B} \otimes u, \quad \mu_{\bar{s}}(s), \quad (1)$$

- • fuzzy observation equation

$$\bar{y} = \bar{C} \otimes \bar{x}, \quad (2)$$

- • fuzzy initial conditions

$$\bar{x}_1(0) = \bar{D}_1, \quad \bar{x}_2(0) = \bar{D}_2, \dots, \bar{x}_n(0) = \bar{D}_n, \quad (3)$$

where  $\otimes, \oplus$  - fuzzy operations, respectively, addition and multiplication;  $u$  - control signal (scalar) that takes fuzzy values;  $\bar{x} = \{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_i, \dots, \bar{x}_n\}$  - a vector of a fuzzy state,  $i = 1, 2, \dots, n$ ;  $\bar{y} = \{\bar{y}_1, \bar{y}_2, \dots, \bar{y}_{\zeta}, \dots, \bar{y}_l\}$  - a vector of fuzzy output variables,  $\zeta = 1, 2, \dots, l$ ;  $\mu_{\bar{s}}(s)$  - indicator of a fuzzy (changing) number of state variables and representing the weight of the  $s$ -th equation of state;

$$\bar{A} = \begin{bmatrix} \bar{A}_1^1 & \dots & \bar{A}_n^1 \\ \dots & \dots & \dots \\ \bar{A}_1^n & \dots & \bar{A}_n^n \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} \bar{B}^1 \\ \dots \\ \bar{B}^n \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} \bar{C}_1^1 & \dots & \bar{C}_n^1 \\ \dots & \dots & \dots \\ \bar{C}_1^l & \dots & \bar{C}_n^l \end{bmatrix} - \text{matrix of fuzzy coefficients of the}$$

model.

Some  $i$ -th variable of the state vector as a function of time  $t$  can be represented by a fuzzy relation (FR) [3,4]:  $\bar{x}_i(t) = \{t, x_i / \mu_{\bar{x}_i}(t, x_i)\}$ ,  $i = 1, 2, \dots, n$ , and at a fixed moment in time, this variable can be expressed by a fuzzy set (FS):  $\bar{x}_i = \{x_i / \mu_{\bar{x}_i}(x_i)\}$ . The  $\zeta$  - th output variable has a similar description:

$$\bar{y}_{\zeta}(t) = \{t, y_{\zeta} / \mu_{\bar{y}_{\zeta}}(t, y_{\zeta})\}, \quad \zeta = 1, 2, \dots, l,$$

$$\bar{y}_\zeta = \{y_\zeta / \mu_{y_\zeta}(y_\zeta)\},$$

where  $\mu_{x_i}, \mu_{y_\zeta}$  are membership functions (MF);  $x_i, y_\zeta$  - values from universal

sets. Matrix elements  $\bar{A}, \bar{B}, \bar{C}$  are given by the FS:

$$\begin{aligned} \bar{A}_1 &= \{A_1^1 / \mu_{A_1^1}(A_1^1)\}, \dots, \bar{A}_n = \{A_n^n / \mu_{A_n^n}(A_n^n)\}, \\ \bar{B}^1 &= \{B^1 / \mu_{B^1}(B^1)\}, \dots, \bar{B}^n = \{B^n / \mu_{B^n}(B^n)\}, \\ \bar{C}_1^1 &= \{C_1^1 / \mu_{C_1^1}(C_1^1)\}, \dots, \bar{C}_n^1 = \{C_n^1 / \mu_{C_n^1}(C_n^1)\} \end{aligned}$$

The initial conditions are described by the FS  $\bar{D}_i = \{x_i / \mu_{\bar{D}_i}(x_i)\}$ , and the number of state vector variables is described by - the FS  $\bar{S} = \{s / \mu_{\bar{S}}(s)\}$ , where  $s = 1, 2, \dots, n$  is the ordinal number of the state vector variable.

Membership functions are specified by analytical dependence [2], for example, for a variable  $\bar{x}_i$ :

$$\begin{aligned} \mu_{\bar{x}_i}(x_i) &= \varphi(x, a_{\bar{x}_i}, b_{1\bar{x}_i}, b_{2\bar{x}_i}, v_{1\bar{x}_i}, v_{2\bar{x}_i}) = \\ &= \left( (b_{1\bar{x}_i}(a_{\bar{x}_i} - x))^{v_{1\bar{x}_i}} \frac{\text{sign}(b_{1\bar{x}_i}(a_{\bar{x}_i} - x)) + 1}{2} + (b_{2\bar{x}_i}(a_{\bar{x}_i} - x))^{v_{2\bar{x}_i}} \frac{\text{sign}(b_{2\bar{x}_i}(a_{\bar{x}_i} - x)) + 1}{2} + 1 \right)^{-1} \end{aligned} \quad (4)$$

In formula (4), the coefficient  $a_{\bar{x}_i}$  represents the MF mode, the coefficients  $b_{1\bar{x}_i}$  and  $b_{2\bar{x}_i}$  set the MF width,  $v_{1\bar{x}_i}$  and  $v_{2\bar{x}_i}$  is the slope of the MF to the  $x_i$  axis, i.e. contrast. Coefficients  $b_{1\bar{x}_i}, b_{2\bar{x}_i}, v_{1\bar{x}_i}, v_{2\bar{x}_i}$  allow to form any form of MF and can act as indicators of uncertainty.

The quality indicators of the control system (transient time, overshoot, tracking error, etc.) are set in the form of utility functions:

$$\begin{aligned} \bar{Q}_k^3 &= \{Q_k^3 / \mu_{Q_k^3}(Q_k^3)\}, k = 1, 2, \dots, K, \\ \mu_{Q_k^3}(Q_k^3) &= \varphi(Q_k^3, a^3_{\bar{x}_i}, b^3_{1\bar{x}_i}, b^3_{2\bar{x}_i}, v^3_{1\bar{x}_i}, v^3_{2\bar{x}_i}), \end{aligned} \quad (5)$$

where  $K$  - is the number of quality indicators of the control system.

A reference model was determined based on the specified indicators of management quality:

$$\dot{x}_M = A_M x_M + B_M u_M, \quad (6)$$

where  $u_M$  - is the setting action of the system;  $x_M(t)$  - vector of reference states.

Constraints on the variables of the state vector and constraints on control are set;

$$\begin{aligned} g_1(\bar{x}, u, \gamma, t) &< x_{1\max}, g_2(\bar{x}, u, \gamma, t) < x_{2\min}, \dots, \\ g_{2n-1}(\bar{x}, u, \gamma, t) &< x_{n\max}, g_{2n}(\bar{x}, u, \gamma, t) < x_{n\min}, \dots, \\ g_{m-1}(\bar{x}, u, \gamma, t) &< u_{\max}, g_m(\bar{x}, u, \gamma, t) < u_{\min}. \end{aligned} \quad (7)$$

Let the minimization of the average deviation of the state vector variables of the real behavior of the controlled object from the standard be chosen as the control goal.

It is necessary to synthesize control systems and adjust the regulator so that all signals to the control systems are limited, i.e.  $|x(t)| < x_{don}$ ,  $|u(t)| < u_{don}$ , and the transient processes in the system met the specified quality indicators (5).

To determine the quality indicator of the control system  $a_{x_i}^-(t)$ ,  $a_{y_i}^-(t)$ , both the time characteristics of the control object and the fuzzy parameters  $b_{1x_i}^-(t)$ ,  $b_{2x_i}^-(t)$ ,  $b_{1y_k}^-(t)$ ,  $b_{2y_k}^-(t)$ , are used, determined by the width of the membership function.

In order to reduce the ambiguity of the data about the object and improve the control quality indicators, the values  $b_{1x_i}^-(t)$ ,  $b_{2x_i}^-(t)$ ,  $b_{1y_k}^-(t)$ ,  $b_{2y_k}^-(t)$ .

To impart robustness to the control algorithm, it is proposed to use a discrete velocity gradient algorithm in a parametric form, which will provide a minimum of complexity and take into account the constraints on the control signal and the rate of its change [5, 6].

In this case, the control signal is formed on the basis of a fuzzy set of values of state variables, which is possible when the behavior of the object is varied.

Due to the fact that the object model is an averaging summation filter, the synthesis of the control law based on the fuzzy model additionally enhances the robustness of the velocity gradient algorithm, while maintaining the boundedness of phase trajectories in a certain area under conditions of uncompensated uncertainty of values about the object.

In order to improve the quality and accuracy of the formation of transient processes in control systems, an algorithm for making decisions on the choice of a compromise control control signal by embedding it in control algorithms is implemented.

The modified control law is as follows:

$$u = k_u(t) \cdot u_m(t) + \sum_{i=1}^n k_x^\Sigma(t) \cdot x_i^\Sigma(t), \quad (8)$$

$$k_{x1}^\Sigma[t+1] = k_{x1}^\Sigma[k](1 - h\gamma_3) + h(\gamma_5 - \gamma_4)\delta \cdot [t]x_1^\Sigma[t] - h\gamma_5\delta \cdot [t+1]x_1^\Sigma[t+1], \quad (9)$$

$$k_{xn}^\Sigma[t+1] = k_{xn}^\Sigma[k](1 - h\gamma_3) + h(\gamma_5 - \gamma_4)\delta \cdot [t]x_n^\Sigma[t] - h\gamma_5\delta \cdot [t+1]x_n^\Sigma[t+1], \quad (10)$$

$$k_u[t+1] = k_u[k](1 - h\gamma_1) + h(\gamma_6 - \gamma_2)\delta \cdot [t]u_m[t] - h\gamma_6\delta \cdot [t+1]u_m[t+1], \quad (11)$$

where  $t = mh$ ,  $h > 0$  is the sampling step,  $m = 0, 1, 2, \dots, m$ ;  $\gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6\}$

- parameters of the adaptive controller;  $e_i^\Sigma = \int_{x_i} (x_i - x_{im}) \mu_{e_i}^-(e_i) dx_i$  - mismatch between

state vector variables and reference state variables;  $\mu_{e_i}^-(e^i) = (e_i, a_{e_i}^-, b_{1e_i}^-, b_{2e_i}^-, v_{1e_i}^-, v_{2e_i}^-)$  -

MF of the error,  $\varphi$  - is the analytical form of the membership function in the form (4),  $a_{ei}^- = a_{xi}^- - x_{im}$ ,  $v_{1ei}^- = v_{1xi}^-$ ,  $v_{2ei}^- = v_{2xi}^-$ ,  $b_{1ei}^- = b_{1xi}^-$ ,  $b_{2ei}^- = b_{2xi}^-$ ,  $x_i^\Sigma = \int_{X_i} x_i dx_i$  - is the

integrated variable of the state vector;  $\delta \cdot [t] = \sum_{i=1}^n k_i \cdot e_i^\Sigma$ ,  $h_i$  - are the coefficients obtained from the coefficients of the matrix and the solution of the Lyapunov equation and the matrix of the reference model  $B_M$ .

The problem of determining the parameters of an adaptive controller  $\gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6\}$ , when the quality indicators of the control system (5) and the behavior of the control object (1), (2), (3) are given in a fuzzy form, can be reduced to the classical clear problem of finding the extremum of the objective function in the presence of constraints (7)

For this, a generalized criterion is proposed, which, firstly, combines the quality indicators of the control system, and secondly, implements defuzzification of fuzzy values of the quality indicator of the control system:

$$I(\gamma) = K - \sum_{k=1}^K \frac{\mu_{Q_k}^1 \wedge \mu_{Q_k}^3}{\mu_{Q_k}^1} \rightarrow \min, \quad (12)$$

where  $\mu_{Q_k}^1 = \int_{-\infty}^{\infty} \mu_{Q_k}^1(Q_k) \cdot dQ_k$  - is the energy of the FS  $\bar{Q}_k = \{Q_k / \mu_{Q_k}^1(Q_k)\}$

expressing the value of the k-th quality indicator of the control system predicted on the basis of the fuzzy model;  $\mu_{Q_k}^3 = \int_{-\infty}^{\infty} \mu_{Q_k}^3(Q_k) \cdot dQ_k$  - energy of FS, expressing the utility function.

The study of the effectiveness of the proposed approach in the operation of a fuzzy adaptive system was carried out with temperature and level control in a rectification unit [1].

The following results were obtained:

1. Operational formation of control signals helps to reduce the time of the transient process by 20%,

2. the average deviation of the vector of the state variables of the real process from the reference one is reduced by 15% in comparison with the indicators of the known adaptive control systems.

The implementation of the proposed control system synthesis algorithm expands the area of application of control systems in production conditions in the presence of a wide range of disturbances.

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