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FUNKSIYA GRAFIGINING ASIMTOTALARI

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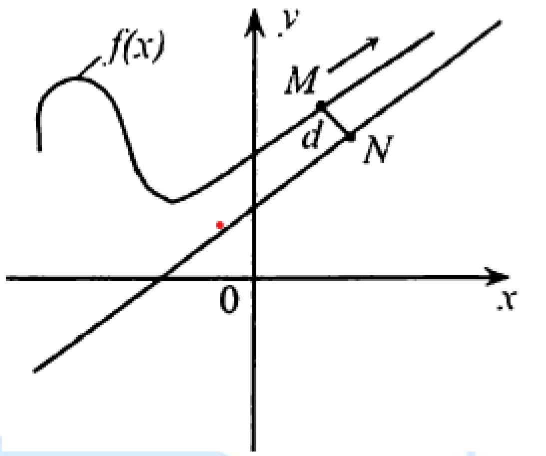
Annotatsiya. Ushbu maqolada funksiya grafigining asimtotalari mavzusiga oid ta`riflar, tushunchalar va tasdiqlar berilgan bo`lib,alohida misollar bilan yoritilgan.

**Kalit so`zlar.** vertikal asimptota, gorizontal asimptota,ikki tomonlama og`ma asimptota,o`ng og`ma asimptota, chap og`ma asimptota.

Funksiyani tekshirayotganda uning grafigi koordinatalar boshidan cheksiz uzoqlashganda, yoki boshqacha aytganda, uning o`zgaruvchi nuqtasi cheksizlikka intilganda grafikning ko`rinishini bilib olish muhim.

1 – ta'rif. Agar o`zgaruvchi M$(x;y)$ nuqta funksiya grafigi bo`yicha koordinatalar boshidan cheksiz uzoqlashganda $y=f(x)$ funksiya grafigidagi o`zgaruvchi $M(x;y)$ nuqtadan to`g`ri chiziqdagi $N(x_{1;}y_1)$ nuqtagacha bo`lgan $d=MN$ masofa nolga intilsa, bu to`g`ri chiziq $y=f(x)$ funksiya grafigining asimtotasi deyiladi.



1 – rasm.

Oy va Ox o`qlarga parallel hamda koordinata o`qlariga parallel bo`lmagan asimptotalarni qaraymiz.

1.Vertikal asimptotalar. $y=f(x)$ fuksiyada α nuqtning biror $\varepsilon>0$ atrofida aniqlangan, ya'ni $x\in U_\varepsilon(\alpha)$ bo`lsin.

2 – ta'rif. Agar

$$\lim_{x\to\alpha-0} f(x), \quad \lim_{x\to\alpha+0} f(x)$$

lardan biri yoki ularning ikkalasi ham cheksiz bo`lsa, $x=a$ to`g`ri chiziq $f(x)$ funksiya grafigining vertikal yoki $Oy$ o`qqa parallel asimptotasi deyiladi

(2-a,b,d,e rasmlar).

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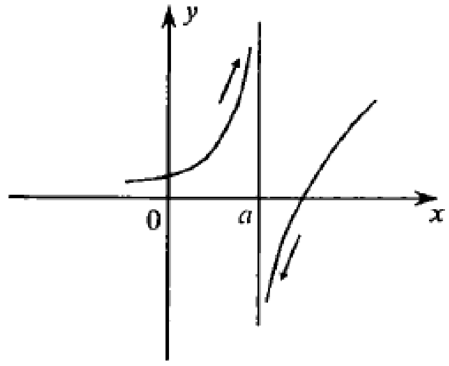
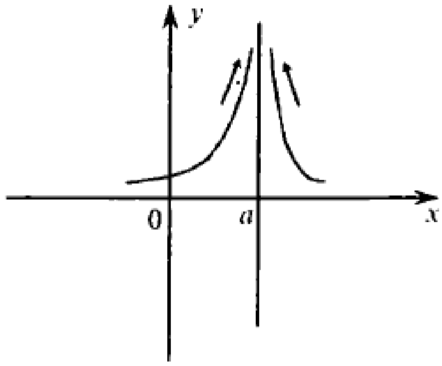


a)



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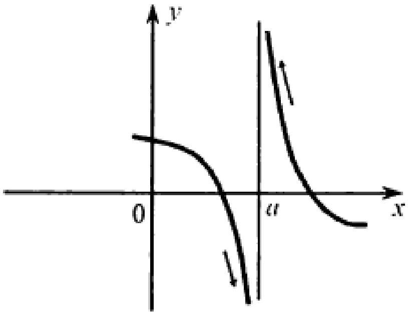

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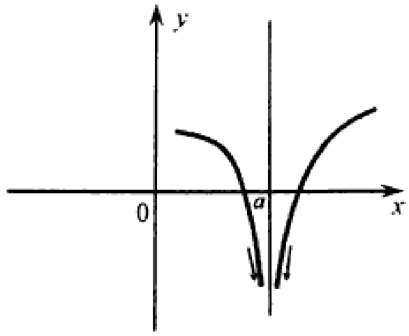

e)

Demak, $y = f(x)$ fuksiya grafining vertikal asimptotalarini izlash uchun funksiyaning qiymatini cheksizlikka aylantiradigan (cheksiz uzilishga ega bo`lgan) $x = a$ nuqtani topish kerak ekan. Bunda $x = a$ to`g`ri chiziq vertikal asimptota bo`ladi.

Eslatma. Umuman aytganda, $y = f(x)$ funksiyaning grafigi bir nechta vertikal asimtotalarga ega bo`lishi ham mumkin.

1 – misol .$f(x) = \frac{1}{x-2}$,$x \in [-2; 3]$ funksiya grafigi ning vertikal asimptotasini toping.

Yechilishi. Berilgan funksiyaning maxraji $x = 2$ nuqtada nolga aylanadi. $x \to 2 \pm 0$ da berilgan funksiyaning limitini hisoblaymiz:

$$\lim_{x\to 2-0} f(x) = \lim_{x\to 2-0} \frac{1}{x-2} = -\infty, \lim_{x\to 2+0} f(x) = \lim_{x\to 2+0} \frac{1}{x-2} = +\infty.$$

Demak, 2 – ta`rifga ko`ra, berilgan funksiyasining grafigi uchun $x = 2$ to`g`ri chiziq vertikal asimptota bo`ladi (3-rasm).

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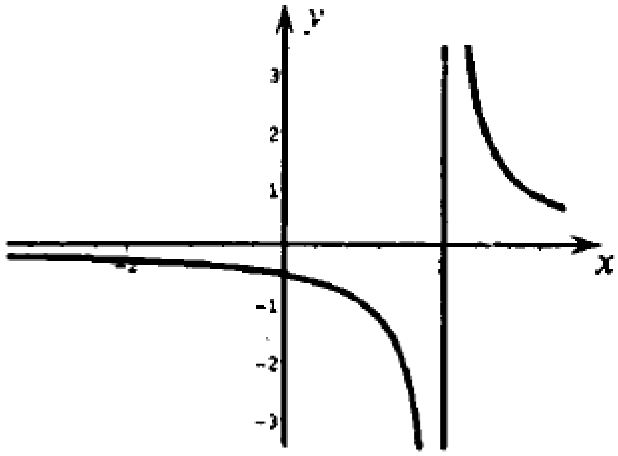
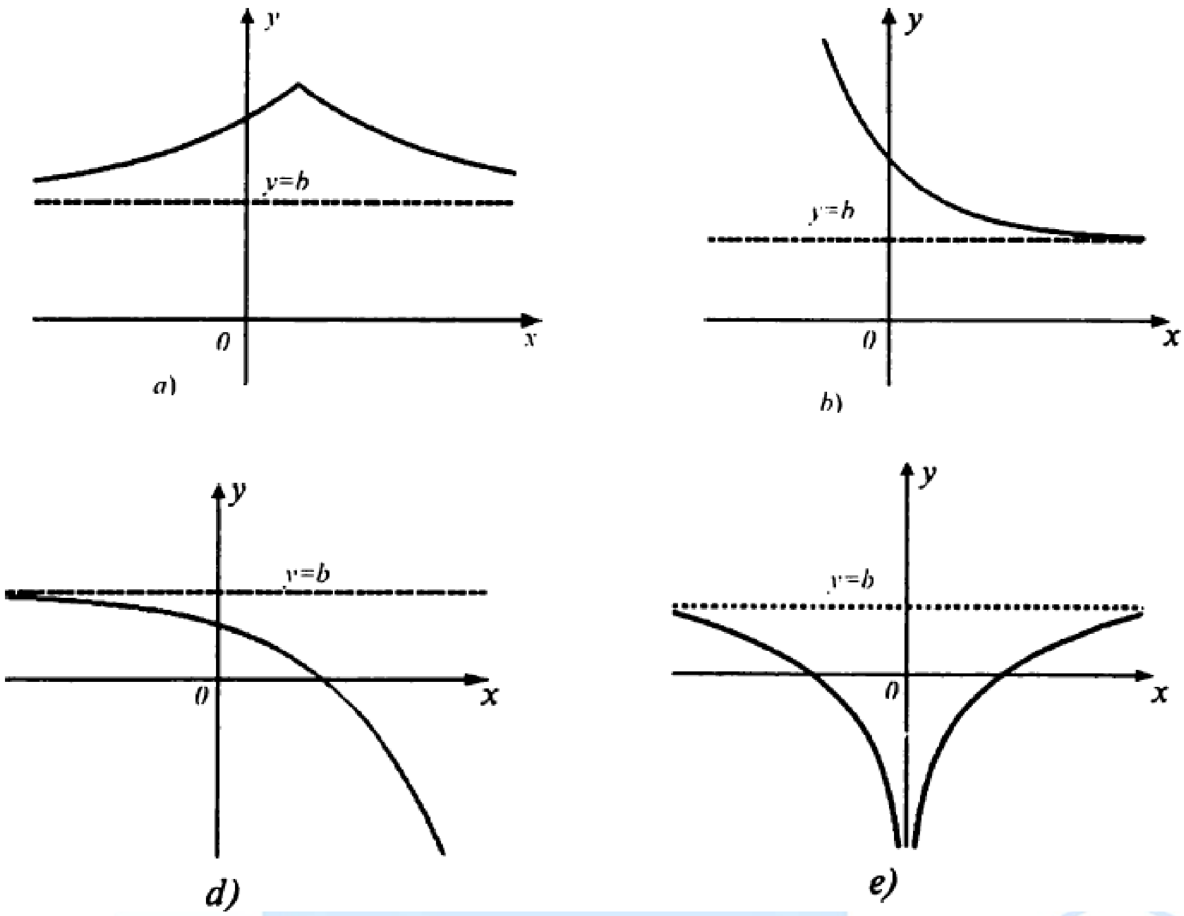


Gorizontal asimtotalar.

3-ta'rif. Agar

$$\lim_{x \to +\infty} f(x) = b \qquad (b \in R^1)$$
$$(x \to \infty)$$

bo`lsa, $y = b$ to`g`ri chiziq $x \to +\infty$ $(x \to -\infty)$ da $y = f(x)$ funksiya grafigining gorizontal yoki $Ox$ o`qqa parallel asimptotasi deyiladi (4-a,b,d,e rasmlar).



2 – misol. $f(x) = \frac{x^2}{x^2+2}$ funksiya grafining gorizantal asimptotasini toping.

Yechilishi. Berilgan funksiya R^1 da aniqlangan.$x \to \pm\infty$ da berilgan funksiyaning limitini hisoblaymiz:

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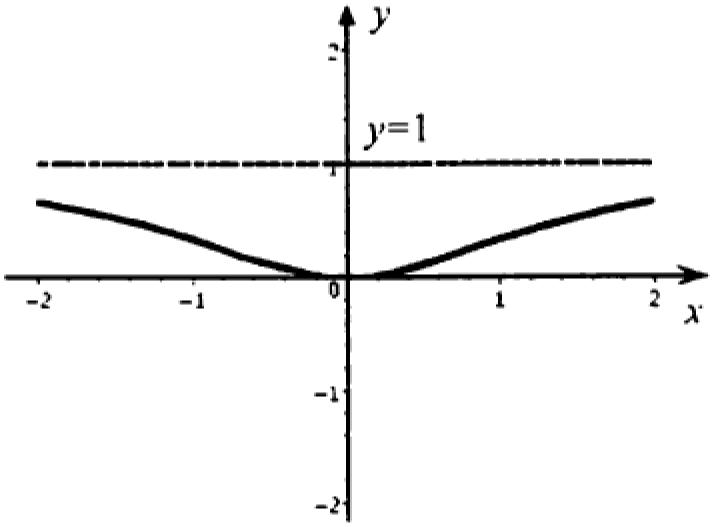
$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty}\frac{x^2}{x^2+2} = \lim_{x\to\pm\infty}\frac{1}{1+\dfrac{2}{x^2}} = 1.$$

Demak, 3 – ta`rifga ko`ra, berilgan funksiyasining grafigi uchun $y = 1$ to`g`ri chiziq gorizontal asimptota bo`ladi (5-rasm).



3.Og`ma asimptotalar.

4 – ta'rif. Shunday k va b chekli sonlar mavjud bo`lib, $x \to +\infty$ $(x \to \infty)$ da $f(x)$ funksiya quyidagi

$$f(x) = kx + b + \alpha(x)$$

ko`rinishda ifodalansa (bunda $\lim_{x\to\pm\infty} \alpha(x) = 0$), $y = kx + b$ to`g`ri chiziq

$y = f(x)$ funksiya grafigining og`ma asimptotasi deyiladi. Xususiy holda, $k = 0$ bo`lsa,$y = b$ to`g`ri chiziq gorizontal asimptota bo`ladi.

1 – teorema. $y = f(x)$ funksiya grafigi $x \to \pm\infty$ da $y = kx + b$ og`ma asimptotaga ega bo`lishi uchun

$$\lim_{x\to\pm\infty}\frac{f(x)}{x} = k,\quad \lim_{x\to\pm\infty}[f(x) - kx] = b$$

Munosabatlar o`rinli bo`lishi zarur va yetarli.

(1)limitlarni hisoblashda quyidagi xususiy holler bo`ladi:

1 – hol. Argumentning ishorasiga bog`liq bo`lmagan holda, ushbu

$$\lim_{x\to+\infty}\frac{f(x)}{x} = \lim_{x\to-\infty}\frac{f(x)}{x} = k$$

$$\lim_{x\to+\infty}[f(x) - kx] = \lim_{x\to-\infty}[f(x) - kx] = b$$

ikkala limit ham mavjud va chekli. Bu hol $y = kx + b$ to`g`ri chiziq funksiya grafigining ikki tomonlama og`ma asimptotasi bo`ladi.

2-hol. Argument x ham musbat, ham manfiy ishorali cheksizlikka intilganda ,ushbu

$$\lim_{x\to+\infty}\frac{f(x)}{x}=k_1,\quad \lim_{x\to-\infty}\frac{f(x)}{x}=k_2,$$
$$\lim_{x\to+\infty}[f(x)-kx]=b_1,\quad \lim_{x\to-\infty}[f(x)-kx]=b_2$$

Limitlar mavjud, lekin ular o`zaro har xil (hech bo`lmaganda $k_1 \neq k_2$ yoki $b_1 \neq b_2$ teng emas). Bu holda $y_1 = k_1x + b_1$ va $y_2 = k_2x + b_2$ to`g`ri chiziqlar funksiya grafigining mos ravishda ikkita bir tomonli (o`ng va chap) og`ma asimptotalari bo`ladi.

3-hol. Faqat $x \to +\infty$ da

$$\lim_{x\to-\infty}\frac{f(x)}{x}=k,\quad \lim_{x\to-\infty}[f(x)-kx]=b$$

Ikkala limit ham mavjud. Bu holda $y = kx + b$ to`g`ri chiziq funksiya grafigining faqat o`ng og`ma asimptotasi bo`ladi.

4-hol. Faqat $x \to -\infty$ da

$$\lim_{x\to+\infty}\frac{f(x)}{x}=k,\quad \lim_{x\to+\infty}[f(x)-kx]=b$$

Ikkala limit ham mavjud. Bu holda $y = kx + b$ to`g`ri chiziq funksiya grafigining faqat chap og`ma asimptotasi bo`ladi.Agar yuqoridagi hollarning barchasida $k = 0$ bo`lsa, $y = b$ to`g`ri chiziq gorizontal asimptota bo`ladi.

3-misol. $y = \sqrt[3]{x^3+1}$ funksiya grafining og`ma asimptotalarini toping.

Yechilishi. Og`ma asimptotalarni topamiz:

$$k=\lim_{x\to\pm\infty}\frac{f(x)}{x}=\lim_{x\to\pm\infty}\frac{\sqrt[3]{x^3+1}}{x}=1,$$
$$b=\lim_{x\to\pm\infty}[f(x)-kx]=\lim_{x\to\pm\infty}\left[\sqrt[3]{x^3+1}-x\right]=$$
$$=\lim_{x\to\pm\infty}\frac{x^3+1-x^3}{\sqrt[3]{(x^3+1)^2+x\sqrt[3]{x^3+1}+x^2}}=0.$$

Demak, 1 – teoremaga asosan, $y = x$ to`g`ri chiziq berilgan funksiyasining grafigi og`ma asimtotasi bo`ladi (6-rasm).

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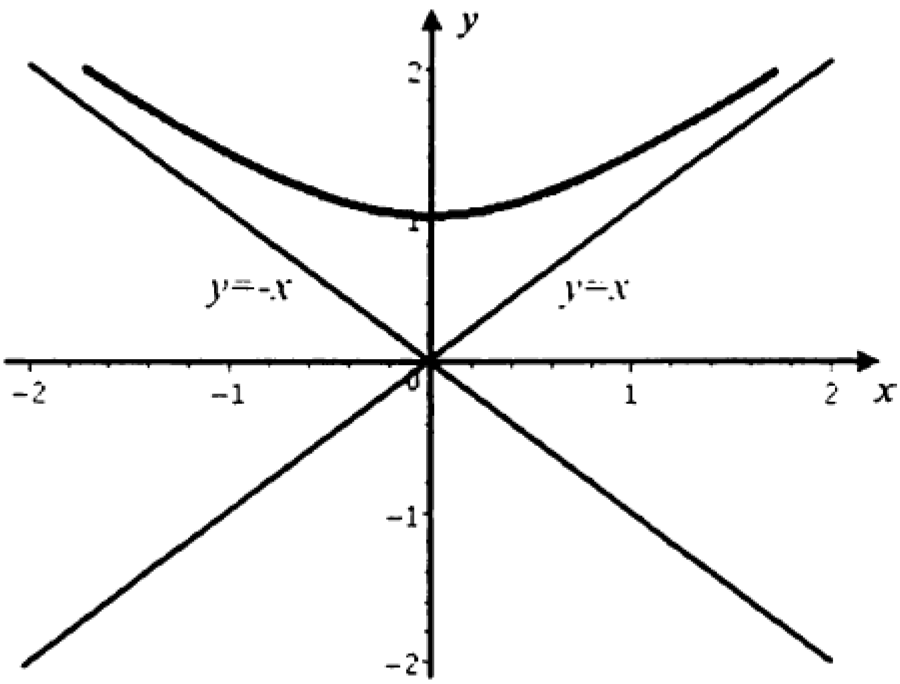

2- teorema. $y = f(x)$ funksiyaning grafi $x \to +\infty$ da

$$y = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

Asimptotaga ega bo`lishi uchun quyidagi $n+1$ ta limit qiymatlarning mavjud bo`lishi zarur va yetarli:

$$\lim_{x\to+\infty}\frac{f(x)}{x^n} = a_n, \quad \lim_{x\to+\infty}\frac{f(x) - a_n x^n}{x^{n-1}} = a_{n-1}, \dots,$$

$$\lim_{x\to+\infty}\frac{f(x) - a_n x^n + a_{n-1}x^{n-1} + \cdots a_2 x^2}{x} = a_1,$$

$$\lim_{x\to+\infty}[f(x) - (a_n x^n + a_{n-1}x^{n-1} + \cdots a_1 x)] = a_0$$

6-misol. $f(x) = \frac{x^3+2x-3}{x-3}$ funksiya grafigining asimptotasini toping.

Yechilishi. Berilgan funksiyani

$$f(x) = x^2 + 3x + 11 + \frac{30}{x-3}$$

ko`rinishda tasvirlaymiz, bunda $\lim_{x\to\infty} a(x) = \lim_{x\to\infty}\frac{30}{x-3} = 0.$

Demak, $y = x^2 + 3x + 11$ chiziq, ta`rifga ko`ra, berilgan funksiyasining grafigining asimptotasi bo`ladi. Ravshanki, $x = 3$ to`g`ri chiziq berilgan funksiya grafigining vertikal asimptotasi bo`ladi .

Oy va Ox o`qlarga parallel hamda koordinata o`qlariga parallel bo`lmagan asimptotalarni ko`rib chiqildi. Ular uch turga bo`linadi: vertikal asimptotalar, gorizontal asimtotalar va og`ma asimtotalar.Va ularni alohida – alohida ko`rib chiqildi. Ularga doir misollar va grafiklar bilan mavzu mustahkamlandi.

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