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AYIRMALI TENGLAMA HAQIDA QISQACHA TUSHUNCHALAR

Ediyev Sherdor - *Shahrisabz davlat pedagogika instituti talabasi*
Ahmatov Rustam- *Shahrisabz davlat pedagogika instituti talabasi*
Shahnoza Umarova Xolmurod qizi – *Shahrisabz davlat pedagogika instituti, matematika o'qitish metodikasi kafedrasi o'qituvchisi*
Mirzayeva Shahlo Abdurahmonovna – *Shahrisabz davlat pedagogika instituti, matematika o'qitish metodikasi kafedrasi o'qituvchisi*
E-mail: shaxnozau22@gmail.com
ORCID raqami:0009-0002-8686-088X

Ushbu $x_j(n)=f_j\left(n,x_1(n),x_2(n),...,x_k(n)\right),\ j=\overline{1,k}$, (1.1)

ko'rinishdagi sistema **ayirmali tenglamalarning normal sistemasi** deb ataladi, bu yerda $n\in\mathbb{Z}_+$; $f_j\left(n,x_1(n),x_2(n),...,x_k(n)\right),\ f_j:\mathbb{Z}_+\times D\to\mathbb{R},\ j=\overline{1,k}$, ,lar esa berilgan funksiyalar, D - $\mathbb{R}^k$ dagi soha; $x_i(n), i=\overline{1,k}$ - diskret o'zgaruvchi n ning noma'lum funksiyalari.

Ushbu $k-$tartibli $x(n+k)=f\left(n,x(n),x(n+1),...,x(n+k-1)\right)$ bitta tenglamada $x(n)=x_1(n),\ x(n+1)=x_2(n),\ \ldots,\ x(n+k-1)=x_k(n)$ belgilashlarni kiritsak,u

$$\begin{cases} x_1(n+1)=x_2(n) \\ x_2(n+1)=x_3(n) \\ \ldots \\ x_{k-1}(n+1)=x_k(n) \\ x_k(n+1)=f\left(n,x_1(n),x_2(n),\ldots,x_k(n)\right) \end{cases}$$

sistemaga o'tadi. k - tartibli tenglama va hosil bo'lgan sistema yechish nuqtai nazaridan ekvivalentdir.

Demak, k - tartibli chiziqli tenglamani yechishni normal sistemaga va aksincha, k ta birinchi tartibli tenglamadan iborat normal sistemani yechishni bitta k - tartibli tenglamani yechishga keltirish mumkin ekan.

Yuqoridagi (2.1) sistemani vektor ko'rinishida

$$x(n+1)=f\left(n,x(n)\right) \qquad (1.2)$$

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kabi yozish mumkin, bu yerda $f:\mathbb{Z}_+\times D\to D,\ D-\mathbb{R}^k$ dagi soha. (1.2) ko'rinishidagi tenglama **ayirmali tenglama** (yoki normal ko'rinishdagi ayirmali tenglama) deb ataladi. Agar f funksiyada n oshkor holda qatnashmasa, ya'ni tenglama

$$x(n+1)=f(x(n)) \tag{1.2}$$

ko'rinishda bo'lsa, uni **avtonom ayirmali tenglama** deb ataladi.

$$x(n_0)=x^0,\ n_0\in\mathbb{Z}_+,\ x^0\in D \tag{1.3}$$

shartni qanoatlantiruvchi yechimi

$$x(n;n_0,x^0)\quad \left(x(n_0;n_0,x^0)=x^0\right) \tag{1.4}$$

ko'rinishda yoziladi. Qisqalik uchun $x(n;n_0,x^0)=x(n)$ yozuvdan ham foydalanamiz.

1.4 yechimni rekurrent usulda quyidagicha topiladi:

$$x(n_0+1,n_0,x^0)=f(n_0,x(n_0))=f(n_0,x^0)$$

$$x(n_0+2,n_0,x^0)=f(n_0+1,x(n_0+1))=f(n_0+1,f(n_0,x^0))$$

$$x(n_0+3,n_0,x^0)=f(n_0+2,x(n_0+2))=f(n_0+2,f(n_0+1,f(n_0,x^0)))$$

$$\dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots$$

$$x(n,n_0,x^0)=f(n-1,x(n-1))=\dots$$

Bu yerda shuni ta'kidlab o'tamizki, $x(n,n_0,x^0)$ yechim barcha $n\ge n_0$ larda aniqlangan, chunki $f(n,x(n)):Z_+\times D\to D$.

Misol. $x(n+1)=2x(n)+n-3$ tenglamani yechaylik.

Rekurrent ravishda quyidaglarni topamiz:

$$x(n+1)=2\underline{x(n)}+n-3=2\left(\underline{2\underline{\underline{x(n-1)}}+(n-1)-3}\right)+n-3=$$

$$=2\left(2\left(\underline{\underline{2x(n-2)+(n-2)-3}}\right)+(n-1)-3\right)+(n)-3=$$

$$=\dots=$$

$$=2^{n+1}x(0)+\sum_{i=0}^{n}2^i(n-i)-3\left(2^{n+1}-1\right).$$

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Agar boshlang'ich $x(0)$ qiymat berilgan bo'lsa, $x(n)$ yechim bir qiymatli topiladi.

Ta'rif. Ushbu

$$x(n+k)=f(n,x(n),x(n+1),...,x(n+k-1)) \tag{2.5}$$

tenglama k - tartibli ayirmali tenglama deb ataladi; bunda o'ng tomonda $x(n+k-1)$ qatnashgan bo'lishi shart.

Masalan, $x(n+6)=7nx(n)\cdot x(n+5)+n^2$ tenglama 6-tartibli, $x(n+5)=x(n+4)$ tenglama esa 5-tartibli emas, u 1-tartibli tenglamaga keltiriladi: $x(n+4)=y(n) \Rightarrow y(n+1)=y(n)$.

Ushbu

$$x(n+1)=A(n)x(n),\ n\geq 0,$$

bu yerda

$$x(n)=\begin{pmatrix} x_1(n) \\ x_2(n) \\ \cdots \\ x_k(n) \end{pmatrix},\quad A(n)=\begin{pmatrix} a_{11}(n) & a_{12}(n) & \ldots & a_{1k}(n) \\ a_{21}(n) & a_{22}(n) & \ldots & a_{2k}(n) \\ \ldots & \ldots & \ldots & \ldots \\ a_{k1}(n) & a_{k2}(n) & \ldots & a_{kk}(n) \end{pmatrix},$$

ko'rinishdagi sistemani **bir jinsli ayirmali tenglamalar sistemasi** deb ataladi. Uning yechimini rekurrent usulda topiladi:

$$x(n+1)=A(n)x(n)=A(n)[A(n-1)x(n-1)]=A(n)A(n-1)[A(n-2)x(n-2)]=...=A(n)A(n-1)...A(0)x(0)=$$
$$=\prod_{i=0}^{n}A(n-i)x(0).$$

Bir jinsli sistemaning k dona $x^1(n)$, $x^2(n)$, ..., $x^k(n)$ yechimlarini qaraylik, bu yerda k - sistemadagi noma'lum skalyar funksiyalar soni. Quyidagi

$$x^1(n)=\left(x_1^1(n),x_2^1(n),\ldots,x_k^1(n)\right)^*\in F^k$$
$$\ldots\ \ldots\ \ldots\ \ldots\ \ldots$$
$$x^k(n)=\left(x_1^k(n),x_2^k(n),\ldots,x_k^k(n)\right)^*\in F^k$$

vektorlarning **Kazorati matritsasi** deb, $k\times k$ tartibli

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$$K(n)=K\left(x^1(n),x^2(n),...,x^k(n)\right)=\begin{pmatrix} x_1^1(n) & x_1^2(n) & ... & x_1^k(n) \\ x_2^1(n) & x_2^2(n) & ... & x_2^k(n) \\ ... & ... & ... & ... \\ x_k^1(n) & x_k^2(n) & ... & x_k^k(n) \end{pmatrix}$$

matritsaga aytiladi. Uning determinanti $x^1(n), x^2(n),..., x^k(n)$ vektor funksiyalarning **Kazoratiani** deb ataladi va

$$K\left[x^1(n),x^2(n),...,x^k(n)\right]$$

bilan belgilanadi:

$$K\left[x^1(n),x^2(n),...,x^k(n)\right]=\begin{vmatrix} x_1^1(n) & x_1^2(n) & ... & x_1^k(n) \\ x_2^1(n) & x_2^2(n) & ... & x_2^k(n) \\ ... & ... & ... & ... \\ x_k^1(n) & x_k^2(n) & ... & x_k^k(n) \end{vmatrix}.$$

Teorema. $x(n+1)=A(n)x(n),\ n\in Z_+$ sistemaning yagona $x(n,n_0,x^0)$ yechimi

$$x(n,n_0,x^0)=N(n,n_0)x^0$$

formula bilan aniqlanadi. Agar sistemaning koeffitsiyentlari o'zgarmas, ya'ni

$$A(n)=A$$

bo'lsa, normal fundamental matritsa

$$N(n,n_0)=A^{n-n_0},$$

umumiy yechimi esa

$$x(n)=A^n c$$

ko'rinishda bo'ladi. $x(n+1)=A(n)x(n)+b(n),\ n\ge 0$ bu yerda $A(n)$ - $k\times k$, $b(n)$ - $k\times 1$ tartibli matritsalar, ko'rinishdagi sistema **chiziqli ayirmali tenglamalar sistemasi** deb ataladi. Ya'ni,

$$x(n)=\begin{pmatrix} x_1(n) \\ x_2(n) \\ ... \\ x_k(n) \end{pmatrix},\ A(n)=\begin{pmatrix} a_{11}(n) & a_{12}(n) & ... & a_{1k}(n) \\ a_{21}(n) & a_{22}(n) & ... & a_{2k}(n) \\ ... & ... & ... & ... \\ a_{k1}(n) & a_{k2}(n) & ... & a_{kk}(n) \end{pmatrix},\ b(n)=\begin{pmatrix} b_1(n) \\ b_2(n) \\ ... \\ b_n(n) \end{pmatrix}$$

bo'lsa, yuqoridagi sistema

$$\begin{cases} x_1(n+1) = a_{11}x_1(n) + a_{12}x_2(n) + \ldots + a_{1k}x_k(n) + b_1(n) \\ x_2(n+1) = a_{21}x_2(n) + a_{22}x_2(n) + \ldots + a_{2k}x_k(n) + b_2(n) \\ \ldots \\ x_k(n+1) = a_{k1}x_k(n) + a_{k2}x_2(n) + \ldots + a_{kk}x_k(n) + b_k(n) \end{cases}$$

ko'rinishni oladi. Bunda $\det A(n) \neq 0$ deb hisoblanadi. $x(n+1) = A(n)x(n)$ sistema unga mos bir jinsli sistemadir. Berilgan sistemaning yechimini $x(n) = \tilde{x}(n) + \Phi(n)c$ ko'rinishda qidiramiz. Bir jinsli bo'lmagan sistemaning xususiy yechimini $\tilde{x}(n) = N(n, n_0)c(n)$ko'rinishda Lagranj usuli bilan izlash mumkin, bu yerda $c(n)$ - noma'lum vektor funksiya. $N(n+1, n_0) = A(n)N(n, n_0)$ekanligini hisobga olib, noma'lum $c(n)$ vektor funksiya uchun quyidagilarni hosil qilishimiz mumkin:

$$N(n+1, n_0)c(n+1) = A(n)N(n, n_0)c(n) + b(n)$$

$$N(n+1, n_0)c(n+1) = N(n+1, n_0)c(n) + b(n)$$

$$c(n+1) - c(n) = N^{-1}(n+1, n_0)b(n).$$

$N^{-1}(n, m) = N(m, n)$ formulaga ko'ra, $c(n_0) = 0$ deb, $c(n) = \sum_{j=n_0}^{n-1} N(n_0, j+1)b(j)$ni aniqlaymiz.

Endi esa, $N(n, m) = N(n, s)N(s, m)$ formuladan foydalanib, $\tilde{x}(n)$ xususiy yechimni aniqlaymiz: $\tilde{x}(n) = \sum_{j=n_0}^{n-1} N(n, j+1)b(j)$,bu yerda $\sum_{j=n_0}^{n_0-1} N(n, j+1)b(j) = 0$ deb olinadi. Teorema. $x(n+1) = A(n)x(n) + b(n)$ sistemaning $n = n_0$ da x^0 qiymatni oluvchi yagona $x(n, n_0, x^0)$ yechimi

$$x(n, n_0, x^0) = \sum_{j=n_0}^{n-1} \left(\prod_{s=j+1}^{n-1} A(s) \right) b(j) + \prod_{j=n_0}^{n-1} A(j) x^0$$

formula bilan topiladi.

Agar $x(n+1) = A(n)x(n) + b(n)$ sistemada $A(n) = A$ bo'lsa, hosil bo'lgan $x(n+1) = Ax(n) + b(n)$ sistema **o'zgarmas koeffitsiyentli chiziqli ayirmali tenglamalar sistemasi** deb ataladi. Bu holda mos bir jinsli tenglama $x(n+1) = Ax(n)$.Shu tenglamani yechish bilan shug'ullanamiz, bir jinsli bol'magan holda Lagranj usulini qo`llash mumkin.

Bu tenglamaning umumiy yechimi:

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$$x(n+1)=Ax(n)=A(Ax(n-1))=A^2(Ax(n-2))=...=A^{n-1}x(1)=A^n x(0)$$

bo'ladi. A^n ning qiymatini esa matritsaviy funksiyaning spektral yoyilmasidan foydalanib hisoblash mumkin.

Spektri kompleks tekislikdagi $|z|<R$ doirada joylashgan $A-(k\times k)$ matritsa va shu doirada analitik $f(z)=a_0+a_1z+a_2z^2+...+a_nz^n+...$funksiya berilgan bo'lsin. Bu qatorning koeffitsiyentlari $a_n=\frac{f^{(n)}(0)}{n!}=\frac{1}{2\pi i}\int_K\frac{f(z)}{z^{n+1}}dz,\ n\in Z_+$ Koshi formulasi yordamida aniqlanadi. Bu yerda K – $|z|<R$ doirada yotuvchi va A matritsaning spektrini o'z ichiga oluvchi yopiq kontur.Matritsaning spektral radiusi sifatida barcha $\lambda_i,\ i=\overline{1,k}$ xos sonlar orqali tuzilgan $\max\limits_{1\le i\le k}|\lambda_i|$ sonni olamiz va $\rho(A)$ orqali belgilaymiz.

$\|A\|$ norma uchun $\|A^n\|\le\|A\|^n$ va $\rho(A)\le\|A\|<R$ bo'lganligi sababli Ushbu $f(A)=a_0E+a_1A+a_2A^2+...+a_nA^n+...=\sum\limits_{i=0}^{\infty}a_iA^i$ qiymat aniqlangan. formulada $a_i,i\in Z_+$ koeffitsiyentlarni (2) formuladagi $a_i,i\in Z_+$ ning mos qiymatlari bilan almashtirsak, $f(A)$ qiymat

$$f(A)=\sum_{n=0}^{\infty}a_nA^n=\sum_{n=0}^{\infty}\left(A^n\cdot\frac{1}{2\pi i}\int_K\frac{f(z)}{z^{n+1}}dz\right)=\frac{1}{2\pi i}\sum_{n=0}^{\infty}\left(A^n\cdot\int_K\frac{f(z)}{z^{n+1}}dz\right)=\frac{1}{2\pi i}\int_K A^nz^{-n-1}f(z)dz$$

ko'rinishni oladi; K kontur $|z|<R$ doirada joylashganligi uchun, qatorni hadma – had integralladik.

Ushbu $(E-B)^{-1}=E+B+B^2+...+B^n+...$, $\|B\|<1$, yoyilmadan foydalanib, ushbu $f(A)=\frac{1}{2\pi i}\int_K A^nz^{-n-1}f(z)dz=\frac{1}{2\pi i}\int_K(Ez-A)^{-1}f(z)dz$ Riss formulasini hosil qilamiz. A matritsaning xarakteristik ko'phadini $\chi(z)=\det(Ez-A)=(z-\lambda_1)^{k_1}(z-\lambda_2)^{k_2}...(z-\lambda_s)^{k_s}$ ko'rinishda yozaylik, λ_j – xarakteristik ko'phadning k_j karrali ildizi va $k_1+k_2+...+k_s=k$. Ravshanki, $(Ez-B)^{-1}$ matritsani $\frac{1}{\chi(z)}B(z)$ ko'rinishda yozish mumkin, bu yerda $B(z)$ – $(Ez-A)$ matritsa elementlarining algebraik to'ldiruvchilaridan tuzilgan matritsa. Ular z ga nisbatan $k-1$ dan yuqori bo'lmagan darajali ko'phadlardir. Shuning uchun $(Ez-A)^{-1}$ matritsa elementlari maxrajlari $(z-\lambda_k)^{k_j}$ ko'rinishidagi kasrlardir: $(Ez-A)^{-1}=\sum\limits_{j=1}^{s}\left(\frac{A_{j1}}{z-\lambda_j}+\frac{A_{j2}}{(z-\lambda_j)^2}+...+\frac{A_{jk}}{(z-\lambda_j)^k}\right)$

$$f^{(m)}(\lambda_j)=\frac{m!}{2\pi i}\int_K\frac{f(z)}{(z-\lambda_j)^{m+1}}dz,\quad j=\overline{0,k_j-1}$$

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Koshi formulasidan, A matritsaning $f(A)$ funksiyasi uchun $f(A)=\sum_{j=1}^{s}\left(f(\lambda_j)A_{j_1}+f'(\lambda_j)A_{j_2}+\ldots+\frac{f^{(k_j-1)}(\lambda_j)}{(k_j-1)!}A_{j_{k_j}}\right)$ ni hosil qilamiz.

Agar xarakteristik sonlar oddiy, ya'ni $s=k$ va $k_1=k_2=\ldots=k_S=1$ bo'lsa,

$f(A)=f(\lambda_1)A_1+f(\lambda_2)A_2+\ldots+f(\lambda_k)A_k$ yoyilma hosil bo'ladi. **Matritsa funksiyasining spektral yoyilmasi** deyiladi. A_j matritsalar A matritsaning **komponentlari** deb ataladi va ular $f(z)$ ga emas, A matritsagagina bog'liq.

$$f(\lambda_j),\, f'(\lambda_j),\ldots, f^{(k_j-1)}(\lambda_j),\; j=\overline{1,s}$$

qiymatlar $f(z)$ analitik funksiyaning A **matritsa spektridagi qiymatlari** deb yuritiladi.

Spektral yoyilmadan A^n, e^{At} matritsalarni hisoblashda va demak, ayirmali hamda differentsial tenglamalarni yechishda foydalanish qulay.

Misol. $A=\begin{pmatrix}5 & 1\\ 8 & 3\end{pmatrix}$ matritsa uchun A^n ni hisoblaylik.

Ravshanki,

$$\chi(z)=\begin{vmatrix}5-z & 1\\ 8 & 3-z\end{vmatrix}=0 \Rightarrow z_1=1,\; z_2=7\,.$$

Bu xarakteristik sonlar oddiy, shu sababli (***) spektral yoyilma

$$f(A)=f(1)A_1+f(7)A_2$$

ko'rinishda bo'ladi.

$f(z)=1-z$ va $f(z)=7-z$ funksiyalardan foydalanish qulay. Ular $f(A)=E-A$ va $f(A)=7E-A$ matritsalarni quyidagicha aniqlaydi:

$$\begin{cases}E-A=0\cdot A_1+(-6)\cdot A_2\\ 7E-A=6\cdot A_1+0\cdot A_2\end{cases} \Rightarrow \begin{cases}A_2=-\frac{1}{6}(E-A)\\ A_1=\frac{1}{6}(7E-A)\end{cases}.$$

Bundan,

$$A_1=\frac{1}{6}\left[\begin{pmatrix}7 & 0\\ 0 & 7\end{pmatrix}-\begin{pmatrix}5 & 1\\ 8 & 3\end{pmatrix}\right]=\frac{1}{6}\begin{pmatrix}2 & -1\\ -8 & 4\end{pmatrix}$$

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hamda

$$A_2 = -\frac{1}{6}\left[\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - \begin{pmatrix}5 & 1\\ 8 & 3\end{pmatrix}\right] = -\frac{1}{6}\begin{pmatrix}-4 & -1\\ -8 & -2\end{pmatrix} = \frac{1}{6}\begin{pmatrix}4 & 1\\ 8 & 2\end{pmatrix}.$$

Demak,

$$f(A) = \frac{f(1)}{6}\begin{pmatrix}2 & -1\\ -8 & 4\end{pmatrix} + \frac{f(7)}{6}\begin{pmatrix}4 & 1\\ 8 & 2\end{pmatrix}.$$

Endi $f(z) = z^n$ deb, A^n ni hisoblaymiz:

$$A^n = \frac{1}{6}\begin{pmatrix}2 & -1\\ -8 & 4\end{pmatrix} + \frac{7^n}{6}\begin{pmatrix}4 & 1\\ 8 & 2\end{pmatrix};$$

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