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GRAPHICAL SOLUTIONS TO LINEAR PROGRAMMING PROBLEMS OF ECONOMIC CONTENT USED BY DIFFERENT METHODS

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Abstract. In this article, the problems of economic content given in different ways are presented in a linear programming problem, and finding solutions in a graphical way will be discussed. Also, at first, linear programming is discussed, its solution methods are given, and special emphasis is placed on solving the given problem in a mathematical interpretation: by means of a linear function. As an example, the issue of optimal production planning for the furniture shop "FAYZLI BAGHDAD" is given.

Key words: Linear function, linear constraints, linear programming, extremum, graphical method, mathematical modeling, objective function.

Enter

Many management decisions require solving the issues of effective distribution of material, economic and personnel resources of the organization and enterprise. Regardless of the organization and enterprise, production or service activities, the continuous solution of problems is among the issues of demand for trade:

• determining the optimal production plan;
• determination of optimal sales and purchase plans;
• execution of the optimal transport plan;
• optimal warehouse management (how much to buy or sell?);
• their optimal processing and management;
• optimal organization and integration and direction of work of social service systems.

L.V. Kantorovich is recognized as the founder of the linear programming method. His 1939 monograph "Production Organization and Planning" introduced the first method of linear programming. L.V. Kantorovich's previous opinion was ignored at the time.

World War II-era American scientist TS Koopmans rediscovered linear programming problems and the file for science and practice.

Another American scientist D. Danzig in 1947 developed an effective method for solving linear programming problems - the simplex method. "Linear programming" was introduced by this scientist in the mid-1940s, and programming should be understood in the sense of planning (in English - another meaning of the word programming).

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After the 1950s, the advent of electronic machines laid the foundation for the rapid development of the linear programming method. At the same time, the research of practical economic issues led to a new era in the development of a new science and the creation of mathematical methods in economics.

Analysis and results

An enterprise has an important role in making effective management decisions of material, labor or time resources. Resources – equipment, raw materials, manpower, time, funds, cost allocation machines or organization are the guarantee of high economic efficiency indicators.

Linear programming is a widely used mathematical modeling tool for decision making that helps managers solve resource planning and allocation problems.

Description. Finding the extremum (maximum or minimum) of a multi-argument linear function is called linear programming.

Such as,

$$f(x, y) = 3x + 4y$$

of a two-argument linear function

$$\begin{cases} 2x + y \le 100 \\ x - 3y \le 75 \end{cases}$$

$$x \ge 0, \quad y \ge 0$$

find the maximum value using the satisfaction of linear inequalities. The given example could be a programming problem. In this case, the function is the objective function, the inequalities are non-negativity settings, and the system of linear inequalities are not basic linear programming structures.

A linear programming problem is a mathematical model of many economic problems.

Currently, linear programming problems are widely used in solving manufacturing, agricultural, production, military, and marketing problems. In addition to being very broad in scope, these issues have a general selection.

The structure of all linear programming problems involves maximizing or minimizing an exponent. Define this quantity as an indicator objective function. In a linear programming problem, the objective must be clearly defined and the objective function must be mathematically expressed.

The objective function depends on the variables, and optimization involves finding the value of the variables that achieves the largest (or smallest) value of the objective function.

In matters of production, the profit or profit placed on the purpose of production may be the production of production. In this case, the objective function may be profit, revenue, or production output. In agricultural issues, it is possible to aim to achieve high productivity indicators (objective function) in return for efficient transportation of land, water, fertilizer or technical resources. Service provision can be seen, for example, in traffic distribution issues. In this case, with the objective function, one can see such things as traffic safety (minimization problem), total profit from transportation (maximization problem) or vehicle transfer time (minimization problem). For example, it is necessary to allocate bank funds (through finance) in such a way that the income from investments (objective function) is maximal.

Implementation of an arbitrary optimization problem on data with a set of variables. It can be defined by restrictions:

• secondary (profit is expected in solving the problem of minimizing the risk of the investment portfolio);

• material, labor, savings, or time constraints;

• services defined in the course of activity (market inspections, regulatory documents, requirements for the decision-making entity, etc.).

A possible solution of an arbitrary set of powers to satisfy the constraints (conditions) of the variables of the problem is considered. A decision must be made by the individual among possible solutions.

Description. Among all possible solutions, the solution that achieves the largest or smallest values of the objective function is called the optimal solution.

In most cases, there is only one optimal solution, but in practice there are models with a large number of optimal solutions.

In linear programming, the objective function must be linearly dependent on the problem variables, and the constraints must be represented by linear equations and inequalities.

In addition to unknown variables, linear programming models also involve fixed quantities. Such quantities are called model parameters. In terms of production, the price of products, the stock of raw materials are parameters of the model, and the production volumes of products are unknown variables of this model. Model parameters determine the appearance and value of the objective function, affect the optimal solution. A change in model parameters leads to a change in the optimal solution. But in the process of solving the problem of linear programming, the parameters of the model are assumed to be constant. In addition to finding the optimal solution to the problem, the linear programming method provides information about how the optimal solution changes with changes in the model parameters.

The basis of expressing economic problems in the form of a linear programming problem is to correctly choose the parameters of the problem and express the goal through them through a linear function, and to express the limits through linear inequalities and equalities.

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To solve the problem of linear programming, it is necessary to formalize the problem, that is, to create a mathematical model of the problem. Creating a mathematical model of the problem includes the following steps:

• identifying the existing problem;
• determining the goal;
• to determine the limitations of the issue;
• to determine the variables of the problem;
• determining the parameters of the problem;
• mathematical representation of the objective function and constraints using problem variables.

Problem setting. Below is a mathematical model for the practical problem of optimal planning of production and the solution of the resulting linear programming problem in a special program.

Production for "FAYZLI BAGHDAD" furniture store optimal planning problem

Production of two products in the workshop: a furniture and a shelf for products. 3.5 m to make one piece of furniture. standard DSP, 1 m. standard bottle and one day's work of one worker. 1 m DSP for one shelf , 2 m. bottle and a day's labor of one worker is spent.

The profit from the sale of one furniture is $200, and from the shelf - $100. The material and labor resources of the factory are limited, and the factory employs a total of 150 workers. The daily stock of DSP is 350 m, and the stock of glass is 240 m. How many sideboards and shelves should the factory produce in a day to maximize profit?

To create a mathematical model of the problem, we define the following:

Management problem	In the production process, using from factory raw materials
The purpose	The purpose maximizing the profit of the furniture factory **"FAYZLI BAGHDAD"** from the reserve.
Restrictions	Limitations on material and labor resources: the daily supply of DSP is 350 m, the stock of glass is 240 m, the factory employs a total of 150 workers.
Problem variables	The number of furniture and shelves that should be produced daily in the enterprise.

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Problem parameters	The amount of material and labor resources allocated for the production of one furniture and one shelf, and the amount of profit from the sale of one furniture and one shelf. The problem parameters are listed in Table 1.

RESOURCE	FURNITURE	SHELF	RESERVE SIZE
DSP	3,5 m.	1,0 m.	350 m.
BOTTLE	1,0 m.	2,0 m.	240 m.
WORKER	1 ta	1 ta	150 ta
PROFIT	200	100	

Table 1. Problem parameters

Mathematical model of the problem. Before writing the mathematical model of the problem, we introduce some. Let be the number of furniture and the of shelves produced daily by the factory. In this case, the daily total profit of the factory is the sum of the profits from both products. If the profit per furniture is $200, the profit from furniture x is $200 \cdot x\$$. Similarly, if the profit from one shelf is $100, then the profit from shelf y is $100 \cdot y\$$. The profit from the total product produced in one day is $F = 200x + 100y$ dollars.

Function of the purpose. The profit from the total product will be equal to $F = 200x + 100y$.

From the point of view of the objective function, it is known that the larger the quantities x and y of the products produced, the greater the profit F of the factory. But the number of furniture and shelves produced every day cannot be increased as desired, because the material and labor resources of the factory are limited.

Restrictions. In order to determine under what constraints we need to optimize the objective function, we define conditions for the number of workers and the amount of DSP in the factory stock.

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	3.5 m for one furniture 1 m for DSP and one shelf As the chip is spent, the total DSP consumption is $3,5x+y$ equal to the meter. The DSP stock of a factory of this size should not exceed 350 meters.	$3,5x+y\le 350$
	1.0 m per furniture 2.0 m for bottle and one shelf the total spending of bottle is $x+2y$ equal to a meter. The bottle stock of this factory must not more than 240 meters.	$x+2y\le 240$
	One worker can make one furniture or one shelf per day. So the total number of products $x+y$ cannot exceed the total number of workers.	$x+y\le 150$
	Finally, the number of furnitures and the number of shelves produced x and the number of shelves y cannot be minus.	$x\ge 0,\ y\ge 0$

So, the mathematical model of the problem is:

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$$\begin{cases} 3,5x + y \le 350 \\ x + 2y \le 240 \\ x + y \le 150 \end{cases}$$

$$x \ge 0,\ y \ge 0$$

$$F = 200x + 100y \to \max$$

Now let's consider how to determine the solution of this problem graphically. Solving a linear programming problem graphically consists of two steps:

- Identifying a set satisfying linear inequalities in the plane;
- Finding a point in the set that gives an extremal value to the objective function.

$$\begin{cases} 3,5x + y \le 350 & (a) \\ x + 2y \le 240 & (b) \\ x + y \le 150 & (c) \end{cases}$$

$$x \ge 0,\ y \ge 0$$

$$F = 200x + 100y \to \max$$

First, we find a set of feasible solutions. If we take into account the conditions of negativity, the set of admissible solutions to $x \ge 0,\ \ y \ge 0$ the Cartesian coordinate system, with inequalities, we are sure to produce space in the first quarter.

We replace the inequality $3,5x + y \le 350$ with an equality and draw the graph of the straight line $3,5x + y = 350$ Using the graph of a straight line, it is convenient to find the points where it intersects with the coordinate axes. $x = 0$ gives $y = 350$. $x = 100$ when $y = 0$.

Drawing a straight line through the points (0; 350) and (100; 0) in the Cartesian coordinate system, we create a graph of the function $3,5x + y = 350$. This divides the straight plane into two parts. Since the half-plane on the plane does not lie, we can take an arbitrary point on a straight line in the plane, for example, the point with coordinates (0; 0). We put these coordinates in the inequality: $3,5 \cdot 0 + 1 \cdot 0 \le 350$, which means $0 \le 350$. So, we choose the half-plane containing the coordinate origin from the 2 half-planes. Thus, the area satisfying the inequality () consists of the triangle OAB (Fig. 1(a)).

Inequality 2: We replace the inequality $x + 2y \le 240$ with an equality and draw the graph of the straight line $x + 2y = 240$. This straight line intersects the coordinate axes at the points (0; 120) and (240; 0). If we put the point (0; 0) in the inequality, we see that $0 \le 240$ satisfies the inequality. So, we choose the one that contains the coordinate origin from the half-planes. As a result, the area satisfying inequality (b) consists of triangle OCD (Fig. 1(b)).

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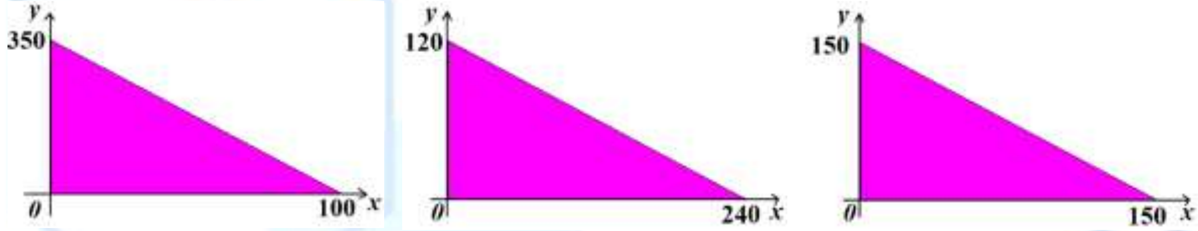
Inequality 3: We replace the inequality $x+y \leq 150$ with an equality and draw the graph of this straight line $x+y=150$. This straight line intersects the coordinate axes at the points (0; 150) and (150;0). If we put the point (0;0) in the inequality, we see that $0 \leq 150$ satisfies the inequality. Therefore, the area satisfying inequality (c) consists of the triangle OFG (Fig. 1(c)).



(a) (b) (c)

Figure 1. The process of determining the acceptable set

If we take the general part of all the above areas, we will have a set of permissible solutions consisting of a rectangle $OAED$ (Fig. 2).

Now we move on to find the point where the objective function $F = 200x + 100y$ reaches its maximum value in the set of feasible solutions. For this, we first construct the normal vector $\vec{n} = (200, 100)$. If we give an exact value to the objective function, that is, if we set c equal to an exact number, then the resulting straight line will be perpendicular to the vector $\vec{n}$. Since we are looking at the problem of maximization of a straight line, we move it parallel to itself in the direction of $\vec{n}$ vectors. The last point where our straight line leaves the desired area is point E, which gives the maximum value to the objective function.

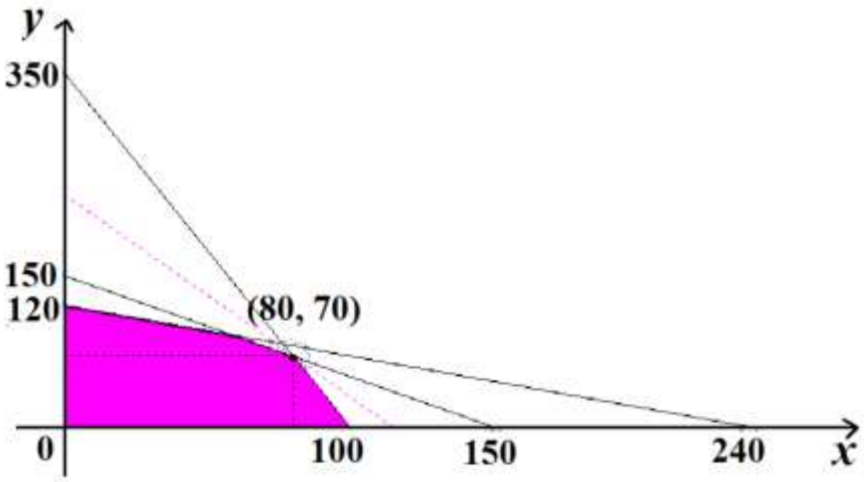


Figure 2. Finding a solution to the problem graphically

To find the coordinates of point E, we solve it by creating a system using the equations of straight lines AB and CD:

$$\begin{cases} 3{,}5x + y = 350 \\ x + y = 150 \end{cases} \Rightarrow (x;\ y) = (80;\ 70)$$

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So, solving the system of equations, we found the coordinates of the intersection point E of the straight lines AB and CD: $(x;\ y) = (80;\ 70)$.. The optimal value of the objective function is found as follows:

$$F_{\max} = F(x;\ y) = F(80;\ 70) = 200\cdot 80 + 100\cdot 70 = 23000$$

It can be seen that if the factory produces 80 furniture and 70 shelves in one day, the maximum profit will be $23,000.

Summary

In the process of finding the optimal solution of a linear programming problem, the following situations may occur: there are situations when the solution of the problem is unique or has infinitely many solutions.

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