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KARRALI TUGUNLAR BO'YICHA INTERPOLYATSIYALASH

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Kalit so'zlar: Lagranj interpolyatsion ko'pxadi, Teylor ko'pxadi, Ermit formulasi. Hisoblash usullari amaliyotda uchraydigan masalalarni taqribiy yechish bilan shug'ullanadi. Ma'lumki, tabiiy fanlar hamda texnika fanlarida uchraydigan ko'pgina masalalar chiziqsiz differensial tenglamalarga keltiriladi, ya'ni ularning analitik yechimini topish nihoyatda murakkab masala, shu sababli taqribiy yechish usullaridan foydalanish ko'proq samara beradi. Hozirgi zamon matematikasi boshqa tabiiy fanlar bilan birga yangi muammolarni hal qilmoqda. Masalan, mexanikada karrali integrallar yordamida og'irlik markazi, inersiya momentlari va boshqa kattaliklari hisoblash osonroq bo'ladi.

Ключевые слова: коэффициент интерполяции Лагранжа, разложение Тейлора, формула Эрмита.Методы расчета имеют дело с приближенным решением задач, встречающихся на практике. Известно, что многие задачи, встречающиеся в естественных и технических науках, сводятся к нелинейным дифференциальным уравнениям, то есть найти их аналитическое решение чрезвычайно сложно, поэтому использование приближенных методов решения более эффективно. Современная математика вместе с другими естественными науками решает новые задачи.

Key words: Lagrange interpolation coefficient, Taylor expansion, Hermite's formula.Calculation methods deal with the approximate solution of problems encountered in practice. It is known that many problems encountered in natural sciences and technical sciences are reduced to nonlinear differential equations, that is, finding their analytical solution is extremely difficult, so the use of approximate solution methods is more effective. Modern mathematics, together with other natural sciences, solves new problems. For example, in mechanics, it is easier to calculate the center of gravity, moments of inertia and other quantities using multiple integrals.

Bu maqolada interpolyasion masalaning Ermit tomonidan ko`rsatilgan quyidagi umumlashgan holini ko`rib chiqamiz. . Faraz qilaylik, [a,b] oraliqda interpolyasiyaning (m+ 1) ta har xil tugunlari berilgan bo`lsin. Shu oraliqda aniqlangan funksiyani olaylik va x = $x_i$ (i= $\overline{0,m}$) nuqtalarda f(x) ning hamda uning ketma-ket hosilalarining qiymatlari f $(x_i)$ , $f'$ $(x_i)$, . . . , $f^{(a_i-1)}$ $(x_i)$ (i= $\overline{0,m}$) berilgan bo`lsin. Bu erda a_0, a_1 . . .

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, a_m mos ravishdagi tugunlarning karra ko`rsatkichlari deyiladi, f(x) funksiya haqidagi barcha dastlabki ma'lumotlarning sonini n+1 orqali belgilaymiz: $a_0 + a_1 + \ldots.. + a_m$ =n+1. Endi darajasi (n+1) dan ortmaydigan

$$H_n^{(i)}(x_k) = f^{(i)}(x_k) \qquad (k = \overline{0,m}; i = \overline{0, a_k - 1}) \qquad (1)$$

shartlarni qanoatlantiruvchi

$$H_n(x) = a_0 x^n + a_1 x^{n-1} + \ldots. + a_n \quad (2)$$

ko`phadni quraylik. Bu shartlar $a_i (i = \overline{o,n})$ noma'lumlarni topish uchun ( n + 1) ta chiziqli tenglamalar sistemasini beradi. Bu istema yechimining mavjudligi va yagonaligini ko`rsatish uchun

$$H_n^{(i)}(x_k) = 0 \quad (k = \overline{0,m}; i = \overline{0, a_k - 1}) \qquad (3)$$

bir jinsli sistemaning faqat trival echimga ega ekanligini ko`rsatish kifoyadir. (3)sistema shuni ko`rsatadiki $x_0, x_1, \ldots., x_m$ tugunlar $H_n(x)$ ko`phad uchun mos ravishda $a_0, a_1, \ldots., a_m$ lardan kichik bo`lmagan tartibli ka $H_n(x)$ ko`phad ildizlarining karra ko`rsatkichlari yig`indisi $a_0 + a_1 + \ldots. + a_m = n+1$ ga teng yoki undan katta ildizlardir. Darajasi n dan ortmaydigan va ildizlar karra ko`rsatgichlari yig'indisi n dan kata bo`lgan $H_n(x)$ ko`phad faqat aynan nolga teng bo`lishi kerak. Bundan esa uning barcha $a_i$ koeffisientlarining nolga tengligi va bir jinsli sistemaning faqat trivial echimga egaligi kelib chiqadi. Shunday qilib, (3) dagi $f^{(i)}(x_k)$ qiymatlarning qanday bo`lishidan qat'i nazar, qo`yilgan masala yagona echimga ega. $H_n(x)$ ko`phadning (x_k) tugunlar va $f^{(i)}(x_k)$ qiymatlar orqali oshkor ko`rinishini determinantlar yordamida ifodalash mumkin. Lekin bunday ifodaning tuzilishi juda murakkabdir. Shuning uchun bu yerda ham Lagranj interpolyasion ko`phadini tuzgandek, boshqacha yo`l tutamiz. Buning uchun fundamental qo`phadlar deb ataluvchi n-darajali $Q_{ij} = (x) \quad (i = \overline{0,m};\ j = \overline{0, a_i - 1})$ ko`phadlarni, ya'ni

$$Q_{ij} = (x_k) = Q_{ij}' = (x_k) = \ldots = Q_{ij}^{(a_k - 1)}(x_k) = 0, k \neq i \quad (4)$$

$$Q_{ij} = (x_i) = Q_{ij}' = (x_i) = \ldots = Q_{ij}^{(j-1)}(x_i) = Q_{ij}^{(j+1)}(x_i) = Q_{ij}^{(a_j - 1)}(x_i) = 0$$
$$Q_{ij}^{(j)} = (x_j) = 1 \qquad (5)$$
$$(i = \overline{0,m};\ j = \overline{0, a_i - 1})$$

shartlarni qanoatlantiruvchi ko`phadlarni tuzamiz. U holda izlanayotgan ko`phadni quyidagicha yozish mumkin:

$$H_n(x) = \sum_{i=0}^{m} \sum_{j=0}^{a_{i-1}} f^{(j)}(x_i) Q_{ij} = (x) \qquad (6)$$

(4) tengliklardan ko`ramizki, $Q_{ij} = (x)$ $x_0, x_1, \ldots.., x_m$ nuqtalarda mos ravishda $a_0 + a_1 + \ldots. + a_m$ karrali nollarga ega bo`lib, (5) tengliklarga asosan x_i nuqtada u karrali nolga ega.

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Demak,

$$Q_{ij}(x)=(x-x_0)^{a_0}(x-x_1)^{a_1}...(x-x_{i-1})^{ai-1}(x-1)^j(x-x_{i+1})^{a_{i+1}}...(x-x_m)^{a_m}q_{ij}(x) \quad (7)$$

Buyerda $q_{ij}(x)$ $x=x_i$ nuqtada nolga aylanmaydigan

$$n-(a_0+...+a_{i-1}+j+a_{i+1}+...+a_m)=a_i-j-1$$

darajali ko`phaddir. Quyidagi belgilashni kiritaylik

$$\Omega(x)=(x-x_0)^{a_0}...(x-x_m)^{a_m} \quad (8)$$

U holda (7) — (8) dan ushbu

$$Q_{ij}(x)=\frac{\Omega(x)}{(x-x_i)^{a_{i-j}}}q_{ij}(x) \quad (9)$$

formulaga ega bo`lamiz. $q_{ij}(x)$ ni aniqlash uchun (5) shartlarga murojaat qilamiz. Bulardan $Q_{ij}(x)$ ning $x_i$ nuqta atrofidagi Teylor yoyilmasi quyidagi ko`rinishga ega ekanligi kelib chiqadi:

$$Q_{ij}(x)=\frac{1}{j!}(x-x_i)^j+(x-x_i)^{j+1}+...+a_{ij}^{(n-1)}(x-x_i)^n=$$
$$\frac{(x-x_i)^j}{j!}[1+b_{ij}^{(1)}(x-x_i)+...+b_{ij}^{(n-j)}(x-x_i)^{n-j}] \quad (10)$$

Bu va (9) dan $q_{ij}(x)$ ko`phad uchun quyidagi

$$q_{ij}(x)=\frac{(x-x_i)^{a_j}}{j!\Omega(x)}+C_{ij}(x-x_i)^{a_j-j}+... \quad (11)$$

ifodaga ega bo`lamiz.

Ushbu $\frac{(x-x_i)^{a_j}}{j!\Omega(x)}$, rasional funksiya x_i nuqta atrofida regulyar bo`lganligi uchun $x-x_i$ ning darajalari bo`yicha Teylor qatoriga yoyiladi. Ikkinchi tomondan $q_{ij}(x)$ darajasi a_i-j-1 ga teng bo`lgan ko`phad bo`lganligi uchun u $\frac{(x-x_i)^{a_j}}{j!\Omega(x)}$ qatoriga yoyilmasining darajasi $a_i-j-1$ larining yig`indisiga tengdir:

$$q_{ij}(x)=\frac{1}{j!}\sum_{k=0}^{a_i-j-1}\frac{1}{k}[\frac{(x-x_i)^{a_i}}{\Omega(x)}]_{x=x_i}^{(k)}(x-x_i)^k \quad (2.12)$$

Aksincha, $q_{ij}(x)$ ning shunday tanlanishi $Q_{ij}(x)$ uchun (10) yoyilmani va demak, (4) - (2.5) shartlarning bajarilishini ta’minlaydi. (12) ni (9) ga qo`yib,

$$Q_{ij}(x)=\frac{\Omega(x)}{j!(x-x_i)^{a_j-j}}\sum_{k=0}^{a_i-j-1}\frac{1}{k}[\frac{(x-x_i)^{a_i}}{\Omega(x)}]_{x=x_i}^{(k)}(x-x_i)^k$$

ni hosil qilamiz va nihoyat, (6) dan

$$H_h(x)=\sum_{i=0}^{m}\sum_{j=0}^{a_i-1}\frac{1}{j!}\sum_{k=0}^{a_i-j-1}\frac{1}{k!}f^{(i)}(x_i)[\frac{(x-x_i)^{a_i}}{\Omega(x)}]_{x=x_i}^{(k)}\frac{\Omega(x)}{(x-x_i)^{a_i=j-1}} \quad (13)$$

Ermit formulasini hosil qilamiz. Bu formulaning xususiy holi sifatida Lagranj interpolyasion ko`phadi hamda ko`phad uchun Teylor formulasini chiqarish mumkin.

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Hozir Ermit formulasining boshqa bir xususiy holiki ko`rib chiqaylik. Barcha $a_i$ lar 2 ga teng bo`lsin, ya'ni shunday n-darajali ko`phadni topish kerakki, u

$$\begin{cases} H_{n(x_i)} = f(x_i) \\ H_{n(x_i)} = f'(x_i) \end{cases} \quad (i=0,1,\dots m)$$

shartlarni qanoatlantirsin. Bu shartlarning geometrik ma'nosi quyidagidan iborat: interpolyasion egri chiziq berilgan $y = f(x)$ egri chiziq bilan interpolyasiya tugunlarida umumiy urinmalarga ega. U holda (13) quyidagi ko'rinishda bo'ladi:

$$H_h(x) = \sum_{i=0}^{m}\{f(x_i)[\frac{(x-x_i)^2}{\Omega(x)}]_{x=x_i}\frac{\Omega(x)}{(x-x_i)^2} + f(x_i)[\frac{(x-x_i)^2}{\Omega(x)}]'_{x=x_i}\frac{\Omega(x)}{(x-x_i)^2} + f'(x_i)[\frac{(x-x_i)^2}{\Omega(x)}]_{x=x_i}\frac{\Omega(x)}{(x-x_i)^2}\} \quad (14)$$

Odatdagidek

$$\omega_{m+1}(x) = (x-x_0)(x-x_1)\dots(x-x_m)$$

belgilash kiritsak, u holda

$$\Omega'(x) = \omega^2_{m+1}(x), \frac{(x-x_i)^2}{\Omega(x)} = [\frac{(x-x_i)}{\omega_{m+1}(x)}]^2$$

ga ega bo`lamiz.

Oxirgi limitni topish uchun Lopital qoidasini qo`lladik. Bularni hisobga olganda (14) formula quyidagi ko`rinishga ega bo`ladi:

$$H_h(x) = \sum_{i=0}^{m}\frac{\omega^2_{m+1}(x)}{\omega^2_{m+1}(x_i)(x-x_i)^2}[f(x_i)(1-\frac{\omega''_{m+1}(x)}{\omega''_{m+1}(x)}(x-x_i)) + f(x_i)(x-x_i)] \quad (15)$$

M i s o l. Quyidagi shartlarni qanoatlantiradigan beshinchi darajali $H_5(x)$ ko`phad topilsin:

$$H_5(-1) = -1, H_5(0) = 1, H_5(1) = 1$$
$$H'_5(-1) = 0, H'_5(0) = 1, H'_5(1) = 0$$

Bu erda

$$\omega_3 = (x+1)x(x-1), \omega'_3 = 3x^2 - 1, \omega''_3 = 6x.$$

Endi (15) formuladan

$$H_5(x) = \frac{x^2(x-1)^2}{4}[-1(1-\frac{6(-1)}{2}(x-1)] + \frac{(x-1)^2(x+1)^2}{1}x + \frac{x^2(x+1)^2}{4}[-(1+3(x-1)] = \frac{1}{2}(-x^5 + x^3 + 2x)$$

natija kelib chiqadi.

Endi Ermit formulasining qoldiq hadini tekshiramiz.

Teorema. Agar $f(x)$ funksiya [a,b]oraliqda (n+1)-tartibli uzluksiz hosilaga ega bo`lsa, u holda Ermit interpolyasion formulasining qoldiq hadini

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$$R_n(x) = f(x) - H_n(x) = f^{n+1}(\xi)\frac{\Omega(x)}{(n+1)!} \quad (16)$$

ko`rinishda ifodalash mumkin. Bu erda $\xi$ [a,b] oraliqqa tegishli nuqta bo`lib, umuman x ning funksiyasidir.

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Isbot. x ning interpolyasiya tugunlaridan farqli biror qiymatini olib, $K=\frac{R(x)}{\Omega(x)}$ deb belgilaylik. U holda ushbu

$\phi(z)R_n(z)-K\Omega(z)$ (17)

funksiya x_0 nuqtada a_0 karrali nolga, x_1 nuqtada a_1 karrali va h. k. x_m nuqtada a_m karrali nolga ega. Bundan tashqari y x nuqtada ham nolga aylanadi. Demak, funksiya $\phi(z)$ [a,b] oralig`ining m+2 ta $x_0,x_1,...x_m$ nuqtalarida nolga aylanib, bu nollar karraliklarining yig`indisi ga $a_0+a_1...+a_m+1=n+2$ teng. Demak,[a,b] oraliqda kamida shunday ξ bitta nuqta topiladiki,

$f^{n+1}(\xi)=0$ (18)

bo`ladi. Lekin

$f^{n+1}(z)=f^{n+1}(z)-K(n+1)!$ (19),

chunki $H_n(z)$ darajasi n dan ortmaydigan ko`phad, demak, $H_n^{(n+1)}(z)=0$ va $\Omega(x)$ bosh hadi 1 ga teng bo`lgan (n + 1)-darajali ko`phad, uning (n + 1) -tartibli hosilasi

(n + 1)! ga teng.(18) -(19) dan

$$K=\frac{f^{n+1}(\xi)}{(n+1)!}.$$

Demak,

$$R_n(x)=\frac{f^{n+1}(\xi)}{(n+1)!}\Omega(x).$$

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