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MATEMATIKA DARSLARINI TASHKIL ETISHDA INTERFAOL METODLARNING AMALIY AHAMIYATI

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Annotatsiya. Koshi integral formulasi yordamida yechilishi mumkin bo'lgan misollar, ushbu maqolada chegirmalar nazariyasidan foydalanib yechimlari bayon etilgan.

Tayanch so'zlar: maxsus nuqtalar, golomorf funksiyalar , chegirmalar, Koshining integral formulasi.

ПРАКТИЧЕСКОЕ ЗНАЧЕНИЕ ИНТЕРАКТИВНЫХ МЕТОДОВ В ОРГАНИЗАЦИИ УРОКОВ МАТЕМАТИКИ

Аннотация: Примеры, которые можно решить с помощью интегральной формулы Коши, в этой статье рассматриваются решения этих примеров с использованием вычет.

Ключевые слова: особые точки, голоморфная функция, вычет, интегральная формула Коши.

PRACTICAL SIGNIFICANCE OF INTERACTIVE METHODS IN ORGANIZING MATHEMATICS LESSONS

Abstract: Examples that can be solved using the Cauchy integral formula, this article deals with solving these examples using residue.

Keywords: singular points, holomorphic function, residue, Cauchy integral formula.

Hozirgi kunda barcha ta‟lim muassasalarida o'qitish jarayonida interfaol usullardan foydalanishga erishilmoqda. Bu esa interfaol ta‟lim asosida tashkil etilayotgan pedagogik jarayonlarning mazmun-mohiyatini to'liq tushunib yetishga va ularni samarali, qiziqarli, sifatli bo'lishini ta'minlashga ko'maklashadi. ”Interfaol usullaridagi darslar o'quvchini ijodiy fikrlashga, olingan axborotlarni faollikda hal etishga, fikrni erkin bayon etishga, tashabbuskorlikka, guruhlarda masalalar yechimini topishga, hamkorlikda ish yuritishga, fikrni yozma ravishda bayon etishga chorlaydi.

Talabalar 2 ta guruhga bo'linadi.

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1. O'tilgan mavzu asosida bir nechta muammoli vaziyatlar yaratuvchi misollar beriladi.

2. 1-guruhga $\int\limits_{|z+1|=3} \frac{z-1}{(z-3)^2(z+i)^2}dz$ integralni Koshining integral formulasidan foydalanib hisoblash topshiriladi.

3. 2-guruhga xuddi shu integralni chegirmalar yordamida hisoblash topshiriladi.

4. Keyin 2 ta guruhga ham $\int\limits_{|z|=2,5} \frac{z-1}{(z+2)(z-3)}dz$ integralni 2 xil usulda ishlash topshiriladi.

5. Oxirida natijalar solishtiriladi.

1-guruh $\int\limits_{|z+1|=3} \frac{z-1}{(z-3)^2(z+i)^2}dz$ integralni Koshining integral formulasidan foydalanib hisoblagach, natijani doskaga yozadi. 2-guruh esa xuddi shu integralni chegirmalar yordamida hisoblab, natijani ular ham doskaga yozishadi. Keyin ikki xil usul tahlil qilinadi. Talabalar 2 usulning bir-birlaridan farqlarini o'rganishadi. Keyinchalik o'zlariga qaysi usul qulay bo'lsa shu usuldan foydalanishadi.

Quyida misollarning 2 xil usulda ishlanishi keltirilgan:

1- misol. Koshining integral formulasidan foydalanib quyidagi

$$\int\limits_{|z-2|=5} \frac{e^{z^2}dz}{(z+4)(z-6)}$$

integralni hisoblang.

$|z-2|=5$ aylana bilan chegaralangan sohani D deb belgilaymiz.

$F(z)=\frac{e^{z^2}}{(z+4)(z-6)}$ deb belgilasak, $z_1 = 6 \in D$ va $z_2 = -4 \notin D$

$$\Rightarrow F(z)=\frac{e^{z^2}}{(z+4)(z-6)}=\frac{f(z)}{z-6}$$

$f(z)$ funksiya D da golomorf bo'lganligi uchun Koshining integral formulasiga muvofiq

$$\int\limits_{|z-2|=5} \frac{e^{z^2}dz}{(z+4)(z-6)}=\int\limits_{|z-2|=5} F(z)dz=\int\limits_{|z-2|=5} \frac{f(z)}{z-6}dz=2\pi i f(6)=2\pi i\frac{e^{36}}{10}=\frac{e^{36}\pi i}{5}.$$

2- misol. Koshining integral formulasidan foydalanib quyidagi

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$$\int\limits_{|z|=2,5}\frac{z-1}{(z+2)(z-3)}dz$$

integralni hisoblang.

$|z| = 2,5$ aylana bilan chegaralangan sohani D deb belgilaymiz.

$F(z)=\dfrac{z-1}{(z+2)(z-3)}$ deb belgilasak, $z_1 = -2 \in D,\ z_2 = 3 \notin D.$

$\Rightarrow F(z)=\dfrac{z-1}{(z+2)(z-3)}=\dfrac{f(z)}{z+2}$

$f(z)$ funksiya D da golomorf bo'lganligi uchun Koshining integral formulasiga muvofiq

$$\int\limits_{|z|=2,5}\frac{z-1}{(z+2)(z-3)}dz=\int\limits_{|z|=2,5}\frac{f(z)dz}{z+2}=2\pi i f(-2)=2\pi i\cdot\frac{3}{5}=\frac{6\pi i}{5}.$$

3- misol. Koshining integral formulasidan foydalanib quyidagi

$$\int\limits_{|z+1|=3}\frac{z-1}{(z-3)^2(z+i)^2}dz$$

integralni hisoblang.

$|z+1| = 3$ aylana bilan chegaralangan sohani D deb belgilaymiz.

$F(z)=\dfrac{z-1}{(z-3)^2(z+i)^2}$ deb belgilasak, $z_1 = -i \in D,\ z_2 = 3 \notin D.$

$\Rightarrow F(z)=\dfrac{z-1}{(z-3)^2(z+i)^2}=\dfrac{f(z)}{(z+i)^2}.$

$f(z)$ funksiya D da golomorf bo'lganligi uchun Koshining integral formulasiga muvofiq

$$\int\limits_{|z+1|=3}\frac{z-1}{(z-3)^2(z+i)^2}dz=\int\limits_{|z+1|=3}\frac{f(z)}{(z+i)^2}dz=2\pi i\cdot f'(-i)=\frac{2\pi i(i+1)}{(i+3)^3}$$

Koshining integral formulasidan foydalanib hisoblagan integrallarimizni chegirmalar yordamida hisoblaymiz:

1.1-misol. Ushbu

$$\oint_{|z-2|=5} \frac{e^{z^2} dz}{(z+4)(z-6)}$$

integralni hisoblang.

Bu holda integral ostidagi funksiya

$$f(z) = \frac{e^{z^2}}{(z+4)(z-6)}.$$

Integrallash konturi $\{z \in C : |z-2| = 5\}$ aylana, D soha esa $D = \{z \in C : |z-2| < 5\}$ doiradan iborat.

$a_1 = -4$, $a_2 = 6$ funksiyaning 1-tartibli qutb nuqtalari ekanini aniqlaymiz. Ravshanki, a_2 maxsus nuqtalar D sohaga tegishli bo'ladi. Koshi teoremasining barcha shartlari bajarilib, shu teoremaga ko'ra

$$\oint_{|z-2|=5} \frac{e^{z^2} dz}{(z+4)(z-6)} = 2\pi i \operatorname*{res}_{z=a_2} \frac{e^{z^2}}{(z+4)(z-6)}$$

bo'ladi.

O'ng tomondagi chegirmani (1) formulaga ko'ra hisoblaymiz:

$$\operatorname*{res}_{z=a_2} f(z) = \operatorname*{res}_{z=6} \frac{e^{z^2}}{(z+4)(z-6)} = \lim_{z\to 6}(z-6)\cdot f(z) = \lim_{z\to 6} \frac{e^{z^2}}{z+4} = \frac{e^{36}}{10},$$

Natijada

$$\oint_{|z-2|=5} \frac{e^{z^2} dz}{(z+4)(z-6)} = 2\pi i \cdot \frac{e^{36}}{10} = \frac{e^{36}\pi i}{5}$$

bo'lishini topamiz.

1.2-misol. Ushbu

$$\oint_{|z|=2,5} \frac{z-1}{(z+2)(z-3)} dz$$

integralni hisoblang.

Bu holda integral ostidagi funksiya

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$$f(z)=\frac{z-1}{(z+2)(z-3)}.$$

Integrallash konturi $\{z\in C:|z|=2,5\}$ aylana, D soha esa $D=\{z\in C:|z|<2,5\}$ doiradan iborat

$a_1=-2,\ a_2=3$ funksiyaning 1-tartibli qutb nuqtalari ekanini aniqlaymiz. Ravshanki, a_1 maxsus nuqtalar D sohaga tegishli bo'ladi. Koshi teoremasining barcha shartlari bajarilib, shu teoremaga ko'ra

$$\oint_{|z|=2,5}\frac{z-1}{(z+2)(z-3)}dz=2\pi i\operatorname*{res}_{z=a_1}\frac{z-1}{(z+2)(z-3)}$$

bo'ladi.

O'ng tomondagi chegirmani (1) formulaga ko'ra hisoblaymiz:

$$\operatorname*{res}_{z=a_1}f(z)=\operatorname*{res}_{z=-2}\frac{z-1}{(z+2)(z-3)}=\lim_{z\to-2}(z+2)\cdot f(z)=\lim_{z\to-2}\frac{z-1}{z-3}=\frac{3}{5},$$

Natijada

$$\oint_{|z|=2,5}\frac{z-1}{(z+2)(z-3)}=2\pi i\cdot\frac{3}{5}=\frac{6\pi i}{5}$$

bo'lishini topamiz.

1.3-misol. Ushbu

$$\oint_{|z+1|=3}\frac{z-1}{(z-3)^2(z+i)^2}dz$$

integralni hisoblang.

Bu holda integral ostidagi funksiya

$$f(z)=\frac{z-1}{(z-3)^2(z+i)^2}.$$

Integrallash konturi $\{z\in C:|z+1|=3\}$ aylana, D soha esa $D=\{z\in C:|z+1|<3\}$ doiradan iborat

$a_1 = 3,\ a_2 = -i$ funksiyaning 2-tartibli qutb nuqtalari ekanini aniqlaymiz. Ravshanki, a_2 maxsus nuqtalar D sohaga tegishli bo'ladi. Koshi teoremasining barcha shartlari bajarilib, shu teoremaga ko'ra

$$\oint_{|z+1|=3} \frac{z-1}{(z-3)^2(z+i)^2}dz = 2\pi i \operatorname*{res}_{z=a_2} \frac{z-1}{(z-3)^2(z+i)^2}$$

bo'ladi.

O'ng tomondagi chegirmani (3) formulaga ko'ra hisoblaymiz:

$$\operatorname*{res}_{z=a_2} f(z) = \operatorname*{res}_{z=-i} \frac{z-1}{(z-3)^2(z+i)^2} = \lim_{z\to -i} \frac{d}{dz}\left[(z+i)^2 \cdot f(z)\right] = \lim_{z\to -i}\left[\frac{z-1}{(z-3)^2}\right]' = \frac{i+1}{(i+3)^3}$$

,

Natijada

$$\oint_{|z+1|=3} \frac{z-1}{(z-3)^2(z+i)^2} = 2\pi i \cdot \frac{i+1}{(i+3)^3} = \frac{2\pi(i+1)}{(i+3)^3}$$

bo'lishini topamiz.

Foydalanilgan adabiyotlar:

1. Sadullayev A., Xudoyberganov G., Mansurov X., Vorisov A., Tuychiyev T. Matematik analiz kursidan misol va masalalar to'plami. 3-qism (kompleks analiz) "O'zbekiston", 2000.
2. Волковыский Л.И., Лунц Г.Л., Араманович И.Г. Сборник задач по теории функций комплексного пременного. 3-nashri. – M. "Наука", 1975.
3. Xudoyberganov G., Vorisov A., Mansurov X. Kompleks analiz. (ma'ruzalar). T, "Universitet", 1998.
4. Шабат Б.В. Введение в комплексный анализ. 2-nashri, 1-ч.-M, "Наука", 1976.
5. E. M. Mahkamov, S. D. Eshmetova. CHEGIRMALAR YORDAMIDA XOSMAS INTEGRALLARNI HISOBLASH USULLARI// ACADEMIC RESEARCH IN EDUCATIONAL SCIENCES VOLUME 2 | ISSUE 9 | 2021 p. 91-100