METHOD OF HAUSDORF ON MEASURING FRACTAL DIMENSIONS

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Fractal geometry provides a powerful framework for quantifying the irregular and self-similar structures prevalent in natural and artificial phenomena. One of the fundamental concepts in fractal analysis is the measurement of fractal dimensions, which captures the complexity and self-replicating patterns exhibited by fractal objects. Among the various methods for calculating fractal dimensions, the Hausdorff method stands out as a widely used and versatile approach due to its simplicity and applicability across diverse domains.

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B. Mendelbrot made the following point: "one of the main characteristics of a fractal is its measure”. The difference between fractal measure and other measures is that the measure obtained must be smaller than the total measure of fractal, that is $l_{\min} < l < l_{\max}$, if the inequality is satisfied, the length obtained must be greater than the smallest of the pieces and less than the total length[3].

Is a topological measure on space that, D_T is said to be the number of linearly independent coordinates. A topological measure is an integer measure, and a topological measure of a circle is $D_T = 1$, measure of circle which has area $D_T = 2$, topological measure of a sphere or cube $D_T = 3$, these measures are considered whole measures, and Fractal measure is fractional. Hausdorf-Bezikovich measure. To induce this measure, the length of a given fractal is scaled to, which is equal to:

$$l = \frac{L}{N}, \tag{1.1}$$

There N number of iterations l is the number of slices of a straight line N the number can be obtained optionally, but l cannot be changed, c because it consists of the initial length. Accordingly,

$$L(l) = l * N, \tag{1.2}$$

then the area of the field is

$$S = L(l) * l, \tag{1.3}$$

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because (1.1) from

$$S = l^2 * N = \left(\frac{L(l)}{N}\right)^2 * N = \frac{L(l)}{N} * \frac{L(l)}{N} * N = L(l) * l. \qquad (1.4)$$

For volume

$$V = l^3 * N = L(l) * l^2. \qquad (1.5)$$

Thus, this rule reduces the given measure to k times. So, $l = \frac{L(l)}{k}$, (1.2) by substituting the formula for:

$$S = L(l)^2 * \frac{N}{k^2}, \qquad (1.6)$$

$$V = L(l)^3 * \frac{N}{k^3}, \qquad (1.7)$$

hence, the similarity coefficient is $\chi = \frac{1}{k}$, $\chi = \frac{1}{k^2}$, $\chi = \frac{1}{k^3}$,...... $\chi = \frac{1}{k^n}$

then l if length $l = 1$

$$l = k^d, \qquad (1.8)$$

(1.4) the formula arises then if both sides are expressed in terms of natural logarithm it follows:

$$d = \lim_{k \to \infty} \frac{\ln(N)}{\ln\left(\frac{1}{k}\right)}. \qquad (1.9)$$

This measure was determined differently from each other by Hausdorf-Bezikovich, on the basis of which the Fractal measure of geometric objects is determined. The above measure differs from other measures in its fractional nature.

Minkowski-Buligan measure:

$$d_M = \lim_{r \to \infty} \frac{\ln(S(r))}{\ln\left(\frac{1}{r}\right)} + 2. \qquad (1.10)$$

For all solid-like fractals d_M Minkowski measure d_H It turned out that the calculation error from the hausdrof-Bezikovich measure is large. Because it does not take into account some small structures of the object.

It is possible to develop algorithms and block schemes for determining fractal measurements of fractal geometric objects. Also, in determining fractal measurements of geometric shapes, the Hausdorf-Bezikovich and Minkovsky-Buligan measurements were used. It was proposed that a numerical algorithm should be used to determine the Fractal measure of fractal structured representations, and measurements of geometrical objects in nature were determined based on fractal measures[9].

Summary

In summary, the Hausdorff method represents a cornerstone in fractal geometry, providing a robust framework for quantifying the fractal dimensions of complex structures across disciplines. Its versatility, computational efficiency, and mathematical rigor make it indispensable for analyzing and interpreting the fractal nature of natural and artificial systems, offering profound insights into their underlying organization, dynamics, and emergent properties.

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