

## PROBLEMS AND SOLUTIONS IN WORKING PROBABILITY AND COMBINATORICS PROBLEMS

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*Annotatsiya.* An issue that appears difficult in one discipline may become simple to tackle when seen from a different perspective. In this note, a combinatorial problem is presented as a probability problem, making it easier to solve.

*Keywords:* Combinatorics, probability, history of combinatorics, mathematic, pedagogical community.

Combinatorics is one of the traditional divisions of discrete mathematics, dealing with the questions about how many different combinations associated with certain conditions can be made of given objects.

The history of combinatorics development testifies to its ever-increasing scientific and practical importance in the life of society. and practical importance in the life of society with the tasks that later received the name combinatorial, people were familiar with several millennia ago. In ancient China, people were fascinated

with magic squares, in ancient Greece. They counted the number of different combinations of long and short syllables in poetic and so on. The term combinatorics was introduced into mathematics by the German scientist G. Leibniz. In 1666.<sup>1</sup>

The scientist published a work called "Discourse on the Art of Combinatorics". He understood combinatorics very broadly, as a component of any research, any creative act. Research, any creative act, which implies first analysis (dissection of the whole into parts), and then synthesis (combining the parts into a whole).

Working on probability and combinatorics problems can be challenging, but understanding common problems and how to approach them can make these tasks more manageable. Here are some typical problems you might encounter when working on such problems, along with solutions or strategies to overcome them:

### **Problems:**

**Misunderstanding the Question:** Probability and combinatorics problems can be intricate, and misunderstanding the question is common.

**Combining Events Incorrectly:** Confusion between independent, mutually exclusive, and non-exclusive events can lead to incorrect calculations.

**Factorial Complexity:** Large factorials in combinatorics problems can be difficult to compute and may lead to errors.

**Permutations vs. Combinations Confusion:** Not understanding when to use permutations (order matters) versus combinations (order does not matter).

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<sup>1</sup> Беляева И.С. Комбинаторный подход и его применение в преподавании математики в восьмилетней школе: автореф. дис. ... канд. пед. наук. – Ярославль, 1971. – 19 с.

Overlooking Cases: In combinatorics, it's easy to overlook possible cases or double-count scenarios.

Conditional Probability Missteps: Misinterpreting or incorrectly applying the concept of conditional probability.

Probability Distributions Confusion: Difficulty in understanding and applying different probability distributions (e.g., binomial, normal).

Iterative or Recursive Relationships: Struggling with problems that require understanding iterative or recursive relationships.

### **Solutions:**

Careful Reading: Take the time to read the question multiple times. Break it down into parts and try to rephrase it in your own words.

Event Classification: Clearly define whether events are independent, mutually exclusive, or non-exclusive before performing any calculations.

Use of Technology: Utilize calculators or software for handling large factorial computations to reduce manual errors.

Understand Definitions: Review and understand the definitions and formulas for permutations and combinations.

Systematic Listing: Make a systematic list of all possible cases to ensure none are overlooked. Use tree diagrams if helpful.

Learn Conditional Probability: Study the formula and typical problems to get a firm grasp on conditional probability.

Study Distributions: Spend time learning the properties and applications of different probability distributions.

Practice Recursive Problems: Work on simpler recursive problems and build up to more complex ones to understand how to deal with them.

### **General Tips:**

Practice regularly to become familiar with various types of problems. Write down all known information and what you're trying to find. Use diagrams or tables to organize information and visualize the problem. Check your work by verifying that your answers make sense in the context of the problem. Learn from mistakes by reviewing solutions and understanding where you went wrong. Finally, don't hesitate to seek help from textbooks, online resources, or educators if you're consistently encountering difficulties. Practice and persistence are key in mastering probability and combinatorics.

Re-evaluation of combinatorics. Let us note one common harmful tendency:

Many math teachers tend to overestimate the role of combinatorics in teaching probability theory. Often the teacher formally states combinatorial facts and formulas,

and then offers problems with the word "probability" as an example of application of probability theory. "probability" as an example of how to apply?

Example: Pinocchio has 4 silver and 2 gold coins in his right pocket. Pinocchio moves three coins to his left pocket without looking. What is the probability that both gold coins are in the same pocket? in the same pocket?

This task, appearing in different formulations in different collections and on exams, caused a lot of discussion on examinations, caused a lot of discussions. As it turned out, most experts see it as a combinatorial problem with the following solution.

The total number of combinations of 6 coins of 3 is equal to  $C_6^3$ . Choose two gold coins out of two and one silver coin out of four and put them in the left pocket can be

$$C_2^2 \cdot C_4^1$$

in a number of ways. There are as many ways to put these selected coins in the right pocket. Gain:

$$p = \frac{2C_2^2 C_4^1}{C_6^3} = \frac{2 \cdot 1 \cdot 4}{20} = 0,4.$$

Here is a typical manifestation of the problem: probability in universities is often taught as an application of combinatorics, and this approach is projected onto schools. We are not against combinatorics. But it's more important to teach probability thinking. Let's teach it first of all. Thinking briefly can dramatically simplify the multitude of possible outcomes. We don't care where of the first gold coin. Let's look at the possible placements of the second.

Solution. Mentally assign numbers 1 and 2 to the gold coins. The first gold coin will be in some pocket. In this pocket, besides it, two more coins out of the five remaining coins will fall into this pocket. So, the probability that the second gold coin will happen to be in the same pocket is equal to

$$\frac{2}{5} = 0,4.$$

Importantly, the absence of combinatorics does not narrow the class of possible problems. In the study and teaching of probability theory, combinatorics should play a supporting role, and it is needed where the probability spaces are vast and cannot be done without it. It is necessary to use good and important problems with simple probability sets, and not to fill the pupil's head with combinatorial relations, passing them off as the essence of science.

As we can see, the problem of including elements combinatorics in the school program is not new, and therefore requires constant and close attention both on the part of the pedagogical community and on the part of researchers of pedagogical problems. Familiarization with combinatorics should be subordinated not only to the desire to professional training, but also to educate students to a new view of the phenomena of the surrounding reality, to educate them to see the application of mathematical apparatus and new ideas, which are considered, to solve a variety of problems.

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