ISSN 2181-3187

BRUVY QATORI YORDAMIDA BIR JINSLI DIFFERENTSIAL - FUNKTSIONAL TENGLAMALARNI YECHISH

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Annotatsiya: Maqolada o'zgarmas koeffitsientli tenglamaning yechimini mavjudligi va yagonaligi, hamda boshlang'ich qiymat yoki funktsiyaga bog'liqligi haqidagi tushunchalar to'la ma'noda o'rganilgan. Bruvy qatorlarini o'zgarmas va o'zgaruvchi koeffitsiyentli, chiziqli, bir jinssiz yoki bir jinsli differentsial-funktsional tenglamalarning yechimini aniqlashdagi tadbiqi bo'lib hisoblanadi.

Аннотация: В статье полностью изучены понятия существования и единственности решения уравнения с постоянными коэффициентами, а также зависимости от начального значения или функции. Ряды Бруви рассматриваются как приложение для решения линейных, неоднородных или однородных дифференциально-функциональных уравнений с постоянными и переменными коэффициентами.

Annotation: In the article, the concepts of the existence and uniqueness of the solution of the equation with constant coefficients, as well as the dependence on the initial value or function, are fully studied. Bruvy series is considered as an application in determining the solution of linear, inhomogeneous or homogeneous differential-functional equations with constant and variable coefficients.

Kalit so'zlar: differenatsial-funktsional tenglama, haqiqiy sonlar maydoni, Stiltiyes integrali, matritsali funksiya, chekli ayirmali differentsial.

Ключевые слова: дифференциально-функциональное уравнение, поле действительных чисел, интеграл Стилтиса, матрица-функция, конечно-разностный дифференциал.

Keywords: differential functional equation, field of real numbers, Stilthies integral, matrix function, finite difference differential.

1. Faraz qilaylik, birinchi tartibli, o'zgarmas koeffitsiyentli, bir jinsli

$$y'(x) + ay(x-r) = 0 \qquad (1.1)$$

differentsial-funktsional (ayirmali) tenglama berilgan bo'lsin, bu yerda a, r lar ihtiyoriy o'zgarmas sonlar va $a \neq 0, r \neq 0$.

(1.1) tenglamaning

$$y(x_0) = y_0 \qquad (1.2)$$

shartni qanoatlantiruvchi yechimini topish talab etilsin.

ISSN 2181-3187

(1.1) tenglamaning (1.2) shartni qanoatlantiruvchi yechimini

$$y(x)=\sum_{k=0}^{\infty}\frac{d_k}{k!}(x-x_0-kr)^k \qquad (1.3)$$

ko'rinishda qidiramiz, bu yerda d_k –noma'lum koeffitsiyentlar, $k=0,1,2,\ldots$.

d_k – noma'lum koeffitsiyentlarni aniqlash uchun, ayrim hisoblashlarni hisobga olib, (1.3) ni (1.1) ga qo'yamiz. Ya'ni:

$$y'(x)=\sum_{k=0}^{\infty}\frac{kd_k}{k!}(x-x_0-kr)^{k-1}=$$

$$=\sum_{k=1}^{\infty}\frac{kd_k}{k!}(x-x_0-kr)^{k-1}=\sum_{k=0}^{\infty}\frac{d_{k+1}}{k!}[x-x_0-(k+1)r]^k$$

yoki

$$y'(x)=\sum_{k=0}^{\infty}\frac{d_{k+1}}{k!}[x-x_0-(k+1)r]^k; \qquad (1.3_1)$$

$$y(x-r)=\sum_{k=0}^{\infty}\frac{c_k}{k!}[x-x_0-(k+1)r]^k \qquad (1.3_2)$$

kelib chiqadi. $y'(x)$ va $y(x-r)$ larning bu ifodalarini (1.1) ga qo'yib

$$\sum_{k=0}^{\infty}\frac{d_{k+1}}{k!}[x-x_0-(k+1)r]^k+a\sum_{k=0}^{\infty}\frac{d_k}{k!}[x-x_0-(k+1)r]^k$$

yoki

$$\sum_{k=0}^{\infty}\frac{d_{k+1}+ad_k}{k!}[x-x_0-(k+1)r]^k=0$$

kelib chiqadi. Qatorning nolga tengligidan d_k koeffitsiyetnlarni aniqlash uchun $d_{k+1}+ad_k=0$

ko'rinishdagi rekkurent sistema kelib chiqadi. Bu sistema

$$d_k=(-1)^k a^k\cdot d_0$$

yechimga ega, bu yerda $k=0,1,2,\ldots$, $d_0\neq 0$ bo'lgan ihtiyoriy o'zgarmas son.

Shunday qilib, (1.1)ning yechimi

$$y(x)=d_0\sum_{k=0}^{\infty}\frac{(-1)^k a^k}{k!}(x-x_0-kr)^k \qquad (1.4)$$

ko'rinishda yoziladi. Bu yechim, (1.2) shartni qanoatlantirishi talab qilinsa, d_0

$$y(x)=\frac{y_0}{1+\sum_{k=1}^{\infty}\frac{(r\cdot a\cdot k)^k}{k!}}\sum_{k=0}^{\infty}\frac{(-1)^k a^k}{k!}(x-x_0-kr)^k \qquad (1.5)$$

ko'rinishda yoziladi.

(1.1) ning xarakteristik tenglamasi

$$\lambda+ae^{-\lambda r}=0 \quad \text{yoki} \quad \lambda e^{\lambda r}=-a \qquad (1.6)$$

ko'rinishda yoziladi. (1.5) tenglama kompleks sonlar maydonida cheksiz ko'p yechimlarga ega. Bu ildizlarning karralilarini ham hisobga olib, ular

ISSN 2181-3187

$\lambda_0, \lambda_1, \lambda_2, \ldots$ (1.7)

lar bilan belgilansa, (1.1) ning bitta hususiy yechimi

$y_j(x) = c_j^* e^{\lambda_j x}$ (1.8)

ko'rinishda yoziladi. Agar (1.6) ning (1.7) ko'rinishdagi ildizlari har hil bo'lsa, u holda (1.1) ning umumiy yechimi

$$y(x) = \sum_{j=0}^{\infty} c_j^* e^{\lambda_j x} \qquad (1.9)$$

ko'rinishda bo'ladi, bu yerda $c_0^*, c_1^*, \ldots$ – ihtiyoriy o'zgarmas sonlar.

Ikkinchi tomondan, (1.6) ni hisobga olsak, (1.5) xususiy yechim

$$y(x) = d_0 \sum_{k=0}^{\infty} \frac{(\lambda e^{\lambda r})^k}{k!} (x - x_0 - kr)^k$$

ko'rinishda yoziladi yoki (1.6) tenglamaning har bir λ_j ildiziga mos keluvchi (1.1) ning bitta xususiy yechimi

$$y(x) = c_j \sum_{k=0}^{\infty} \frac{(\lambda_j e^{\lambda_j r})^k}{k!} (x - x_0 - kr)^k \qquad (1.10)$$

ko'rinishni oladi.

Uchunchidan, $e^{\lambda_j x}$ ni Bruvy qatorga yoyilmasi

$$e^{\lambda_j x} = (1 + \lambda_j r) \sum_{k=0}^{\infty} \frac{(\lambda_j e^{\lambda_j r})^k}{k!} (x - x_0 - kr)^k \qquad (1.11)$$

ko'rinishda olinadi va uning o'ng tomonida turgan qator har bir (1.6) tenglamaning λ_j ildizi uchun yaqinlashuvchi bo'ladi, bu yerda $1 + \lambda_j r \neq 0$.

Demak, (1.10) dan

$$y_j(x) = \frac{c_j}{1 + \lambda_j r} (1 + \lambda_j r) \sum_{k=0}^{\infty} \frac{(\lambda_j e^{\lambda_j r})^k}{k!} (x - x_0 - kr)^k = \frac{c_j}{1 + \lambda_j r} e^{\lambda_j x} = c_j^* e^{\lambda_j x}$$

kelib chiqadi yoki (1.1) differentsial tenglamaning Bruvy qatori orqali aniqlangan (1.10) ko'rinishdagi xususiy yechimi, uning xarakteristik ildizi orqali olingan (1.8) xususiy yechim bilan bir xildir.

(1.1) tenglamaning (1.2) shartni qaoatlantiruvchi yechimini yagona emasligi (1.9) dan bevosita kelib chiqadi. Shuningdek, (1.7) ildizlar orasida karralilari mavjud bo'lsa, bu holda ham yuqorida keltirilgan tushunchalarni o'rinli ekanligi to'g'ridan-to'g'ri kelib tasdiqlanadi.

2. Faraz qilaylik, n chi tartibli, o'zgarmas koeffitsiyentli, bir jinsli, chiziqli

$$\begin{aligned} &y^{(n)}(t) + a_{n-1} y^{(n-1)}(t - r) + a_{n-2} y^{(n-2)}(t - 2r) + \\ &+ \cdots + a_1 y'(t - (n-1)r) + a_0 y(t - nr) = 0 \end{aligned} \qquad (1.12)$$

differentsial-funktsional (ayirmali) tenglama berilgan bo'lsin, bu yerda $a_0, a_1, \ldots, a_{n-1}$ va r lar qandaydir haqiqiy sonlar, $r \neq 0$.

(1.12) tenglamaning

ISSN 2181-3187

$$y(x_0)=y_0, y'(x_0)=y_1,\ldots,y^{n-1}(x_0)=y_{n-1} \qquad (1.13)$$

shartlarni qanoatlantiruvchi yechimini topish talab qilinsin.

(1.12) tenglamaning (1.13) shartlarni qanoatlantiruvchi yechimini

$$y(x)=\sum_{k=0}^{\infty}\frac{d_k}{k!}(x-x_0-kr)^k \qquad (1.14)$$

ko'rinishda Bruvy qatori orqali qidiramiz, bu yerda bu yerda d_k –noma'lum koeffitsiyentlar, $k=0,1,2,\ldots.$ bu yerda d_k –noma'lum koeffitsiyentlarni aniqlash uchun, (1.14) qatorni yaqinlashuvchi qator deb faraz qilamiz va unda ayrim hisoblashlarni bajaramiz: Yuqoridagi misolda ko'rdikki

$$y'(x)=\sum_{k=0}^{\infty}\frac{d_{k+1}}{k!}[x-x_0-kr]^k$$

ko'rinishda yoziladi; shunga o'hshash

$$y''(x)=\sum_{k=0}^{\infty}\frac{kd_{k+1}}{k!}[x-x_0-(k+1)r]^{k-1}=$$

$$=\sum_{k=1}^{\infty}\frac{kd_{k+1}}{k!}[x-x_0-(k+1)r]^{k-1}=$$

$$=\sum_{k=0}^{\infty}\frac{d_{k+2}}{k!}[x-x_0-(k+2)r]^k$$

yoki

$$y''(x)=\sum_{k=0}^{\infty}\frac{d_{k+2}}{k!}[x-x_0-(k+2)r]^k$$

kelib chiqadi va hokazo

$$y^{(n-1)}(x)=\sum_{k=0}^{\infty}\frac{d_{k+n-1}}{k!}[x-x_0-(k+n-1)r]^k$$

ekanligidan

$$y^{(n)}(x)=\sum_{k=0}^{\infty}\frac{d_{k+n}}{k!}[x-x_0-(k+n)r]^k$$

hosil bo'ladi.

Bu tengliklar va (1.14) dan

$$y'(x-(n-1)r)=\sum_{k=0}^{\infty}\frac{d_{k+1}}{k!}[x-x_0-(k+n)r]^k,$$

$$y''(x-(n-2)r)=\sum_{k=0}^{\infty}\frac{d_{k+2}}{k!}[x-x_0-(k+n)r]^k$$

va hokazo

$$y^{(n-1)}(x-r)=\sum_{k=0}^{\infty}\frac{d_{k+n-1}}{k!}[x-x_0-(k+n)r]^k,$$

$$y^{(n)}(x)=\sum_{k=0}^{\infty}\frac{d_{k+n}}{k!}[x-x_0-(k+n)r]^k$$

va

ISSN 2181-3187

$$y(x-nr)=\sum_{k=0}^{\infty}\frac{d_k}{k!}[x-x_0-(k+n)r]^k$$

lar kelib chiqadi.

$y(x-nr), y^{'}(x-(n-1)r),\ldots, y^{n-1}(x-r), y^{n}(x)$

larning, yuqorida hisoblab chiqilgan, ifodalarini (1.12) ga qo'yib va ayrim hisoblashlardan keyin

$$\sum_{k=0}^{\infty}\frac{b_{k+n}+a_{n-1}b_{k+n-1}+\cdots+a_1b_{k+1}+a_0b_k}{k!}[x-x_0-(k+n)r]^k=0$$

tenglikka kelamiz. Bu tenglik o'rinli bo'lishi uchun $[x-x_0-(k+n)r]^k$ oldida turgan koeffitsiyentlar nolga teng bo'lishi yetarlidir. Demak,

$$b_{k+n}+a_{n-1}b_{k+n-1}+\cdots+a_1b_{k+1}+a_0b_k=0, \quad k=0,1,2,\ldots \qquad (1.15)$$

hosil bo'ladi. Bu esa b_{k-} noma'lumlarni aniqlovchi, $a_0\neq 0$ bo'lganda, n chi tartibli rekkurent sistemadan iboratdir.

(1.15) rekkurent sistemaning biror xusuisy yechimini

$$b_k=\xi\cdot\lambda^k \qquad (1.16)$$

ko'rinishda qidiramiz, bu yerda ξ – ihtiyoriy o'zgarmas va λ – noma'lum son va $\lambda\neq 0$.

(1.16) ni (1.15) ga qo'yib va λ^k ga nisbatan gruppalab, so'ngra λ^k oldida turgan koeffitsiyentni nolga tenglashtirib, quyidagi n – darajali λ noma'lumga nisbatan algebraik tenglama hosil bo'ladi:

$$\lambda^n+a_{n-1}\lambda^{n-1}+\cdots+a_1\lambda+a_0=0. \qquad (1.17)$$

(1.17) tenglama, algebraning asosiy teoremasiga asosan, kompleks sonlar maydonida faqat n ta ildizga (karralilarini ham hisobga olganda) ega. U ildizlarni

$$\lambda_1,\lambda_2,\ldots\lambda_n \qquad (1.18)$$

bilan belgilaylik. Bu holda, (1.18) ildizlarni har xil yoki ular orasida karralilari bo'lishiga qarab, (1.17) tenglamaning yechimi o'ziga hos ko'rinishga ega bo'ladi.

a) (1.18) ildizlar har hil yoki barcha $s\neq k$ uchun $\lambda_s\neq\lambda_k$ $(s=1,2,\ldots,n;\ k=1,2,\ldots,n)$ bo'lsin. Bu holda, (1.18) ning umumiy yechimi

$$b_k=\xi_1\lambda_1^k+\xi_2\lambda_2^k+\cdots+\xi_n\lambda_n^k \qquad (1.19)$$

ko'rinishda yoziladi, bu yerda $\xi_1,\xi_2,\ldots,\xi_n$ – ihtiyoriy o'zgarmas sonlar.

(1.15) tenglamaga ahamiyat berilsa, $b_0,b_1,b_2,\ldots,b_{n-1}$ – erkin noma'lum(parametr)lar bo'lib hisoblanadi. Bu erkin noma'lumlar orqali (1.15) ning (1.19) yechimini yozaylik. Buning uchun, (1.19) dagi k ga $0,1,,2,\ldots,n-1$ qiymatlarni ketma-ket beramiz. Ya'ni

ISSN 2181-3187

$$\begin{cases} \xi_1 + \xi_2 + \dots + \xi_n = b_0 \\ \xi_1\lambda_1 + \xi_2\lambda_2 + \dots + \xi_n\lambda_n = b_1 \\ \lambda_1^2\xi_1 + \lambda_2^2\xi_2 + \dots + \lambda_n^2\xi_n = b_2 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ \lambda_1^{n-1}\xi_1 + \lambda_2^{n-1}\xi_2 + \dots + \lambda_n^{n-1}\xi_n = b_{n-1} \end{cases} \qquad (*)$$

$\xi_1, \xi_2, \dots, \xi_n$ larga nisbatan n noma'lumli n ta chiziqli tenglamalar sistemasi hosil bo'ladi. Bu tenglamalar sistemasining noma'lumlar oldida turgan koeffitsiyentlardan tuzilgan

$$\Delta_n = \begin{vmatrix} 1 & 1 & \dots & 1 \\ \lambda_1 & \lambda_2 & \dots & \lambda_n \\ \dots & \dots & \dots & \dots \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \dots & \lambda_n^{n-1} \end{vmatrix}$$

anniqlovchi $s \neq k$ bo'lganda, $\lambda_s \neq \lambda_k \ (s = 1,2,\dots,n;\ k = 1,2,\dots,n)$ bo'lib, $\Delta_n \neq 0$ bo'ladi va u aniqlovchisi deb nomlanadi.

Demak, (*)- Kroneker-Kapelli teoremasiga ko'ra aniq sistemadan iborat va u yagona yechimga ega yoki Kramer qoidasi asosida, (*) ning yechimlari

$$\xi_1 = \frac{\Delta_{n,1}}{\Delta_n}; \ \xi_2 = \frac{\Delta_{n,2}}{\Delta_n}; \dots; \ \xi_n = \frac{\Delta_{n,n}}{\Delta_n}$$

yoziladi, bu yerda

$$\Delta_{n,j} = \begin{vmatrix} 1 & \dots & 1 & b_0 & 1 & \dots & 1 \\ \lambda_1 & & \lambda_{j-1} & b_1 & \lambda_{j+1} & \dots & \lambda_n \\ \dots & \dots & \dots & & \dots & & \\ \lambda_1^{n-1} & \dots & \lambda_{j-1}^{n-1} & \underbrace{b_{n-1}}_{j-\text{устун}} & \lambda_{j+1}^{n-1} & \dots & \lambda_n^{n-1} \end{vmatrix}.$$

Bu aniqlovchini j – ustun elementlari bo'yicha yoysak

$$\Delta_{n,j} = \sum_{k=0}^{n-1} \begin{vmatrix} 1 & \dots & 1 & 1 & \dots & 1 \\ \lambda_1 & \dots & \lambda_{j-1} & \lambda_{j+1} & \dots & \lambda_n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \lambda_1^{k-1} & \dots & \lambda_{j-1}^{k-1} & \lambda_{j+1}^{k-1} & \dots & \lambda_n^{k-1} \\ \lambda_1^{k+1} & \dots & \lambda_{j-1}^{k+1} & \lambda_{j+1}^{k+1} & \dots & \lambda_n^{k+1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \lambda_1^{n-1} & \dots & \lambda_{j-1}^{n-1} & \lambda_{j+1}^{n-1} & \dots & \lambda_n^{n-1} \end{vmatrix} \cdot b_k = \sum_{k=0}^{n-1} \Delta_{n-1,j}^k b_k$$

ko'rinishda yozish imkoni bo'ladi, bu yerda $j = 1,2,\dots,n,$ $\Delta_{n-1,j}^k$ esa $n-1$ chi tartibli aniqlovchi.

$\Delta_{n,j}$ ning ohirgi ifodasini (1.19) ga qo'yib, so'ngra hosil bo'lgan ifodani

ISSN 2181-3187

$b_0, b_1, b_2, \ldots, b_{n-1}$ larga nisbatan gruppalab quyidagini hosil qilamiz:

$$b_k = \frac{\Delta_{n,1}}{\Delta}\lambda_1^k + \frac{\Delta_{n,2}}{\Delta}\lambda_2^k + \cdots + \frac{\Delta_{n,n}}{\Delta}\lambda_n^k =$$

$$= \frac{1}{\Delta}\left\{\sum_{s=0}^{n-1}\Delta_{n-1,1}^s b_s \lambda_1^k + \sum_{s=0}^{n-1}\Delta_{n-1,2}^s b_s \lambda_2^k + \cdots + \sum_{s=0}^{n-1}\Delta_{n-1,n}^s b_s \lambda_n^k\right\} =$$

$$= \frac{1}{\Delta}\Big[(\Delta_{n-1,1}^0 \lambda_1^k + \Delta_{n-1,2}^0 \lambda_2^k + \cdots + \Delta_{n-1,n}^0 \lambda_n^k) b_0 +$$

$$+ (\Delta_{n-1,1}^1 \lambda_1^k + \Delta_{n-1,2}^1 \lambda_2^k + \cdots + \Delta_{n-1,n}^1 \lambda_n^k) b_1 +$$

$$+ \cdots + (\Delta_{n-1,1}^{n-1} \lambda_1^k + \Delta_{n-1,2}^{n-1} \lambda_2^k + \cdots + \Delta_{n-1,n}^{n-1} \lambda_n^k) b_{n-1}\Big] =$$

$$= \frac{1}{\Delta}\left[\sum_{s=0}^{n}\Delta_{n-1,s}^0 \lambda_s^k b_0 + \sum_{s=0}^{n}\Delta_{n-1,s}^1 \lambda_s^k b_1 + \cdots + \sum_{s=0}^{n}\Delta_{n-1,s}^{n-1} \lambda_s^k b_{n-1}\right]$$

yoki

$$b_k = \frac{1}{\Delta}\left[\Delta_{n,1}^{0,k} \cdot b_0 + \Delta_{n,2}^{1,k} \cdot b_1 + \cdots + \Delta_{n,n}^{n-1,k} \cdot b_{n-1}\right] \qquad (1.20)$$

ko'rinishni oladi, bu yerda

$$\Delta_{n,s}^{s-1,k} = \begin{vmatrix} 1 & \ldots & 1 & \lambda_1^k & 1 & \ldots & 1 \\ \lambda_1 & \ldots & \lambda_{s-1} & \lambda_2^k & \lambda_{s+1} & \ldots & \lambda_n \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \lambda_1^{n-1} & \ldots & \lambda_{s-1}^{n-1} & \lambda_n^k & \lambda_{s+1}^{n-1} & \ldots & \lambda_n^{n-1} \end{vmatrix}$$

$s = 1,2,\ldots,n,\ k = 1,2,\ldots,n,\ \ \Delta_{n,s}^{s-1,k} \neq 0$ bo'lgan $n-$tartibli aniqlovchidan iborat.

(1.20) ni (1.14) ga qo'yib, (1.12) ning aniq ko'rinishdagi xususiy yechimni topamiz:

$$y(x) = \sum_{k=0}^{\infty}\frac{1}{k!\cdot\Delta}\left[\Delta_{n,1}^{0,k} \cdot b_0 + \Delta_{n,2}^{1,k} \cdot b_1 + \cdots + \Delta_{n,n}^{n-1,k} \cdot b_{n-1}\right](x - x_0 - kr)^k, \qquad (1.21)$$

bu yerda $b_0, b_1, b_2, \ldots, b_{n-1}-$erkin parametrlar (1.12) tenglamaga qo'yilgan (1.13) boshlang'ich shartlar asosida bir qiymatli aniqlanadi. Haqiqatdan ham, $x = x_0$ da (1.13) shartlar o'rinli bo'lsa, u holda (1.21) dan $y'(x), y''(x), \ldots, y^{(n-1)}(x)$ larni aniqlab, so'ngra ular $x = x_0$ da hisoblansa $b_0, b_1, b_2, \ldots, b_{n-1}$ nomalumlarga nisbatan n noma'lumli n ta quyidagi tenglamalar sistemasi

$$\begin{cases} C_{0,1}b_0 + C_{0,2}b_1 + \cdots + C_{0,n}b_{n-1} = \Delta \cdot y_0 \\ C_{1,1}b_0 + C_{1,2}b_1 + \cdots + C_{1,n}b_{n-1} = \Delta \cdot y_1 \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ C_{n-1,1}b_0 + C_{n-1,2}b_1 + \cdots + C_{n-1,n}b_{n-1} = \Delta \cdot y_{n-1} \end{cases} \qquad (1.22)$$

hosil bo'ladi, bu yerda

$$C_{j-1,s} = \sum_{k=0}^{\infty}\frac{\left[-(k+j-1)r\right]^k}{k!}\Delta_{n,s}^{s-1,k+j-1};$$

ISSN 2181-3187

$j=1,2,\dots,n,\ s=1,2,\dots,n.$

(1.22) tenglamalar sistemasining asosiy aniqlovchisi

$$D_n=\begin{vmatrix} C_{0,1} & C_{0,2} & \cdots & C_{0,n} \\ C_{1,1} & C_{1,2} & \cdots & C_{1,n} \\ \dots & \dots & \dots & \dots \\ C_{n-1,1} & C_{n-1,2} & \cdots & C_{n-1,n} \end{vmatrix}$$

ni $s\neq j$ bo'lganda $\lambda_s\neq\lambda_j$ $(s=1,2,\dots,n$ va $j=1,2,\dots,n)$ bo'lgani uchun $D_n\neq 0$ bo'lishini bevosita tasdiqlash mumkin. Demak, (1.22) aniq sistemadan iborat yoki u yagona yechimga ega yoki u yechim

$$b_0=\frac{D_1}{D}, b_1=\frac{D_1}{D},\dots,b_{n-1}=\frac{D_{n-1}}{D} \qquad (1.23)$$

ko'rinishda yoziladi, bu yerda

$$D_s=\begin{vmatrix} C_{0,1}\dots & C_{0,s-1} & \Delta\cdot y_0 & C_{0,s+1} & \cdots & C_{0,n} \\ C_{1,1}\dots & C_{1,s-1} & \Delta\cdot y_1 & C_{1,s+1} & \cdots & C_{1,n} \\ \dots\ \dots & \dots & \dots & \dots & \dots & \dots \\ C_{n-1,1}\dots & C_{n-1,s-1} & \Delta\cdot y_{n-1} & C_{n-1,s+1} & \cdots & C_{n-1,n} \end{vmatrix},\ s=1,2,\dots,n.$$

(1.23) ni (1.21) ga qo'yib talab qilingan yechimni olamiz.

$b)$ Aniqlik uchun, λ_1 s karrali ildizdan iborat bo'lib, $\lambda_s,\lambda_{s+1},\dots,\lambda_n$ – har xil ildizlardan iborat bo'lsin. Bu holda, λ_1 s karrali ildizga (1.17) ning

$$\lambda_1^k(C_0+C_1k+C_2k^2+\dots+C_{s-1}k^{s-1})$$

xususiy yechimi mos kelgani uchun, uning umumiy yechimi

$$b_k=\lambda_1^k(C_0+C_1k+C_2k^2+\dots+C_{s-1}k^{s-1})+C_s\lambda_s^k+C_{n-1}\lambda_n$$

ko'rinishda yoziladi, bu yerda $C_0,C_1,C_2,\dots,C_{s-1},C_s,\dots,C_{n-1}$ lar n ta ihtiyoriy o'zgarmas sonlardan iborat. Bu holda (1.12) ning (1.13) shartlarni qanoatlantiruvchi yechimini aniqlash $a)$ dagi hisoblashlar asosida olib boriladi.

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