ISSN 2181-3187

INTEGRO-DIFFERENSIAL TENGLAMALAR

Maxammatqulov Fazliddin
NDPI akademik litseyi matematika fani o'qituvchisi
Bobomurodova Lola Valijonovna
NDPI akademik litseyi matematika fani o'qituvchisi
Saparova Guljanat Yerbayevna
NDPI akademik litseyi informatika fani o'qituvchisi

Annotasiya: Ushbu maqolada differensial va integral tenglamalar yechimlarining ayrim usullari qaralgan. Tenglamada funksiya va uning hosilasi hamda integrali birgalikda kelsa integro-differensial tenglamaning xususiy usuli keltirildi.

Kalit so'zlar: Differensial tenglama, integral tenglama, integro-differensial tenglama, funksiya, hosila, boshlang'ich funksiya

Agar tenglamadagi noma'lum funksiya bir vaqtda ham integral ishorasi ostida qatnashsa, ham uning hosilalari qatnashsa, bunday tenglama integro-differensial tenglama deyiladi. Biz bir argumentli va ikki argumentli noma'lum funksiyalar uchun yozilgan eng sodda integro-differensial tenglamalarni yechish bilan chegaralanamiz.

Bir argumentli noma'lum funksiyaning integro-differensial tenglamalari bilan shug'ullanamiz. Bunday tenglamalarning yechimini ushbu

$$u(x) = u_0(x) + \lambda u_1(x) + \lambda^2 u_2(x) + \dots + \lambda^n u_n(x) + \dots$$

funksional qator shaklida izlaymiz, ya'ni ularni ketma-ket yaqinlashish usuli bilan yechamiz.

2.1.1-misol. Ushbu tenglama

$$u'(x) = \lambda \int_{px}^{x} xtu(t)dt \qquad (2.1.2)$$

berilgan, $0 < p < 1, u -$noma'lum funksiya.

Izlanayot yechimni (2.1.1)*qator ko'rinishida olamiz va uni* (2.1.2) *tenglamaga* qo'yamiz. Natijada quyidagi ayniyat hosil bo'ladi:

$$u_0'(x) + \lambda u_1'(x) + \lambda^2 u_2'(x) + \dots + \lambda^n u_n'(x) + \ \dots \equiv$$
$$\equiv \lambda \int_{px}^{x} xt[u_0(t) + \lambda u_1(t) + \lambda^2 u_2(t) + \dots]\, dt.$$

Bu tenglikning ikki tomonidagi teng darajali $\lambda^m(m = 0,1,2,\dots)$ larning koeffisientlarini tenglab, ketma-ket $u_0, u_1, u_2, \dots$ larni topamiz:

$u_0'(x) = 0$, bundan $u_0(x) = c_0$.

$c_0 -$ixtiyoriy o'zgarmas son.

ISSN 2181-3187

$$u_1'(x) = \int_{px}^{x} xtu_0(t)dt$$

$u_0(t) - c_0$ ga teng.

$$u_1'(x) = \int_{px}^{x} xtu_0(t)dt = \int_{px}^{x} xtc_0dt = c_0x\int_{px}^{x} t\,dt = \frac{1}{2}c_0(1-p^2)x^3,$$

bundan x bo'yicha integral olinsa,

$$u_1(x) = c_0\frac{x^4}{2\cdot 4}(1-p^2) + c_1,$$

$$A_1 = \frac{1}{2\cdot 4}(1-p^2);$$

$$u_1(x) = c_1 + c_0A_1x^4,$$

Shuningdek,

$$u_2'(x) = \int_{px}^{x} xtu_1(t)dt,$$

$$u_1(t) = c_1 + c_0A_1t^4,$$

$$u_2'(x) = \int_{px}^{x} xt(c_1 + c_0A_1t^4)dt = xc_1\int_{px}^{x} t\,dt + c_0A_1x\int_{px}^{x} t^5dt =$$

$$= xc_1\left(\frac{x^2}{2} - \frac{p^2x^2}{2}\right) + c_0A_1x\left(\frac{x^6}{6} - \frac{p^6x^6}{6}\right) =$$

$$= c_1\frac{x^3}{2}(1-p^2) + c_0A_1\frac{x^7}{6}(1-p^6)$$

bundan x bo'yicha integral olinsa,

$$u_2(x) = c_1\frac{x^4}{2\cdot 4}(1-p^2) + c_0A_1\frac{x^8}{6\cdot 8}(1-p^6) + c_2,$$

$$A_2 = \frac{1}{6\cdot 8}(1-p^6),$$

$$u_2(x) = c_2 + c_1A_1x^4 + c_0A_1A_2x^8,$$

Shu yo'sinda $u_3(x)$ ni topish mumkin:

$$u_3(x) = c_3 + c_2A_1x^4 + c_1A_1A_2x^8 + c_0A_1A_2A_3x^{12},$$

$$A_3 = \frac{1}{10\cdot 12}(1-p^{10})$$

$$u_4(x) = c_4 + c_3A_1x^4 + c_2A_1A_2x^8 + c_1A_1A_2A_3x^{12} + c_0A_1A_2A_3A_4x^{16},$$

$$A_4 = \frac{1}{14\cdot 16}(1-p^{14})$$

va hokazo, umuman,

$$u_n(x) = c_n + c_{n-1}A_1x^4 + c_{n-2}A_1A_2x^8 + \ldots + C_0A_1A_2\cdot\ldots\cdot A_nx^{4n},$$

$$A_n = \frac{1}{(4n-2)\cdot 4n}(1-p^{4n-2})$$

ISSN 2181-3187

$$A_{n+1} = \frac{1}{8(2n+1)(n+1)}\left(1 - p^{2(2n+1)}\right); \qquad n = 0,1,2,3, \ldots\ldots.$$

Mana shu u_n larning ifodalarini (2.1.1) qatorga qo'yib va o'xshash hadlarni ixchamlash natijasida berilgan tenglamaning umumiy yechimi kelib chiqadi:

$$u(x) = c\left[1 + A_1(\lambda x^4) + A_1A_2(\lambda x^4)^2 + \ldots + A_1A_2\ldots A_n(\lambda x^4)^n + \ldots\right] \qquad (2.1.3)$$

Bunda $c = c_0 + \lambda c_1 + \lambda^2 c_2 + \ldots$

Xususiy holda, agar $p = 0$ bo'lsa, (2.1.2) tenglama ushbu

$$u'(x) = \lambda \int_0^x xtu(t)dt$$

ko'rinishga ega bo'lib (2.1.3) yechimdagi koeffisientlar esa

$$A_{n+1} = \frac{1}{8(2n+1)(n+1)}, \qquad n = 0,1,2,3, \ldots$$

bo'ladi.

2.1.2-misol. Ushbu tenglama yechilsin:

$$u'(x) = e^x + \lambda \int_{x-\sigma}^x e^{x-t}u(t)dt,$$

$$\sigma > 0, \quad |\lambda| = \frac{1}{\sigma}$$

Bu tenglamadagi u o'rniga ham (2.1.1) qatorni qo'yib, $u_0, u_1, u_2 \ldots$ larni quyidagicha topamiz:

$$u_0'(x) + \lambda u_1'(x) + \lambda^2 u_2'(x) + \ldots + \lambda^n u_n'(x) + \ \ldots \equiv$$

$$\equiv e^x + \lambda \int_{x-\sigma}^x e^{x-t}\left[u_0(t) + \lambda u_1(t) + \lambda^2 u_2(t) + \ldots + \lambda^n u_n(t)\right]dt$$

Bu tenglikning ikki tomonidagi teng darajali $\lambda^m (m = 0,1,2,3, \ldots)$ larning koeffisientlarini tenglab, ketma-ket $u_0, u_1, u_2 \ldots$ larni topamiz.

$$u_0'(x) = e^x, \qquad \text{bundan} \qquad u_0(x) = e^x + c_0,$$

c_0 –ixtiyoriy o'zgarmas son; hisoblash ishlarini qisqartish maqsadida c_0 ni va bundan keyingi ixtiyoriy o'zgarmas $c_1, c_2, \ldots$ larni nolga teng deb hisoblaymiz. U holda biz tenglamaning bitta xususiy yechimini topgan bo'lamiz. Shu sababli

bundan, x bo'yicha integral olinsa

$u_1(x) = \sigma e^x + c_1; \quad c_1 = 0,$ *bo'lsa* $u_1(x) = \sigma e^x.$

$$u_2'(x) = \int_{x-\sigma}^x e^{x-t}u_1(t)dt = \int_{x-\sigma}^x e^{x-t}\sigma e^t dt = \sigma e^x \int_{x-\sigma}^x dt = \sigma^2 e^x$$

bundan, x bo'yicha integral olinsa,

$u_2(x) = \sigma^2 e^x + c_1 x + c_2, \quad c_1 = 0, \quad c_2 = 0.$

$u_2(x) = \sigma^2 e^x.$

ISSN 2181-3187

$$u_3'(x)=\int_{x-\sigma}^{x} e^{x-t}u_2(t)dt=\int_{x-\sigma}^{x} e^{x-t}\sigma^2 e^t dt=\sigma^2 e^x\int_{x-\sigma}^{x} dt=\sigma^3 e^x$$

bundan, x bo'yicha integral olinsa,

$$u_3(x)=\sigma^3 e^x.$$

Buning u_0 dan farqi σ ko'paytuvchidan iborat. Shuning uchun

$$u_n(x)=\sigma^n e^x, \qquad n=0,1,2,3,\ldots$$

deb yozish mumkin.

Endi u_n larning ifodalarini (2.1.1) qatorga qo'ysak, ushbu yechim hosil bo'ladi:

$$u(x)=\frac{e^x}{1-\lambda\sigma}$$

hosil bo'ladi.

2.1.3-misol. Ushbu tenglama yechilsin:

$$u''(x)=e^{-x}+\lambda\int_{x-\sigma}^{x} e^{-(x-t)}u(t)\,dt, \qquad \sigma>0, \quad |\lambda|<\frac{1}{\sigma}.$$

Yuqoridagi usul bilan ketma-ket $u_n(x)$ larni topamiz:

$$u_0''(x)+\lambda u_1''(x)+\lambda^2 u_2''(x)+\ldots+\lambda^n u_n''(x)+\ldots\equiv$$
$$\equiv e^{-x}+\lambda\int_{x-\sigma}^{x} e^{-(x-t)}[u_0(t)+\lambda u_1(t)+\lambda^2 u_2(t)+\ldots]\,dt.$$

Bu tenglikning ikki tomonidagi teng darajali $\lambda^m(m=0,1,2,3,\ldots)$ larning koeffisientlarini tenglab, ketma-ket $u_0,u_1,u_2,\ldots$ larni topamiz:

$$u_0''(x)=e^{-x}, \qquad u_0'(x)=-e^{-x}+C_0, \qquad u_0(x)=D_0+xC_0+e^{-x}.$$

Hisoblash ishlarini qisqartish maqsadida $C_0=D_0=0$ deb hisoblaymiz. U holda

$$u_0(x)=e^{-x},$$

shu sababli

$$u_1''(x)=\int_{x-\sigma}^{x} e^{-(x-t)}u_0(t)dt=\int_{x-\sigma}^{x} e^{-(x-t)}e^{-t}dt=e^{-x}\int_{x-\sigma}^{x} dt=\sigma e^{-x}.$$

Bundan x bo'yicha integral olinsa,

$$u_1'(x)=-\sigma e^{-x}+C_1, \qquad u_1(x)=D_1+C_1x+\sigma e^{-x};$$

bu yerda ham yuqoridagidek, $D_1=C_1=0$ deb belgilaymiz. U holda

$$u_1(x)=\sigma e^{-x};$$

shu sababli

$$u_2'(x)=\int_{x-\sigma}^{x} e^{-(x-t)}u_0(t)dt=\int_{x-\sigma}^{x} e^{-(x-t)}\sigma e^{-t}dt=\sigma e^{-x}\int_{x-\sigma}^{x} dt=\sigma^2 e^{-x}.$$

Bundan x bo'yicha integral olinsa,

$u_2'(x) = -\sigma^2 e^{-x} + C_2,$ $\qquad u_2(x) = D_2 + C_2 x + \sigma^2 e^{-x};$

bu yerda ham yuqoridagidek, $D_2 = C_2 = 0$ deb belgilaymiz. U holda

$u_2(x) = \sigma^2 e^{-x};$

umuman,

$u_n(x) = \sigma^n e^{-x} \qquad n = 0,1,2,\ldots$

deb yozish mumkin. Bularni (2.1.1) qatorga qo'yib soddalashtirilgandan so'ng quyidagi xususiy yechim hosil bo'ladi:

$$u(x) = \frac{e^{-x}}{1 - \lambda\sigma}.$$

Foydalanilgan adabiyotlar:

1. M.S.Saloxiddinov "Matematika fizika tenglamalari", "Toshkent", "O'zbekiston" 2002 y.
2. Sh.T.Maqsudov "Chiziqli integral tenglamalar elementlari", "Toshkent", "O'qituvchi" 1975 y.
3. Y.U.Soatov "Oliy matematika", "Toshkent", "O'qituvchi", 1-tom. 1992 y.