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O‘QUVCHILARGA YASSI FIGURALARNING YUZALARINI TOPISHDA ANIQ INTEGRALLARDAN FOYDALANIB O‘QITISH

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Annatatsiya. Ushbu maqolada dekart va qutb koordinatalar sistemasida turli xil yassi figuralarning yuzini hisoblashda aniq integrallardan foydalanish usullari haqida ma’lumotlar berilgan.

Kalit so’zlar: Aniq integral, yassi figura, qutub koordinatalar sistemasi, Arximed spirali birinchi.

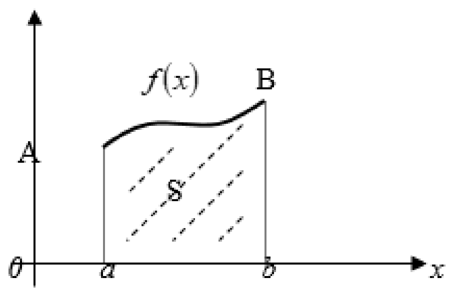
Yassi figuralarning yuzini hisoblashda aniq integralni qo`llashning bir necha hollari mavjud. Bunda chegara funksiyalarining joylashuv vaziyatlari muhim ahamiyatga ega. Ba`zi hollarini ko`rib o`tamiz.

1) Agar $y = f(x)$ funksiya OX o`qining yuqori (manfiy bo`lmagan) qismida joylashgan hamda uzluksiz bo`lib, $x = a$ va $x = b$ to`g`ri chiziq kesmalari bilan chegaralangan bo`lsa, hosil bo`lgan egri chiziqli trapesiya yuzi

y

$$S = \int_a^b y dx \quad \text{yoki} \quad S = \int_a^b f(x)dx \qquad (1)$$

formula yordamida topiladi.



2) ***Misol:*** $y = x^2 + 1$, $y = 0$, $x = -1$, $x = 2$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.

Yechilishi: Shartga asosan figura $y = x^2 + 1$ egri chiziq, absissalar o`qi ( $y = 0$ ) hamda $x = -1$ **va** $x = 2$ to`g`ri chiziqlar bilan chegaralangan. U holda, (1) formuladan foydalanib, quyidagi integralni hisoblaymiz:

$$S = \int_{-1}^{2} (x^2 + 1)dx = \left(\frac{x^3}{3} + x \right)\Bigg|_{-1}^{2} = \frac{2^3}{3} - \frac{(-1)^3}{3} + 2 - (-1) = 6.$$

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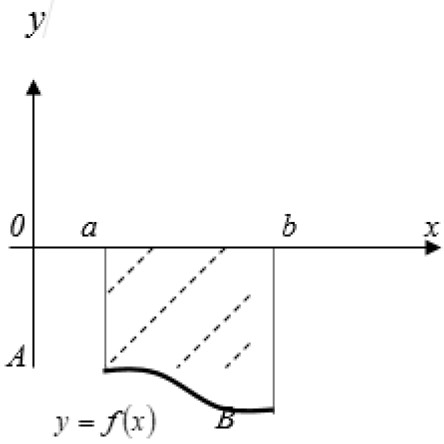
Demak, berilgan egri chiziqli trapesiyasimon figuraning yuzi 6 ga teng ekan.

2) Agar $y = f(x)$ funksiya OX o`qining pastki qismida joylashgan hamda uzluksiz bo`lib, $x = a$ va $x = b$ to`g`ri chiziq kesmalari bilan chegaralangan bo`lsa, hosil bo`lgan egri chiziqli trapesiyasimon figuraning yuzi quyidagi formula yordamida topiladi:



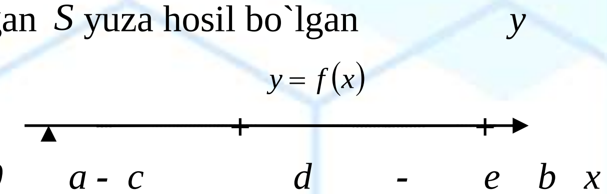
$$S = -\int_a^b y dx \quad \text{yoki} \quad S = -\int_a^b f(x)dx.$$

Misol: $y = -x^2 - 2,\quad y = 0,\ x = -1,\quad x = 1$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.

Yechilishi: Berilgan masalani yechish uchun (2) formuladan foydalanib, chegaralari -1 va 1 dan iborat bo`lgan quyidagi aniq integralni hisoblaymiz:

$$S = -\int_{-1}^{1}\left(-x^2 - 2\right)dx = \int_{-1}^{3}\left(x^2 + 2\right)dx = \left(\frac{x^3}{3} + 2x\right)\Bigg|_{-1}^{1} = 3\frac{2}{3}.$$

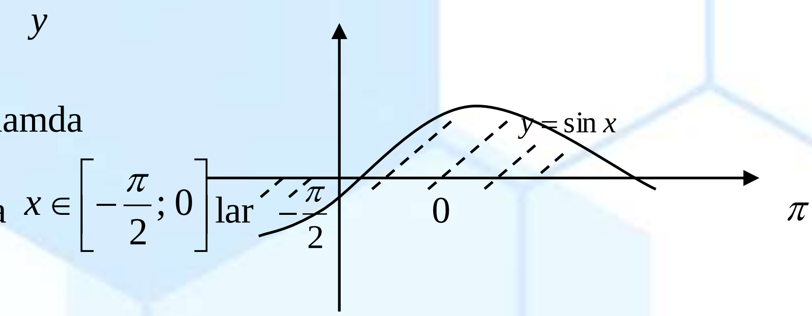
3) $y = f(x)$ uzluksiz funksiya grafigi $[a, b]$ kesmada OX o`qini chekli sondagi nuqtalarda kesib o`tsin. U holda, $[a, b]$ kesma funksiyaning ishorasi almashinishiga asoslanib, bir xil ishorali qismlari alohida –alohida kesmachalarga ajratiladi, ya`ni $[a; c]$, $[c; d]$, $[d; e]$ va $[e, b]$. U holda izlangan $S$ yuza hosil bo`lgan yuzachalarning algebraik yig`indisidan iborat bo`ladi. Bunda qism funksiyalarning ishoralari e`tiborda bo`ladi. Izlanayotgan S yuza quyidagi integrallarning algebraik yiqindilari yordamida topiladi:



$$S = -\int_a^c f(x)dx + \int_c^d f(x)dx - \int_d^e f(x)dx + \int_e^b f(x)dx. \qquad (3)$$

Misol: $y = \sin x,\ y = 0,\ x = -\frac{\pi}{2}$ va $x = \pi$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.

Yechilishi: Berilganlarga hamda chizmalarga asosan barcha $x \in \left[-\frac{\pi}{2}; 0\right]$ lar



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uchun $\sin x \le 0$ va barcha $x \in [0; \pi]$ lar uchun $\sin x \ge 0$ dir. U holda, (3) formulaga asosa:

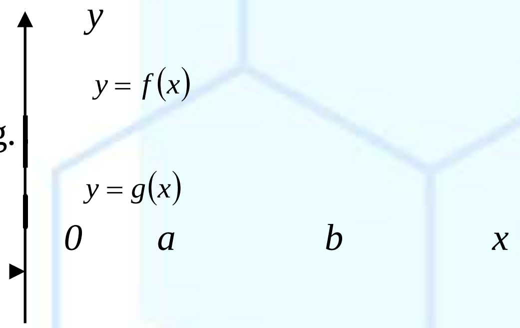
$$S = -\int_{\frac{\pi}{2}}^{0} \sin x dx + \int_{0}^{\pi} \sin x dx = \cos x\Big|_{\frac{\pi}{2}}^{0} - \cos x\Big|_{0}^{\pi} =$$

$$= \cos 0 - \cos\left(-\frac{\pi}{2}\right) - \cos\pi + \cos 0 = 1 - 0 - (-1) + 1 = 3.$$

4) Agar figura $[a, b]$ kesmada ikkita uzluksiz $f(x)$ va $g(x)$ funksiyalar, $x = a$ hamda $x = b$ to\`g\`ri chiziqlar bilan chegaralangan bo\`lsa, uning yuzi quyidagi formula yordamida hisoblanadi:

$$S = \int_a^b (f(x) - g(x))dx. \qquad (4)$$

Bunda $f(x) \ge g(x)$ va $a \le x \le b$ dir.



Misol: $y = x + 2$ va $y = x^2$ chiziqlar bilan chegaralangan figuraning yuzini toping.

Yechilishi: Integrallash chegaralarini, ya\`ni $a$ va $b$ ni berilgan chiziq tenglamalarini o\`zaro tenglashtirib, topamiz:

$$x + 2 = x^2, \quad x^2 - x - 2 = 0.$$

Bundan, $x_1 = -2$, $x_2 = 1$ yani $a = -2$, $b = 1$. U holda, (4) formulaga asosan:

$$S = \int_{-2}^{1} \left((x+2) - x^2\right)dx = \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)\Bigg|_{-2}^{1} = \frac{1}{2} - \frac{1}{2} + 2 - (-4) - \frac{1}{3} - \left(-\frac{8}{3}\right) = 8\frac{1}{3}.$$

Demak, izlanayotgan figuraning yuzasi $S = 8\frac{1}{3}$ dan iborat ekan.

Quyida ba\`zi egri chiziqli figuralarning yuzalarini topish formulalarni qaraymiz.

Ellipsning yuzi

Ma\`lumki, ellipsning tenglamasi

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (5)$$

dan iborat. Ellipsni 4 ta chorakka ajratib, uning bir bo\`lagi, ya\`ni $\frac{1}{4}S$ ni topish yetarlidir. (5) ga asosan

$$y = \frac{b}{a}\sqrt{a^2 - x^2}. \qquad (6)$$

(1) formulaga asosan $\frac{1}{4}S=\int_a^b ydx=\frac{b}{a}\int_0^a\sqrt{a^2-x^2}dx.$

Quyidagi almashtirishlar olamiz: $x=a\sin t,\quad dx=a\sin tdt.$

U holda, integralning yangi chegaralarini aniqlaymiz: $0=a\sin t$ va $a=a\sin t$ lardan $\alpha=0$ va $\beta=\frac{\pi}{2}$. Bulardan ,

$$\frac{1}{4}S=\frac{b}{a}\int_0^{\frac{\pi}{2}}\sqrt{a^2-a^2\sin^2 t}\cdot a\cos tdt=ab\int_0^{\frac{\pi}{2}}\cos^2 tdt=\frac{ab}{2}\int_0^{\frac{\pi}{2}}(1+\cos 2t)dt=$$

$$=\frac{ab}{2}\left(1+\frac{1}{2}\sin 2t\right)\Bigg|_0^{\frac{\pi}{2}}=\frac{ab}{2}\cdot\frac{\pi}{2}=\frac{\pi}{4}ab.$$

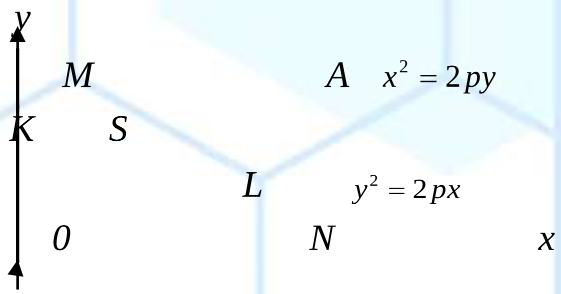
Demak, $\frac{1}{4}S=\frac{\pi}{4}ab.$ Bundan, $S=\pi ab.$ (7)

Ellips yuzini topishning umumiy fomulasi quyidagi ko`rinishda bo`ladi:

$$S=2\int_{-a}^{a}\frac{b}{a}\sqrt{a^2-x^2}dx=\pi ab. \quad (8)$$

Ikkita parbolaning kesishmasidan hosil bo`lgan figuraning yuzi

$y^2=2px$ va $x^2=2py$ parabolalar berilan. S yuza $OKAN$ va $OLAN$ yuzalar ayirmasiga teng. Berilgan paralar $O(0;0)$ va $A(2p;2p)$ nuqtalarda kesishishadi. Shuning uchun



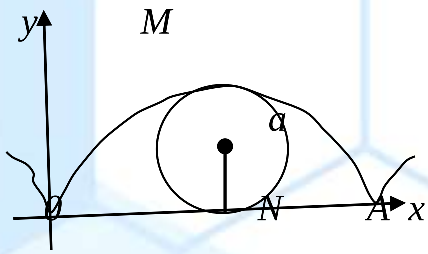
$$S=\int_0^{2p}\sqrt{2px}dx-\int_0^{2p}\frac{x^2}{2p}dx=\frac{4}{3}p^2=\frac{(2p)^2}{3}. \quad (9)$$

Demak, izlangan $OKALO$ yuza $OMAN$ kvadratining uchdan bir qismidan iborat.

Sikloidaning yuzi

$x=a(t-\sin t)$ va $y=a(1-\cos t)$ berilgan bo`lsa,

$$S_{OMANO}=\int_0^{2\pi}ydx=a^2\int_0^{2\pi}(1-\cos t)^2dt=3\pi a^2. \quad (10)$$



Demak, sikloidaning yuzi $S=3\pi a^2$ iborat ekan .

Qutb koordinatalarida yuzani topish

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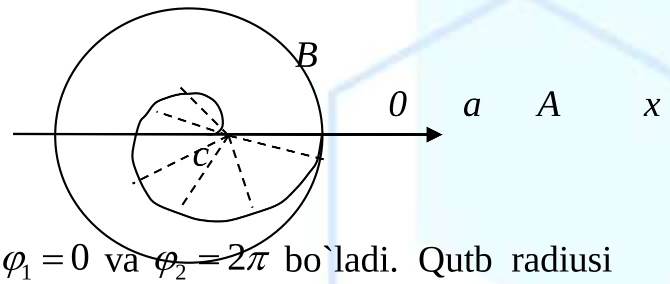
AOB sektor AB yoy, OA va OB nurlar bilan chegaralangan bo`lsin. Bunday sektorning yuzi qutb koordinatalarida quyidagi formula yordamida topiladi:

$$S = \frac{1}{2}\int_{\varphi_1}^{\varphi_2} r^2 d\varphi. \qquad (11)$$

Bunda r -qutb radiusi, φ -qutb radiusining OX o`q bilan tashkil qilgan burchagi, ya`ni qutb burchagi.

Arximed spirali birinchi o`ramining $OX$ o`qi bilan chegaralangan qismining yuzi

Arximed spiralining birinchi o`rami $O$ nuqtadan (qutb markazidan) boshlanib, $A$ nuqtada tugagan bo`lsin.



U holda, qutb burchaklari $\varphi_1 = 0$ va $\varphi_2 = 2\pi$ bo`ladi. Qutb radiusi

$$r = \frac{a}{2\pi}\varphi \qquad (12)$$

dan iboratdir. Bunda a - spiral qadami, ya`ni $OA = a$.

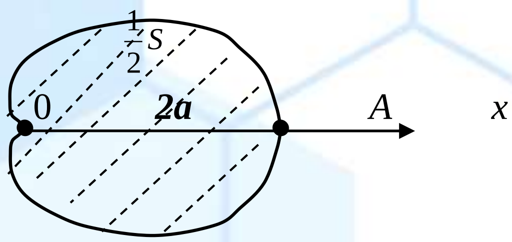
(11) formulaga asosan BCA egri chiziq va OA spiral qadami bilan chegaralangan figuraning yuzi quyidagi formula yoramida hisoblanadi:

$$S = \frac{1}{2}\int_0^{2\pi} r^2 d\varphi = \frac{a^2}{8\pi^2}\int_0^{2\pi} \varphi^2 d\varphi = \frac{1}{3}\pi a^2. \qquad (13)$$

Kardioidaning yuzi

$\rho = a(1+\cos\varphi)$- kardioda bilan chegaralangan yuzani hisoblash talab qilinsin. Ma`lumki, kardoida $\rho = a(1+\cos\varphi)$

OX qutb o`qiga nisbatan simmetrik.



Shuning uchun uning yuqori qismi yuzasini topib, natija ikkilantirilsa, yetarli bo`ladi. U holda,

$$\frac{1}{2}S=\frac{1}{2}\int_0^{\pi}\rho^2 d\varphi. \qquad (14)$$

Bundan,

$$S=\int_0^{\pi}\rho^2 d\varphi=a^2\int_0^{\pi}(1+\cos\varphi)^2 d\varphi=a^2\left(\int_0^{\pi}d\varphi+2\int_0^{\pi}\cos\varphi d\varphi+\int_0^{\pi}\cos^2\varphi d\varphi\right). \qquad (15)$$

$$\int_0^{\pi}d\varphi=\pi, \quad \int_0^{\pi}\cos\varphi d\varphi=\sin\varphi\Big|_0^{\pi}=0 \quad \text{hamda}$$

$$\int_0^{\pi}\cos^2\varphi d\varphi=\frac{1}{2}\int_0^{\pi}(1+\cos2\varphi)d\varphi=\frac{1}{2}\left(\varphi+\frac{1}{2}\sin 2\varphi\right)\Bigg|_0^{\pi}=\frac{\pi}{2}$$ ekanligini hisobga olsak, (15) quyidagidan iborat bo`ladi:

$$S=\int_0^{\pi}\rho^2 d\varphi=a^2\left(\pi+0+\frac{\pi}{2}\right)=\frac{3\pi}{2}a^2. \qquad (16)$$

Demak, kardioidaning yuzi $S=\frac{3\pi}{2}a^2$ ekan.

Xulosa qilib aytganda shuni aytishimiz mumkinki o‘quvchilarga turli xil koordinatalar sistemasida yassi figuralarning yuzasini hisoblashda aniq integrallardan foydalanish bizga qulayliklar yaratadi. Buni yuqoridagi keltirilgan misollarda ham ko‘rsatib o‘tganmiz.

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