BIR O'ZGARUVCHI FUNKSIYANING DIFFERENSIAL HISOBI

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Annotatsiya. Ushbu maqolada matematika fanning asosiy bo'limlaridan biri funksiya hosilasi haqida ma'lumot berilgan. Asosan maqolada, hosila tushunchasi, hosilaga oid misollar, funksiya hosilasi, hosilaning geometrik va mehanik ma'nolari, ikkinchi tartibli hosila, n-tartibli hosilalari keltirilgan.

Kalit so'zlar. Hosila, fuksiya orttirmasi, argument orttirmasi, limit, funksiyaning geometrik ma'nosi, urinma, burchak koeffitsiyenti, tezlik, yo'l, vaqt, ikkinchi tartibli hosila, n-tartibli hosila.

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Абстрактный. В этой статье представлена информация о производной функции, одном из основных разделов математики. В основном в статье представлены понятие производной, примеры производных, производная функции, геометрический и механический смысл производной, производная второго порядка, производная n-го порядка.

Ключевые слова. Производная, произведение функции, произведение аргумента, предел, геометрический смысл функции, испытание, угловой коэффициент, скорость, путь, время, производная второго порядка, производная n-го порядка.

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THE DIFFERENTIAL OF A FUNCTION OF ONE VARIABLE ACCOUNT

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Annotation. This article provides information about the derivative of a function, one of the main branches of mathematics. Basically, the article presents the concept of derivative, derivative examples, function derivative, geometric and mechanical meanings of derivative, second-order derivative, n-order derivative.

Keywords. Derivative, product of a function, product of an argument, limit, geometric meaning of a function, trial, angle coefficient, speed, path, time, derivative of the second order, derivative of the nth order.

Kirish. Hosila tushunchasi bir-biriga o'zaro bog'liq bo'lmagan ikki masala tufayli vujudga kelgan. Bu masalaning birinchisi harakatlanayotgan jismning tezligini aniqlash bo'lsa, ikkinchi masala biror chiziqqa o'tkazilgan urinmani topishdan iborat. Aslida bu ikki masala o'zaro uzviy bog'liqdir, chunki nuqtaning tezligi bu nuqta harakati trayektoriyasiga urinma bo'lgan vektordir. Hosila funksiyalarning turli xossalarini o'rganishda keng qo'llaniladi. Hosila yordamida turli matematik-fizik masalalar, mehanikada harakat, kuch va ishga oid masalalar yechiladi. Hozirgi kunda barcha sohadagi fan-texnika yutuqlarini hosilasiz umuman tasavvur etish qiyin.

Asosiy qism. $y=f(x)$ funksiyaning x_o nuqtadagi orttirmasi Δy ning argument ortirmasi Δx nolga intilgandagi limiti

$$f'(x_o)=\lim_{\Delta x\to 0}\left(\frac{\Delta y}{\Delta x}\right)=\lim_{\Delta x\to 0}\left(\frac{f(x_o+\Delta x)-f(x_o)}{\Delta x}\right) \qquad (1)$$

mavjud bo'lsa, bu limit $y=f(x)$ funksiyaning x_o nuqtadagi *hosilasi* deyiladi.

Agar $y=f(x)$ funksiyaning x_o nuqtadagi hosilasi mavjud bo'lsa, bu funksiyani x_o nuqtada *differensiallanuvchi* deb ataladi. Funksiya hosilasini hisoblash amalini esa *differensiallash amali* deyiladi.

[1] Hosilani geometrik ma'nosi $y=f(x)$ funksiya $[a, b]$ oraliqda aniqlangan bo'lib, x_o , $x_o+\Delta x\epsilon[a, b]$ bo'lsa, ushbu

$$\frac{f(x_o+\Delta x)-f(x_o)}{\Delta x}$$

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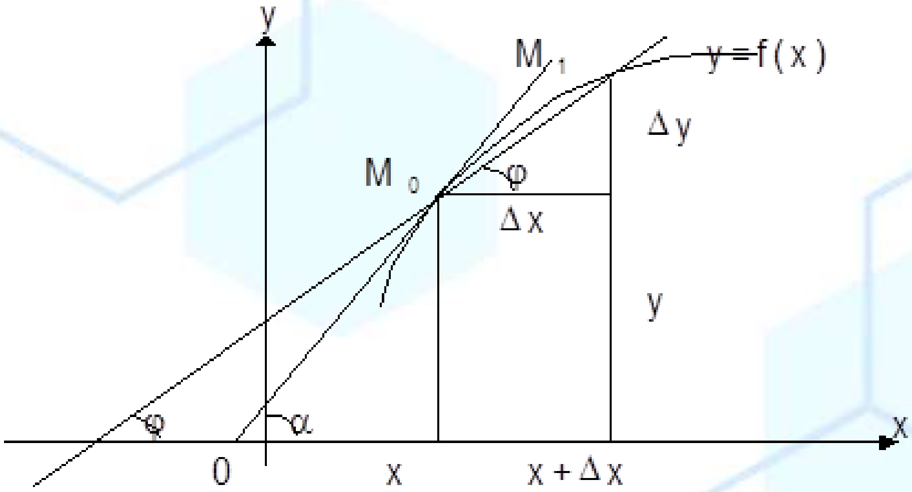
nisbat $(x_o, f(x_o))$ va $(x_o+\Delta x, f(x_o+\Delta x))$ nuqtalardan o'tadigan kesuvchining ($\Delta x \neq 0$ bo'lganda) burchak koeffitsiyentidan iboratdir (1-rasmga qarang). Shuning uchun, agar $y=f(x)$ funksiya x_o nuqtada differensiallanuvchi bo'lsa, $\Delta x \to 0$ da kesuvchi, $(x_o, f(x_o))$ nuqtadan o'tgan urinmaga yaqinlashib boradi. Demak, $f'(x_o)$ hosila bu urinmaning burchak koeffitsiyentidan iborat:

$$tg\beta=\frac{\Delta y}{\Delta x};\ \ y' = \lim_{\Delta x\to 0}\left(\frac{\Delta y}{\Delta x}\right) = \lim_{\Delta x\to 0} tg\beta = tg\alpha.$$



1-rasm

Biror funksiyaning hosilasini, bevosita hosilaning ta'rifi yordamida hisoblash uchun quyidagi amallarni bajarish kerak:

I. x_o nuqtadaga ixtiyoriy Δx orttirma berib, $y=f(x)$ funksiyaning $x_o+\Delta x$ nuqtadagi qiymatini hisoblaymiz:

$$y_o+\Delta y=f(x_o+\Delta x)$$

II. $y=f(x)$ funksiyaning $x_o+\Delta x$ nuqtadagi qiymatidan x_o nuqtadagi qiymatini ayirib, funksiya orttirmasini topamiz:

$$\Delta y=f(x_o+\Delta x)-y_o= f(x_o+\Delta x)-f(x_o)$$

III. Funksiya orttirmasining argument orttirmasiga nisbatini topamiz:

$$\frac{\Delta y}{\Delta x}=\frac{f(x_o+\Delta x)-f(x_o)}{\Delta x}$$

IV. Δx nolga intilganda yuqoridagi nisbatning limitini hisoblaymiz. Bu son, $y=f(x)$ funksiyaning x_o nuqtadagi hosilasi bo'ladi.

$$f'(x_o)=\lim_{\Delta x\to 0}\left(\frac{\Delta y}{\Delta x}\right) = \lim_{\Delta x\to 0}\left(\frac{f(x_o+\Delta x)-f(x_o)}{\Delta x}\right)$$

Quyidagilarni isbotlang:

1. $y'=\cos x$ $\quad$ $y=\sin x$
2. $y'=e^x$ $\quad$ $y=e^x$
3. $(u\cdot v)'=u'v+uv'$
4. $(\arcsin x)'=\frac{1}{\sqrt{1-x^2}}$

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1. y=sinx $\qquad f'(x_o)=\lim\limits_{\Delta x\to 0}\left(\frac{f(x_o+\Delta x)-f(x_o)}{\Delta x}\right)$

y+Δy=sin(x+Δx)

Δy=sin(x+Δx)-y

Δy=sin(x+Δx)-sinx

$$f'(x_o)=\lim_{\Delta x\to 0}\left(\frac{f(x_o+\Delta x)-f(x_o)}{\Delta x}\right)=\lim_{\Delta x\to 0}\left(\frac{\sin(x+\Delta x)-\sin(x)}{\Delta x}\right)=$$

$$\lim_{\Delta x\to 0}\left(\frac{2\sin\frac{1}{2}(\Delta x)\cos\frac{1}{2}(2x+\Delta x)}{\Delta x}\right)=\left|\lim_{\Delta x\to 0}\left(\frac{\sin(½\Delta x)}{½\Delta x}\right)=1\right|=$$

$$\lim_{\Delta x\to 0}\left(cos\,\frac{(2x+\Delta x)}{2}\right)=\cos\frac{2x}{2}=\cos x \quad ▲$$

2. y= e^x $\qquad f'(x_o)=\lim\limits_{\Delta x\to 0}\left(\frac{f(x_o+\Delta x)-f(x_o)}{\Delta x}\right)$

y+Δy=$e^{x+\Delta x}$

Δy=$e^{x+\Delta x}$-y

Δy=$e^{x+\Delta x}$- e^x=$e^x(e^{\Delta x}-1)$

$$y'=\lim_{\Delta x\to 0}\left(\frac{e^x(e^{\Delta x}-1)}{\Delta x}\right)=\left|\lim_{\Delta x\to 0}\left(\frac{(a^{\Delta x}-1)}{\Delta x}\right)=\ln a\right|=\lim_{\Delta x\to 0}(e^x lne)==e^x lne=e^x \quad ▲$$

3. (u·υ)'=u'υ +uυ ' $\qquad f'(x_o)=\lim\limits_{\Delta x\to 0}\left(\frac{f(x_o+\Delta x)-f(x_o)}{\Delta x}\right)$

y+Δy=(u+Δu) (υ +Δυ)

Δy=(u+Δu) (υ+Δυ)-y

Δy=(u+Δu) (υ +Δ υ)-(u· υ)

$$y'=\lim_{\Delta x\to 0}\left(\frac{(u+\Delta u)\,(\upsilon+\Delta\upsilon)-(u\cdot\upsilon)}{\Delta x}\right)=\lim_{\Delta x\to 0}\left(\frac{u\cdot\upsilon+u\cdot\Delta\upsilon+\upsilon\cdot\Delta u+\Delta u\cdot\Delta\upsilon-u\cdot\upsilon}{\Delta x}\right)=$$

$$\lim_{\Delta x\to 0}\left(\frac{u\cdot\Delta\upsilon+\upsilon\cdot\Delta u+\Delta u\cdot\Delta\upsilon}{\Delta x}\right)=\lim_{\Delta x\to 0}\left(u\frac{\Delta v}{\Delta x}+v\frac{\Delta u}{\Delta x}+\frac{\Delta u\cdot\Delta v}{\Delta x}\right)=u\lim_{\Delta x\to 0}\frac{\Delta v}{\Delta x}+v\lim_{\Delta x\to 0}\frac{\Delta u}{\Delta x}+$$

$$\lim_{\Delta x\to 0}\frac{\Delta u\cdot\Delta v}{\Delta x}=u\frac{dv}{dx}+v\frac{du}{dx}+0=u\cdot\upsilon'+\upsilon\cdot u' \quad ▲$$

4.y=arcsinx funksiyaning teskari funksiyasi x=siny funksiyadir.

$$y'(x)=(\arcsin x)'_\chi=\frac{1}{(siny)'_Y}=\frac{1}{cosy}=\frac{1}{\sqrt{1-sin^2y}}=\frac{1}{\sqrt{1-x^2}}.$$

[2] y=f(x) funksiya (a,b) oraliqning barcha nuqtalarida differensiallanuvchi bo’lsa, y'=f'(x) (a, b) oraliqda aniqlangan bo’lsin. Bu funksiyaning x_o nuqtadagi hosilasi, agar u mavjud bo’lsa, y=f(x) funksiyaning x_o nuqtadagi *ikkinchi tartibli hosilasi* deyiladi va quyidagicha belgilanadi:

$$y''(x_o)\ ,\ f''(x_o)\ \text{yoki}\ \frac{d^2y}{dx^2}\ .$$

Qisqacha qilib aytganda, y=f(x) funksiyaning *ikkinchi tartibli* yoki *ikkinchi hosilasi* deb, uning birinchi tartibli hosilasidan olingan hosilaga, ya’ni $(y'(x_o))'$ ga aytiladi.

y=f(x) funksiyaning *n-tartibli* yoki n-*hosilasi* deb, uning (n-1)-tartibli hosilasidan olingan hosilaga, ya’ni $y^{(n)}=\left(y^{(n-1)}\right)'$ ga aytiladi.

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Faraz qilaylik, f(x) va g(x) funksiyalar (a, b) da berilgan bo'lib, ixtiyoriy x∈(a, b) da $f^{(n)}(x)$ va $g^{(n)}(x)$ hosilalarga ega bo'lsin. U holda:

I. $(c \cdot f(x))^{(n)} = c \cdot f^{(n)}(x)$, c=const;

II. $\left(f(x) \pm g(x)\right)^{(n)} = f^{(n)}(x) \pm g^{(n)}(x)$;

III. $\left(f(x) \cdot g(x)\right)^{(n)} = \sum_{k=0}^{n} C_n^k f^{(k)}(x) \cdot g^{(k-n)}(x)$,

($C_n^k = \frac{n(n-1)\ldots(n-k+1)}{k!}$) , $f^{(0)}(x)$ = f(x) bo'ladi.

Quyidagi murakkab funksiyalarni hosilasini toping:

1. $y = x\ln^2(x + \sqrt{1+x^2}) - 2\sqrt{1+x^2}\ln(x + \sqrt{1+x^2}) + 2x$
2. $y = (\arccos x)^2[\ln^2(\arccos x) - \ln(\arccos x) + \frac{1}{2}]$

1.$y = x\ln^2(x + \sqrt{1+x^2}) - 2\sqrt{1+x^2})\ln(x + \sqrt{1+x^2}) + 2x$

$f(x) = x\ln^2(x + \sqrt{1+x^2})$

$f'(x) = \ln^2(x + \sqrt{1+x^2}) + x \cdot 2\ln(x + \sqrt{1+x^2}) \cdot \frac{1}{(x+\sqrt{1+x^2})} \cdot (1 + \frac{1}{2} \cdot 2x \cdot (\sqrt{1+x^2})^{-1}) = \ln^2(x + \sqrt{1+x^2}) + \frac{2x\ln(x+\sqrt{1+x^2})}{(\sqrt{1+x^2})}$;

$g(x) = \sqrt{1+x^2}\ln(x + \sqrt{1+x^2})$

$g'(x) = \frac{1}{2} \cdot 2x \cdot (\sqrt{1+x^2})^{-1}) \cdot \ln(x + \sqrt{1+x^2}) + (\sqrt{1+x^2}) \cdot \frac{1}{(x+\sqrt{1+x^2})} \cdot (1 + \frac{1}{2} \cdot 2x \cdot (\sqrt{1+x^2})^{-1}) = \frac{x\ln(x+\sqrt{1+x^2})}{(\sqrt{1+x^2})} + 1$;

$\varphi(x) = 2x$

$\varphi`(x) = 2$

$y`(x) = f`(x) + g`(x) + \varphi`(x) = \ln^2(x + \sqrt{1+x^2}) + \frac{2x\ln(x+\sqrt{1+x^2})}{(\sqrt{1+x^2})} - 2 \cdot (\frac{x\ln(x+\sqrt{1+x^2})}{(\sqrt{1+x^2})} + 1) + 2 = \ln^2(x + \sqrt{1+x^2}) + 2\frac{x\ln(x+\sqrt{1+x^2})}{(\sqrt{1+x^2})} - 2 \cdot \frac{x\ln(x+\sqrt{1+x^2})}{(\sqrt{1+x^2})} - 2 + 2 = \ln^2(x + \sqrt{1+x^2})$;

2.$y = (\arccos x)^2[\ln^2(\arccos x) - \ln(\arccos x) + \frac{1}{2}]$

u=arccosx

$y = u^2[\ln^2(u) - \ln(u) + \frac{1}{2}]$

$y` = 2u \cdot u`[\ln^2(u) - \ln(u) + \frac{1}{2}] + u^2[2 \cdot \ln(u) \cdot \frac{1}{u} \cdot u` - \frac{1}{u} \cdot u`] = 2u \cdot u` \cdot \ln^2(u) - 2u \cdot u` \cdot \ln(u) + u \cdot u` + u^2 \cdot 2 \cdot \ln(u) \cdot \frac{1}{u} \cdot u` - u^2 \cdot \frac{1}{u} \cdot u` = 2u \cdot u` \cdot \ln^2(u) - 2u \cdot u` \cdot \ln(u) + u \cdot u` + 2u \cdot u` \cdot \ln(u) - u \cdot u` = 2u \cdot u` \cdot \ln^2(u) = 2(\arccos x)[\ln^2(\arccos x)] \cdot (-\frac{1}{(\sqrt{1-x^2})})$;

Quyidagi berilgan murakkab funksiyani ikkinchi tartibli hosilasini toping:

$y = \frac{\ln 3 \cdot \sin x + \cos x}{3^x}$

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$$y` = \frac{(ln3 \cdot cosx - sinx) \cdot 3^x - (sinx \cdot ln3 + cosx) \cdot 3^x \cdot ln3}{3^{2x}}$$

$$= \frac{3^x \cdot ln3 \cdot cosx - 3^x \cdot sinx - 3^x ln^2 3 \cdot sinx - 3^x \cdot ln3 \cdot cosx}{3^{2x}} = \frac{-3^x \cdot sinx - 3^x ln^2 3 \cdot sinx}{3^{2x}} = \frac{-3^x \cdot sinx \cdot (1 + ln^2 3)}{3^{2x}} = \frac{-sinx \cdot (1 + ln^2 3)}{3^x};$$

$$y`` = \left(\frac{-sinx \cdot (1 + ln^2 3)}{3^x}\right)` = \frac{-cosx \cdot (1 + ln^2 3) \cdot 3^x + sinx \cdot (1 + ln^2 3) \cdot 3^x \cdot ln3}{3^{2x}} = \frac{3^x \cdot (1 + ln^2 3) \cdot (sinx \cdot ln3 - cosx)}{3^{2x}} = \frac{(1 + ln^2 3)(sinx \cdot ln3 - cosx)}{3^x};$$

Xulosa: Demak, biz fizikada hosiladan ko'p holatlardan foydalanamiz. Ulardan biz ko'p foydalanganimiz, bosib o'tilgan yo'ldan vaqt bo'yicha olingan birinchi tartibli hosila tezlikni, bosib o'tilgan yo'ldan vaqt bo'yicha olingan ikkinchi tartibli hosila tezlanishni yoki tezlikdan vaqt bo'yicha olingan birinchi tartibli hosila tezlanishni beradi.

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