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LINEAR-FRACTIONAL PROGRAMMING IN SOLVING ECONOMIC PROBLEMS

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Abstract: In the paper, we consider and solve the problem of linear-fractional programming, which can be used to find the optimal solution to an economic problem.

Keywords: linear-fractional programming, optimal production volumes, the objective function, constraints, solution polyhedron, graphical method.

In many situations, economists have to solve problems in which one of the indicators must be maximized, while others must be minimized. The simplest examples of problems of this kind are obtaining the highest ratio of profit to costs, increasing the profitability of production, labor productivity, etc. In such cases, it is convenient to use linear-fractional programming, in which there are methods for solving many optimization problems with concrete quality indicators. In this section of mathematical programming, characteristics are studied, which are mathematically described using linear-fractional functions.

Linear-fractional programming is a generalization of linear programming, and it retains all the terms and methods accepted in linear programming, in particular, the division into a general problem and into special problems.

In general, the problem of linear- fractional programming has the following form:

$$F=\frac{\sum_{j=1}^{n} c_j x_j}{\sum_{j=1}^{n} d_j x_j} \to \max(\min) \qquad (1)$$

under constraints

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i, \ i=1,\ldots,m, \qquad (2)$$

$$x_j \ge 0, \ \ j=1,\ldots,n, \qquad (3)$$

here c_j, d_j, b_i, a_{ij} are known constants, and $\sum_{j=1}^{n} d_j x_j \neq 0$.

In the problems of linear-fractional programming, the volume of output is not predetermined, and this is their value, since we get the optimal production plan for any period and for any volume of output. The linear-fractional objective function, in particular, the variable is determined by the denominator of the objective function $F \neq 0$, and it allows the use of solution methods specific to the linear programming section.

If we consider the problem of determining the optimal production volumes and denote by d_i the profit from the sale of a unit of the *i*-th type of product, then the total profit will be written by the formula $\sum_{j=1}^{n} d_j x_j$. Further, if we denote the production costs of the *i*-th type of product by v_i, then the sum $\sum_{j=1}^{n} v_j x_j$ will represent the total cost of production. If we are interested in maximizing the level of profitability, then the optimality criterion will have the form:

$$F = \frac{\sum_{j=1}^{n} c_j x_j}{\sum_{j=1}^{n} d_j x_j} \to \max$$

when the constraints on the use of resources (2) and (3) are met, assuming that the denominator of the objective function in the domain of determining the system of constraints is not zero.

It is usually assumed that the sum $\sum_{j=1}^{n} d_j x_j > 0$. This condition does not violate the generality of the problem, since if this sum is negative, the minus sign can be attributed to the numerator of the function *F*.

If the problem (1) – (3) has an optimal plan, its objective function (1) takes its maximum (or minimum) value at one of the vertices of the solution polyhedron defined by the system of constraints (2) and (3). If the maximum (respectively, minimum) value of the objective function (1) of problem (1) – (3) takes on more than one vertex of the solution polyhedron, then, as in the case of a linear programming problem, it also reaches it at any point that is a convex combination vertex data.

In the case when a fractional-linear programming problem contains only two variables, it is convenient to use a graphical method to solve it. Consider, for example, the function $F = \frac{6x_1 + 9x_2}{x_1 + x_2}$. This function can represent the cost of two types of products, while the numerator will mathematically express the costs of producing these products. It is natural to minimize this function, that's why we consider the problem:

$$F = \frac{6x_1 + 9x_2}{x_1 + x_2} \to \min .$$

Let it be known that the variables x_1 and x_2 have to satisfy the following constraints:

$$\begin{cases} 4x_1 + 8x_2 \le 27, \\ 2x_1 + 3x_2 \ge 6, \\ 12x_1 + 5x_2 \le 40, \end{cases}$$
$$x_1, x_2 \ge 0.$$

To solve the problem using a graphical method, we will construct a domain of acceptable solutions. In Figure 1, this is the highlighted quadrilateral.

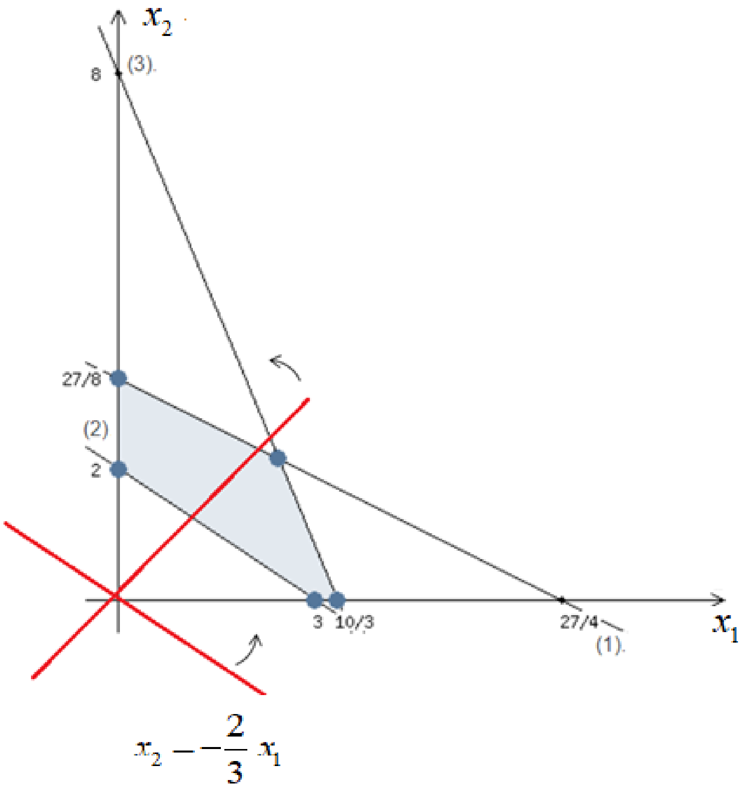

Figure 1.

Now investigate the objective function of the problem. We have

$$x_2 = -\frac{6-F}{9-F}x_1;$$

for $F = 0$ it will be a straight line $x_2 = -\frac{2}{3}x_1$, in the figure it is highlighted in red.

The slope of the objective function as a function of value F, $k(F) = -\frac{6-F}{9-F}$. The derivative of this function $k'(F) = \frac{3}{(9-F)^2} > 0$, hence, the function itself increases with increasing F. Therefore, we find the minimum value of this function by turning the straight line $x_2 = -\frac{2}{3}x_1$ counterclockwise. By performing this action, we obtain that all points of the domain of acceptable solutions lying on its boundary belonging to the axis

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Ox_1 are minimum points, in particular, these are points (3;0) or $\left(\frac{10}{3};0\right)$. Choosing any of them, we define $F_{\min} = 6$.

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