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NAV`E – STOKS TENGAMASIGA QO`YILGAN CHEGARAVIY MASALA

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$\Omega - R^n$ dagi chegaralangan $\partial\Omega$ chegaraga ega bo`lgan soha bo`lsin. $u = (u_1, u_2, ..., u_n)$ orqali $Q = \Omega \times (0,T)$ da aniqlangan u vektorni (vektor tezlikni) belgilaymiz.

$$\frac{\partial u}{\partial t} = u' = \{u_1', u_2', ..., u_n'\} = \left\{\frac{\partial u_1}{\partial t}, ..., \frac{\partial u_n}{\partial t}\right\},$$

$$D_i u = \{D_i u_1, ..., D_i u_n\} = \left\{\frac{\partial u_1}{\partial x_1}, ..., \frac{\partial u_n}{\partial x_n}\right\}, \quad \text{deb olamiz.}$$

$$\Delta u = \{\Delta u_1, ..., \Delta u_n\}.$$

Nav`e – Stoks tenglamasi evolyusion holda quyidagi ko`rinishga ega: $\frac{\partial u}{\partial t} - \nu\Delta u + \sum_{i=1}^{n} u_i D_i u = f - grad\, p \quad (\nu > 0),$ div$u = 0$ (ya`ni $\sum_{i=1}^{n} D_i u_i = 0$), chegaraviy va boshlang`ich shartlar quyidagicha: (ya`ni $\Sigma$ da $u_i = 0,\ i = 1,2,...,n$), $u = 0 \quad \Sigma = \partial\Omega \times (0,T)$ $u(x,0) = u_0(x)$ $\Omega$ da (ya`ni $x \in \Omega$ da $u_i(x,0) = u_{0i}(x),\ i = 1,2,...,n$) Ushbu masala shundan iboratki, yuqoridagi shartlarni qanoatlantiruvchi u va p funksiyalar topilsin(p bu yerda o`zgarmas yig`indi aniqligida topiladi).

Yuqoridagi tenglama Koshi – Kovalevskaya tipidagi tenglama emas, chunki tenglamada $\frac{\partial p}{\partial t}$ hosila qanashmayapti.Endi biz masalaning kuchsiz yechimi yoki turbulent yechimi tushunchasini aniqlaymiz. Buning uchun bizga quyidagi belgilashlar kerak bo`ladi:

$u = 0 \quad \Sigma = \partial\Omega \times (0,T)$ da (ya`ni $\Sigma$ da $u_i = 0,\ i = 1,2,...,n$), $u(x,0) = u_0(x)$ $\Omega$ da (ya`ni $x \in \Omega$ da $u_i(x,0) = u_{0i}(x),\ i = 1,2,...,n$)

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$W_s = V$ ni $(H^s(\Omega))^n$ dagi yopilmasi, $\|u\|_s = ((u,u))_s^{1/2}$. Xususan, $W_1 = W$, $\|u\|_1 = \|u\|$ deb olamiz. U holda agarda $s>1$ bo`lsa, $W_s \subset W \subset H$ . Bu fazolarning har biri keyingisiga zich. $H$ fazoni uning qo`shmasi bilan tenglashtirib olamiz: $H' = H$. Biz H ustida W', W_s' larni ham tenglashtirib olishimiz mumkin edi. Demak, $W_s \subset W \subset H \subset W' \subset W_s'$. W' fazo (W_s' ham) $D'(\Omega)^n$ fazoning faktor fazosi bo`ladi.

Agar $v\in W_s$ bo`lsa, u holda $v_i \in H_0^s(\Omega)$ va demak, $s>\frac{1}{2}$ da $\partial\Omega$ da $v_i = 0$ bo`ladi.Shunday qilib, yuqoridagi shartlarni deyarli barcha t larda W da yotish sharti deb tushunish mumkin. Endi $a(u,v) = \sum_{i,j=1}^{n}\int_\Omega \frac{\partial u_j}{\partial x_i}\frac{\partial v_j}{\partial x_i}dx,\ u,v\in W,$

$b(u,v,w) = \sum_{i,j=1}^{n}\int_\Omega u_k(D_k v_i)w_i dx$ deb olamiz. Bu u,v,w vektorlar uchun mos integrallar yaqinlashadi; bu sababli quyidagi lemmani keltiramiz.

Lemma. Uch chiziqli forma $u,v,w \to b(u,v,w)$ $W\times W\times(W\cap(L_n(\Omega))^n)$ da uzluksiz.

Isboti. Haqiqatan ham, agar $u\in W$ bo`lsa, u holda $u_i \in H_0^1(\Omega)$ va demak, Cobolev teoremasiga ko`ra $u_i \in L_q(\Omega),\ \frac{1}{q} = \frac{1}{2} - \frac{1}{n}$ ($n=2$ da q chekli va ixtiyoriy) . Oson ko`rish mumkinki, $\left|\int_\Omega u_k(D_k v_i)w_i dx\right| \leq \|u_k\|_{L_q(\Omega)}\|D_k v_i\|_{L_2(\Omega)}\|w_i\|_{L_n(\Omega)}$,

Shunday qilib, $((L_q(\Omega))^n \times W \times (L_n(\Omega))^n$ da uzluksiz ekanligi isbotlandi. MasaLan , $n=2$ da quyidagi baholashni olamiz

$$\left|\int_\Omega u_k(D_k v_i)w_i dx\right| \leq \|u_k\|_{L_4(\Omega)}\|D_k v_i\|_{L_2(\Omega)}\|w_i\|_{L_4(\Omega)}$$

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1-masala. Ushbu $f\in L_2(0,T;(H^{-1}(\Omega))^n;\ u_0\in H$ funksiyalar berilgan bo`lsin. $\frac{\partial u}{\partial t}-\nu\Delta u+\sum_{i=1}^{n}u_iD_iu=f-grad\,p\ \ (\nu>0),$ tenglamani $u(0)=u_0$ boshlang`ich shartni qanoatlantiruvchi hamda $p\in D'(Q),\ u\in L_2(0,T;W)\cap L_\infty(0,T;H)$ bo`lgan u va p funksiyalar topilsin.

Eslatma. yuqoridagi shartdagi u ni $L_\infty(0,T;H)$ yotishi suniy ko`rinishi mumkin, haqiqatan ham u masalani qo`yilishiga muhim rol o`ynamaydi; ammo biz yechimni mavjudligini ushbu sinfda isbotlaymiz. Shunday qilib, eslatmada aytilgan ediki $u\in L_2(0,T;W)$ shart qaysidir ma`noda yuqoridagi shartlarni o`z ichiga oladi. $W$ fazoga ikki xil ta`rif berish mumkin:

birinchi ta`r: $W=V$ ni $(\mathrm{H}^1(\Omega))^n$ da yopilmasi ;

ikkinchi ta`rif: $W=\{v:v\in(H_0^1(\Omega))^n,\ \mathrm{div}\ v=0\}$.

Bu ta`riflar ekvivalent. Haqiqatan ham, ikkinchi ta`rifdagi fazoni $\tilde W$ orqali belgilaymiz. Tushunarliki, $W\subset\tilde W;$. $\tilde W\subset W$ ham ekanligini ko`rsatamiz. $L-\tilde W$ dagi uzluksiz chiziqli forma hamda $W$ da nolga teng bo`lsin; Xan-Banax teoremasiga ko`ra $L$ ni (yagona ko`rinishda emas) quyidagi ko`rinishda ifodalash mumkin: $L(v)=\sum_{i=1}^{n}(L_i,v_i),\ \ L_i\in H^{-1}(\Omega)$. Shunday qilib $L$ $W$ da nolga teng, u holda albatta $V$ da ham, u holda de Ram ning qo`shmalik haqidagi teoremasiga ko`ra : $L_i=\frac{\partial S}{\partial x_i},\ \ S=D'(\Omega)$. Ammo ko`rsatish mumkinki, u holda $S\in L_2(\Omega)$ va bu shartlarda $L(v)=\sum_{i=1}^{n}\left(\frac{\partial S}{\partial x_i},v_i\right)=-(S,\mathrm{div}\,v)=0\ \ \forall v\in\tilde W$. Bundan esa $\tilde W\subset W$ kelib chiqadi. Eslatma. 1-masala aniq ta`riflanishiga qaramasdan bitta ikki ma`noli fikr bor: bizda u' va p ga nisbatan hech qanday ma`lumot mavjud emas, faqat $Q$ da $u'+grad\ p=\nu\Delta u+\sum_{i=1}^{n}u_iD_iu+f\ \ (\nu>0)$ munosabat mavjud, bundan shartni ma`nosi yo`q. Agar biz $\varphi\in V$ deb olsak, u holda $(grad\ p,\varphi)=0$ $(D'(0,T)$ da)m va quyidagi tenglikka keladi $(u',\varphi)=-\nu a(u,\varphi)-b(u,u,\varphi)+(f,\varphi)$ . Qiyinchiliksiz ko`rish mumkinki, $b(u,u,\varphi)=-b(u,\varphi,u),$ shunday qilib, ushbu munosabatga ekvivalent $(u',\varphi)=-\nu a(u,\varphi)+b(u,\varphi,u)+(f,\varphi)$

$X-V$ ni $(W^{1,n/2}(\Omega))^n$ dagi yopilmasi bo`lsin, u holda

$$|b(u,\varphi,u)|\le c_1\|u\|_{(L_q(\Omega))^n}\sum_{i,j=1}^{n}\|D_i\varphi_j\|_{L_{n/2}(\Omega)}\le c_2\|u\|^q\|\varphi\|_X$$

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bo`ladi va demak, $b(u,\varphi,u)=(g,\varphi)$, $\|g\|\le c\|u\|^q$ ,bu yerdan, $g\in L_1(0,T;X')$. $u'\in L_2(0,T;V')+L_1(0,T;X')$ kelib chiqadi. Shunday qilib,quyida ma`noga ega (masalan, X' da) Quyida biz bu barchasini aniqlashtiramiz, ko`ramizki, $n=2$ da yuqorida ko`rsatilgan interpretatsiya (talqin) juda soddalashadi.

Eslatma 1-masalani tabiiy ravishda boshqacha ta`riflashga imkon beradi.

2-masala. $f\in L_2(0,T;W')$, $u_0\in H$ ifoda bilan ustma-ust tushadi). Shunday u funksiya qidiriladiki, $u\in L_2(0,T;W)\cap L_\infty(0,T;H)$ ifoda bilan ustma-ust tushadi. $(u',v)+\nu a(u,v)+b(u,u,v)=(f,v)\quad \forall v\in W\cap(L_n(\Omega))^n$, $u(0)=u_0$. 2.3.-eslatma. (2.1.31) shart 2.2.-esatmadagi kabi talqin qilinadi; 1-lemma $b(u,u,v)$ formaga ma`no berish uchun foydalaniladi.

Eslatma. Yuqorida keltirilgan ikkita ta`rifni (formulirovkani) ekvivalent ekanligini isbotlaymiz. Agar $u-$ 1-masalaning yechimi bo`lsa, u holda barcha $\varphi\in V$ lar uchun bajariladi. Bu yerdan $W\cap L_n(\Omega))^n$ da limitga o`tib quyidagini hosil qilamiz; shunday qilib, u 2-masalaning yechimi ekan.

Aksincha, $u-$ 2-masalaning ya`ni tenglamaning yechimi bo`lsin, u holda agar biz $u'-\nu\Delta u+\sum_{i=1}^{n}u_iD_iu-f=S$, deb olsak, u holda S $(D'(\Omega))^n$ da yotadi va $(D'(0,T))^n$ da $\forall\varphi\in V$ uchun $(S,\varphi)=0$ Bu yerdan, $S=-grad\,p$, $p\in D'(Q)$ kelib chiqadi.

Quyida ushbu natijalar isbotlanadi. 2.1-teorema. 2-masalaning u yechimi mavjud (chekli ixtiyoriy $T>0$ da). Bu masalaning yechimini yagonaligi esa ochiq qoladi; shu sababli quyidagi teorema isbotlanadi.2.2-teorema. Agar o`lchov $n=2$ bo`lsa, u holda 2-masala yagona yechimga ega.Bulardan tashqari quyidagi teorema ham isbotlanadi.2.3-teorema.Agar o`chov $n=2$ bo`lsa, u holda 2-masala yechimini nol o`lchovli to`plamda to`g`rilash orqali $[0,T]\to H$ funksiya kabi uzluksiz bo`ladi; bunda $H$ da $t\to 0$ da $u(t)\to u_0$ bo`ladi.

Fazoning o`lchovi $n=2$ bo`lsin. Qo`yilgan masala yechimini yagonaligini isbotlaylik.

Bizga bir nechta lemmalar kerak bo`ladi.

Lemma Agar $n=2$bo`lsa, u holda shunday $c(\Omega)$ o`zgarmas mavjudki, $\forall v\in H_0^1(\Omega)$ uchun $\|v\|_{L_4(\Omega)}\le c(\Omega)\|v\|_{H_0^1(\Omega)}^{1/2}\|v\|_{L_2(\Omega)}^{1/2}$ bo`ladi. Isboti. munosabatni $v\in D(\Omega)$ uchun isbotlash etarli; v ni Ω dan tashqarida nol qilib davom ettiramiz va tengsizlik quyidagi tengsizlikdan kelib chiqadi: $\|v\|_{L_4(R^2)}\le 2^{1/2}\|v\|_{L_2(R^2)}^{1/2}\left(\sum_{i=1}^{2}\|D_iv\|_{L_2(R^2)}^2\right)^{1/2}$. Biz ushbu $v^2(x)=2\int_{-\infty}^{x_i}v(D_iv)dx_i$ tenglikdan quyidagi munosabatlarni hosil qilamiz: Bu yerda $v_1(x_2)=\int_{-\infty}^{\infty}|v||D_1v|dx_1$ va $v_2(x_1)=\int_{-\infty}^{\infty}|v||D_2v|dx_2$. Demak,

$$\int_{R^2} v^4(x)dx \le 4\int_R v_1(x_2)dx_2\int_R v_2(x_1)dx_1 \le 4\|v\|_{L_2(R^2)}\|D_1v\|_{L_2(R^2)}\|v\|_{L_2(R^2)}\|D_2v\|_{L_2(R^2)}.$$

Bu yerdan xususan, yuqoridagi kelib chiqadi.

Lemma. Agar $n=2$ va $u\in L_2(0,T;W)\cap L_\infty(0,T;H)$ bo`lsa, u holda $\sum_{i=1}^{2}u_iD_iu\in L_2(0,T;W')$.

Isboti. Agar $\varphi\in W$ ($\|\varphi\|-W$ dagi norma) bo`lsa, u holda $\left(\sum_{i=1}^{2}u_iD_iu,\varphi\right)=-\left(\sum_{i=1}^{2}u_iD_i\varphi,u\right)$. Bu yerdan $\left|\left(\sum_{i=1}^{2}u_iD_iu,\varphi\right)\right|\le c_1\|u\|^2_{(L_4(\Omega))^2}\|\varphi\|\le c_2\|u\||u|\|\varphi\|$ va, $\left\|\sum_{i=1}^{2}u_i(t)D_iu(t)\right\|_{W'}\le c_2\|u(t)\||u(t)|$,

bundan kelib chiqadi, chunki $t\to\|u(t)\|$ funksiya $L_2(0,T)$ da yotadi, $t\to|u(t)|$ funksiya esa $L_\infty(0,T)$ da yotadi. $n=2$ da $u'\in L_2(0,T;W')$ kelib chiqadi; shunday qilib, quyidagi teorema isbotlandi.

2.4-teorema.2-masalaning ixtiyoriy yechimi uchun $n=2$ da quyidagi munosabat o`rinli: $u'\in L_2(0,T;W')$. 2.3-teorema $u\in L_2(0,T;W)$, va izlar haqidagi teoremalardan (Lions – Madjenes [1], 1-bob) kelib chiqadi. 2.2-teoremaning isboti. $u$ va $u^*$ ikkita yechim va $w=u-u^*$ bo`lsin. U holda $\forall v\in W$ (shunday qilib $W\subset(L_2(\Omega))^2!$) uchun $(w',v)+\nu a(w,v)+b(w,u,v)+b(u,w,v)-b(w,w,v)=0$). bizga ma`lumki, $w'\in L_2(0,T;W')$ , u holda $v=w(t)$ deb olishimiz mumkin va $t$ bo`yicha integrallab, quyidagi tenglikni hosil qilamiz:

$$\frac{1}{2}|w(t)|^2+\nu\int_0^t a(w,w)d\sigma+\int_0^t[b(w,u,w)+b(u,w,w)-b(w,w,w)]d\sigma=0.$$ Ammo $b(u,w,w)=0,\ b(w,w,w)=0$. U holda quyidagi ko`rinishda keladi:

$$\frac{1}{2}|w(t)|^2+\nu\int_0^T\|w(\sigma)\|^2d\sigma=-\int_0^t b(w,u,w)d\sigma.$$ Bu yerdan

$$\left|\int_0^t b(w,u,w)d\sigma\right|\le c_3\int_0^t\|w(\sigma)\|^2_{(L_4(\Omega))^2}\|u(\sigma)\|d\sigma\le c_4\int_0^t|w(\sigma)|\|w(\sigma)\|\|u(\sigma)\|d\sigma\le$$

$$\le\nu\int_0^t\|w(\sigma)\|^2d\sigma+c_5\int_0^t\|w(\sigma)\|^2\|u(\sigma)\|^2d\sigma.$$

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$|w(t)|^2 \leq 2c_5 \int_0^t \|u(\sigma)\|^2 |w(\sigma)|^2 \, d\sigma$, bu yerdan $w = 0$ kelib chiqadi.

FOYDALANILGAN ADABIYOTLAR:

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