ISSN 2181-3187

BINOM FORMULASI

TDPU dotsenti Tajiboyeva X.X.
TAFU bakalavri Karimova N.Q

Annotatsiya: Algebra va analiz asoslari o`qitish texnologiyasi asosida o`quvchilar matematik modellar tuzishni bayon etdi. Bunda amaliy, tadbiqiy masalalar ko`rib o`tildi. Jumladan hayotdagi to`lov hujjatlari, avtomobillarni nomerlash, shaxsiy va boshqa hujjatlarni nomerlash, juda zarur bo`lgan masalalar yechish ko`rsatib o`tildi. Bu esa ishimizning amaliy ahamiyatini ko`rsatadi.

Natural ko'rsatkichli binom formulasi

Quyidagi ifodalar bizga tanish

$(a+b)^1=a+b$ (1)

$(a+b)^2=a^2+2ab+b^2$ (2)

$(a+b)^3=a^3+3a^2b+3ab^2+b^3$ (3)

a va *b* koeffitsientlarga e' tibor beramiz. (1)-formulaning chap tomonida bu son 1,1 bu faktni $C_1^0=1$, $C_1^1=1$ bu yerda C_n^m n elementdan to m gacha kombinatsiya sonidir. (3) formuladagi koeffitsientlar $C_3^0=1$, $C_3^1=3$, $C_3^2=3$, $C_3^3=1$ ko' rinishda yozish mumkin.

Endi (2) va (3) larni quyidagi ko' rinishda yozamiz.

$(a+b)^2=C_2^0a^2+C_2^1ab+C_2^2b^2$

$(a+b)^3=C_3^0a^3+C_3^1a^2b+C_3^2ab^2+C_3^3b^3$

Bu tengliklar bizga n chi darajali N son uchun quyidagi formulani keltirib chiqarishga yordam beradi.

$(a+b)^n=C_n^0a^n+C_n^1a^{n-1}b+C_n^2a^{n-2}b^2+\cdots+C_n^nb^n$ (4)

Buni biz matematik induktsiya orqali isbotlashimiz mumkin.

n= 1 da (4)-quyidagi ko' rinishni oladi

$(a+b)^1=C_1^0a+C_1^1b$

Ya' ni (1)- tenglik.

Faraz qilaylik (4) n=m da isbotlangan, ya' ni quyidagi ko' rinishni oladi.

$(a+b)^m=C_m^0a^m+C_m^1a^{m-1}b+\cdots+C_m^mb^m$ (5)

(4) formula n=m+1 ham to' g' riligini isbotlaymiz.

Buning uchun (5) ning ikkala qismi (a+b) ga ko' paytiriladi.

$(a+b)^{m+1}=(C_m^0a^m+C_m^1a^{m-1}b+C_m^2a^{m-2}b^2+\cdots+C_m^mb^m)(a+b)=$

$=C_m^0a^{m+1}+(C_m^0+C_m^1)a^mb+(C_m^1+C_m^2)a^{m-1}b^2+\cdots+C_m^mb^{m+1}$

C_m^k xossasini isbotlaymiz.

$C_m^0=C_{m+1}^0=1$, $C_m^m=C_{m+1}^{m+1}=1$, $C_m^k+C_m^{k-1}=C_{m+1}^k$

ISSN 2181-3187

Unda (6) – tenglik quyidagi ko' rinishni oladi.

$$(a+b)^{m+1} = C_{m+1}^{0}a^{m+1} + C_{m+1}^{1}a^{m}b + C_{m+1}^{2}a^{m-1}b^{2} + \cdots + C_{m+1}^{m+1}b^{m+1} \qquad (7)$$

(7)- tenglik n=m+1 da (4) – formulani anglatadi. SHu narsani anglatish kerak edi.

(4)- formula Binom formulasi deyiladi.

Kasr va manfiy ko'rsatkichli binom formulasi

SHu narsani aytishimiz kerakki (4)-formula Nьyo`tonga ham ma'lum edi. Bu yo'nalishdagi ishlar o'rta Osiyolik olim G'iyosiddin Jamshid al-Koshiy (1420 yy.) asarlarida uchraydi. Nьyo`tonning xissasi shundaki u (4) ni manfiy va kasrli ko'rsatgichlari bo'yicha isbotlab beradi. Lekin aniq isbotlab bermagan. Bo`tun musbat ko'rsatgichli sonlar uchun Yakobь Bernulli tomonidan isbotlangan manfiy va kasrli ko' rinishlar uchun (4) formula quyidagicha yoziladi.

$$(a+b)^{\alpha} = \binom{0}{\alpha}a^{\alpha} + \binom{1}{\alpha}a^{\alpha-1}b + \binom{2}{\alpha}a^{\alpha-2}b^{2} + \cdots + \binom{n}{\alpha}a^{\alpha-n}b^{n} + \cdots \quad (8)$$

Bu yerda

$$\binom{n}{\alpha} = \frac{\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-(n-1))}{n!}, \qquad \alpha \neq 0$$

(8) formulani isboti.

$$f(x) = (1+x)^{\alpha}$$

funktsiyani ko' rib o' tamiz. Faraz qilaylik bu funktsiya quyidagiko' rinishda ifodalangan bo' lsin.

$$f^{0}(x) = A_0 + A_1x + A_2x^2 + \cdots + A_nx^n + \cdots \qquad (9)$$

Bu yerda . A_0, A_1, A_2 noma' lum koeffitsienlardir. Koeffitsienlarni aniqlash uchun (9) dan xosilalar topamiz

$$\left.\begin{aligned} f'(x) &= A_1 + 2A_2x + 3A_3x^2 + \cdots + nA_nx^{n-1} + \cdots \\ f''(x) &= 1\cdot 2A_2 + 2\cdot 3A_3x + \cdots + n\cdot(n-1)A_nx^{n-2} + \cdots \\ f'''(x) &= 1\cdot 2\cdot 3A_3 + 2\cdot 3\cdot 4A_4x + \cdots \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ f^{(n)}(x) &= n(n-1)(n-2)\cdots 2\cdot 1A_n + \cdots \end{aligned}\right\} \quad (10)$$

x=0 da (9) va (10) dan quyidagilarni topamiz

ISSN 2181-3187

$f(0) = A_0$

$f'(x) = A_1 = 1!\,A_1$

$f''(0) = 1 \cdot 2 \cdot A_2 = 2!\,A_2$

$f'(0) = 1 \cdot 2 \cdot 3 \cdot A_3 = 3!\,A_3$

. . . .

$f^{(n)}(0) = n!\,A_n$

$A_0 = f(0)$

$A_1 = \frac{1}{1!} f'(0)$

$A_2 = \frac{1}{2!} f''(0)$

$A_3 = \frac{1}{3!} f'''(0)$

. . . .

$A_n = \frac{1}{n!} f^{(n)}(0)$

Endi bu $f(x) = (1+x)^{\alpha}$ funktsiyadan xosilalar

$f'(x) = \alpha(1+x)^{\alpha-1}$

$f''(x) = \alpha(\alpha-1)(1+x)^{\alpha-2}$

$f'''(x) = \alpha(\alpha-1)(\alpha-2)(1+x)^{\alpha-3}$

. . . .

$f^{(n)}(x) = \alpha(\alpha-1)(\alpha-2)\cdots[\alpha-(n-1)](1+x)^{\alpha-n}$

$x = 0$ da quyidagilarni topamiz.

$f(0) = 1$

$f'(0) = \alpha$

$$f''(0) = \alpha(\alpha-1) \qquad (12)$$

$f'''(0) = \alpha(\alpha-1)(\alpha-2)$

. . . .

$f^{(n)}(0) = \alpha(\alpha-1)(\alpha-2)\cdots[\alpha-(n-1)]$

(12) va (11) lani qo' yib quyidagini topamiz

$A_0 = 1$

$A_1 = \frac{\alpha}{1!}$

$A_2 = \frac{\alpha(\alpha-1)}{2!}$

$$A_3 = \frac{\alpha(\alpha-1)(\alpha-2)}{3!} \qquad (13)$$

. . . .

ISSN 2181-3187

$$A_n = \frac{\alpha(\alpha-1)(\alpha-2)\cdots[\alpha-(n-1)]}{n!}$$

(13) va (9)ni kuyib kuyidagilarni topamiz.

$$(1+x)^\alpha = 1 + \frac{\alpha}{1!}x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \cdots + \frac{\alpha(\alpha-1)(\alpha-2)\cdots[\alpha-(n-1)]}{n!}x^n + \cdots$$

α ixtiyoriy son bo' lgani uchun unda C_α^n gacha o' zgartirish kiritamiz.

$$\frac{\alpha(\alpha-1)(\alpha-2)\cdots[\alpha-(n-1)]}{n!} = \binom{n}{\alpha}$$

A N sonlarda $\binom{n}{\alpha} = C_\alpha^n$ shunday qilib

$$(1+x)^\alpha = \binom{0}{\alpha} + \binom{1}{\alpha}x + \binom{2}{\alpha}x^2 + \binom{3}{\alpha}x^3 + \cdots + \binom{n}{\alpha}x^n + \cdots$$

olamiz.

Endi $x = \frac{b}{a}$ shundan

$$(a+b)^\alpha = \binom{0}{\alpha}a^\alpha + \binom{1}{\alpha}a^{\alpha-1}b + \binom{2}{\alpha}a^{\alpha-2}b^2 + \cdots + \binom{n}{\alpha}a^{\alpha-n}b^n$$

olamiz (8) formula isbotlandi.

Masalan:

1. Nьyo`ton Binomi formulasi bo'yicha yoyib chiqamiz.

$(1+x)^{-1} \quad a=1, \quad b=x, \quad \alpha=-1$

1) $(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + \cdots$

2) $(1+x)^{\frac{1}{2}} = \sqrt{1+x} \quad a=1, \quad b=x, \quad \alpha=\frac{1}{2}$

$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots$

3) $(1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \cdots$

Binom formulasini umumlashtirish

Endi umumiyroq formulani isbotlaymiz.

$$(a_1 + a_2 + \cdots a_k)^n = \sum_{\alpha_1+\alpha_2+\cdots\alpha_n=n} \frac{n!}{\alpha_1!\,\alpha_2!\cdots\alpha_{k!}} a_1^{\alpha_1}, a_2^{\alpha_2}, \cdots, a_k^{\alpha_k}$$

Isbot: n bir xil to' plamlarni ko' rib chiqamiz.

ISSN 2181-3187

$$n\begin{cases}(a_1+a_2+........+a_k)\\(a_1+a_2+........+a_k)\\.............................\\(a_1+a_2+........+a_k)\end{cases}$$

Ularni ko’ paytirish qoidalari bo’ yicha ko’ paytirib chiqamiz.
Natijada biz quyidagi summaga ega bo’ lamiz va u

$$a_{i_1}a_{i_2}..........a_{i_n} \qquad (15)$$

ko’ rinishga ega bo’ ladi. Indekslar uchun 1,2,………k sonlar o’ rinlidir. k elementdan $a_{1, ..,}$ a_k to n takrorlanishlar bilan hosil bo’ lgan (15) ifodalarning soni tengdir, ya’ ni k^n dan olingan hadlar, qaysiki a_1 α_1 marta, a_2 α_2 marta va boshqalar shuncha marta tashkil etadi. .... marta tashkil etadi va quyidagiga tengdir:

$$\alpha_1+\alpha_2+..........\alpha_n=n$$

Har qaysi had **<u>o’ rinlashtirishlarga</u>** <u>bog’ liq bo’ lib, takrorlanadi:</u>

$$a_{i_1}a_{i_2}..........a_{i_n}$$

Qaysiki bunda a_1 α_1 marta uchraydi, a_2 α_2 marta uchraydi va h.o, a_k α_k marta uchraydi. Bu hadlarning qiymati mumkin bo’ lgan takrorlanishlar bilan o’ rinlashtirishlar soniga tengdir. Bunda $a_{1,}$ $a_{2, ...,}$ a_n elementlarda ko’ rsatilgan son bo’ yicha shuncha marta uchraydi va h.o.

$$\frac{n!}{\alpha_1!\alpha_2!..\alpha_k!} \qquad (16)$$

Mulohazaning ko’ rinishida

$$(a_1+a_2+........+a_k)^n$$

Ko’ paytmalar summa ko’ rinishda

$$a_1^{\alpha_1}a_2^{\alpha_2}............a_k^{\alpha_k}$$

(16) koeffitsient bo’ lib kiradi, bu yerda

$$\alpha_1+\alpha_2+..........\alpha_n=n$$

Bu bilan (14) – formula isboti tugaydi.
(14) formula k=2 da (4) Nьyo`ton-Binomi formulasi hisoblanadi.
Masalan:
Isbotlangan formula bo’yicha hisoblaymiz. $(a_1+a_2+a_3)^3$

ISSN 2181-3187

$$(a_1+a_2+a_3)^3=\frac{3!}{3!0!0!}\cdot a_1^3+\frac{3!}{0!3!0!}\cdot a_2^3+\frac{3!}{0!0!3!}\cdot a_3^3+\frac{3!}{2!1!0!}\cdot a_1^2a_2+\frac{3!}{2!0!1!}\cdot a_1^2a_3+\frac{3!}{0!2!1!}\cdot a_2^2a_3+$$

$$+\frac{3!}{1!2!0!}\cdot a_2^2a_1+\frac{3!}{1!0!2!}\cdot a_1a_3^2+\frac{3!}{0!1!2!}\cdot a_2a_3^2+\frac{3!}{1!1!1!}\cdot a_1a_2a_3=$$

$$=a_1^3+a_2^3+a_3^3+3(a_1^2a_2+a_1^2a_3+a_2^2a_3+a_1a_3^2+(a_1a_2^2+a_2a_3^2)+6a_1a_2a_3$$

FOYDALANILGAN ADABIYOTLAR

1. Айерленд К., Раузен М. Классическое введение в современную теория чисел. Перевод с английского. –М. Мир 1987.
2. Гаимназаров О.Г. Математикадан амалий масалалар ечиш намуналари. Т.: «Фан», 2006 й.
3. Боревич З.И., Шафаревич И.Р. Теория чисел. –М. наука, 1985.