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FUNKSIONAL ANALIZ FANIGA OID BA'ZI MISOLLARNI YECHISH USULLARI

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Iqtisod va buxgalteriya kafedrasi o'qituvchisi

1. $f:[a,b]\to\mathbb{R}, g:[a,b]\to\mathbb{R}$ o'suvchi funksiyalar bo'lsa, u holda

$$(b-a)\int_a^b f(x)\cdot g(x)dx \ge \int_a^b f(x)dx\cdot\int_a^b g(x)dx$$

tengsizlik o'rinli.

Isbot: $f(x)$ va $g(x)$ lar o'suvchi funksiyalar bo'lgani uchun $\big(f(x)-f(y)\big)\big(g(x)-g(y)\big)\ge 0$ munosabat o'rinli bundan $\int_a^b\left(\int_a^b\big(f(x)-f(y)\big)\big(g(x)-g(y)\big)dx\right)dy\ge 0$ ekanligi kelib chiqadi. Kavslarni ochib chiqsak:

$$\int_a^b\left(\int_a^b f(x)g(x)dx\right)dy+\int_a^b\left(\int_a^b f(y)g(y)dy\right)dx-\int_a^b f(x)dx\int_a^b g(y)dy-\int_a^b f(y)dy\int_a^b g(x)dx\ge 0$$

Bundan quyidagi tengsizlikni olamiz:

$$(b-a)\int_a^b f(x)\cdot g(x)dx \ge \int_a^b f(x)dx\cdot\int_a^b g(x)dx$$

Isbot tugadi.

2. Agar T xosmas matritsa bo'lib, $A=\{x\in\mathbb{R}^n:(x,Tx)\le 1\}$ bo'lsa, u holda berilgan $p\in\mathbb{R}^n$ vektor uchun $\max\limits_{x\in A}(p,x)$ ni toping. Bunda (p,x) - p va x vektorlarning skalyar ko'paytmasi. *(20 ball)*

Yechish: Aytaylik, $\max\limits_{x\in A}(p,x)=c$ bo'lsin. U holda $(p,x)=c$ tekislik A ga $x_p\in A$ nuqtada urinadi va shu nuqtada maksimum c ga erishadi. Agar biz A ni vektor maydon sifatida qarasak p vektor yo\`nalishi bo\`yicha oqim eng katta bo\`ladi, shuning uchun $(x,T(x))$ ning gradienti $p$ vektorga kolleniar hamda yo\`nalishi bir xil shuning uchun shunday $\lambda>0$ topiladiki quyidagi $\lambda p=\text{grad}(x,T(x))|_{x=x_p}=2Tx_p$ (biror uchun) o\`rinli bo\`ladi (bu yerda biz

$$T\left(a_{ij}\right)_{i,j=1}^{n}, \text{bo\`lsin, u holda } (x,T(x))=x_1(a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n)+\ldots+x_j(a_{j1}x_1+a_{j2}x_2+\ldots+a_{jn}x_n)+$$

$$\ldots+x_n(a_{n1}x_1+a_{n2}x_2+\ldots+a_{nn}x_n)\Rightarrow\frac{\partial(x,T(x))}{\partial x_j}=2(a_{j1}x_1+a_{j2}x_2+\ldots+a_{jn}x_n)\Rightarrow\text{grad}(x,T(x))=2Tx$$

dan foydalandik) demak

$$x_p = \frac{\lambda}{2}T^{-1}p, \qquad (x_p, Tx_p) = \frac{\lambda^2}{4}(T^{-1}p, p) \le 1.$$

Bundan esa,

$$\lambda \le \frac{2}{\sqrt{(T^{-1}p, p)}}, \qquad \text{va} \qquad x_p = \frac{T^{-1}p}{\sqrt{(T^{-1}p, p)}},$$

ekanligi kelib chiqadi. Demak

$$\max_{x \in A}(p, x) = \sqrt{(T^{-1}p, p)}.$$

Javob: $\sqrt{(T^{-1}p, p)}$

3. Masalani koordinatalar sistemasini kiritish yo'li bilan yechamiz. Koordinatalarni quyidagicha kiritaylik: $A(0,0)$, $B(a,0)$, $C(a,b)$, $D(0,b)$ va $M(x,y)(0 \le x \le a,\ 0 \le y \le b)$ bundan $MA = \sqrt{x^2+y^2}$, $MB = \sqrt{(x-a)^2+y^2}$, $MC = \sqrt{(x-a)^2+(y-b)^2}$, $MD = \sqrt{x^2+(y-b)^2}$ va berilgan masala quyidagi $P_\lambda(x,y)$ ko'phadning maximumini topish masalasiga keladi:

$$P_\lambda(x,y) = \left(x^2+y^2\right)^{\frac{\lambda}{2}} + \left((x-a)^2+y^2\right)^{\frac{\lambda}{2}} + \left((x-a)^2+(b-y)^2\right)^{\frac{\lambda}{2}} + \left(x^2+(b-y)^2\right)^{\frac{\lambda}{2}}$$

1 hol: $x=0$ yoki $x=a$ bo'lsin. Demak:

$$P_\lambda(0,y) = P_\lambda(a,y) = y^\lambda + (b-y)^\lambda + \left(a^2+y^2\right)^{\frac{\lambda}{2}} + \left(a^2+(b-y)^2\right)^{\frac{\lambda}{2}}$$

bo'ladi. Shuning uchun $P_\lambda(0,y)$ ni maximumga tekshirish yetarli. Buning uchun uni y bo'yicha hosilalarini hisoblaymiz:

$$P_\lambda'(0,y) = \lambda y^{\lambda-1} - \lambda(b-y)^{\lambda-1} + \lambda y\left(a^2+y^2\right)^{\frac{\lambda}{2}-1} - \lambda(b-y)\left(a^2+(b-y)^2\right)^{\frac{\lambda}{2}-1},$$

$$P_\lambda''(0,y) = \lambda(\lambda-1)y^{\lambda-2} + \lambda(\lambda-1)(b-y)^{\lambda-2} + \lambda\left(a^2+y^2\right)^{\frac{\lambda}{2}-2}\left(a^2+(\lambda-1)y^2\right) +$$

$$+\lambda\left(a^2+(b-y)^2\right)^{\frac{\lambda}{2}-2}\left(a^2+(\lambda-1)(b-y)^2\right)$$

$\lambda \ge 1$ bo'lgani uchun $P_\lambda''(0,y) \ge 0$. $P_\lambda'\left(0, \frac{b}{2}\right) = 0$ va $P_\lambda''(0,y) \ge 0$ bo'lgani uchun:

$$\max_{y \in [0,b]} P_\lambda(0,y) = P_\lambda(0,b) = a^\lambda + b^\lambda + \left(a^2+b^2\right)^{\frac{\lambda}{2}}.$$

2 hol: $x \in (0,a)$ bo'lsin. $P_\lambda(0,y)$ ni y bo'yicha hosilalarini hisoblaymiz:

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$$P_\lambda'(x,y)=\lambda y\left(x^2+y^2\right)^{\frac{\lambda}{2}-1}+\lambda y\left((x-a)^2+y^2\right)^{\frac{\lambda}{2}-1}-\lambda(b-y)\left((x-a)^2+(y-b)^2\right)^{\frac{\lambda}{2}-1}-\lambda(b-y)\left(x^2+(y-b)^2\right)^{\frac{\lambda}{2}-1}$$

$$P_\lambda''(x,y)=\lambda\left(x^2+y^2\right)^{\frac{\lambda}{2}-2}\left(x^2+(\lambda-1)y^2\right)+\lambda\left((x-a)^2+y^2\right)^{\frac{\lambda}{2}-2}\left((x-a)^2+(\lambda-1)y^2\right)+$$

$$+\lambda\left((x-a)^2+(y-b)^2\right)^{\frac{\lambda}{2}-2}\left((x-a)^2+(\lambda-1)(y-b)^2\right)+\lambda\left(x^2+(y-b)^2\right)^{\frac{\lambda}{2}-2}\left(x^2+(\lambda-1)(y-b)^2\right)$$

$\lambda\geq 1$ bo'lgani uchun $P_\lambda''(x,y)\geq 0$. $P_\lambda'\left(x,\frac{b}{2}\right)=0$ va $P_\lambda''(x,y)\geq 0$ bo'lgani uchun:

$$\max_{x\in[0,a]}P_\lambda(x,y)=P_\lambda(x,b)=\left(x^2+b^2\right)^{\frac{\lambda}{2}}+\left((x-a)^2+b^2\right)^{\lambda}+(x-a)^{\lambda}+x^{\lambda}.$$

Oxirgi kelgan ifoda **1 hol**da ko'rilgan ifodaga o'xshash, shuning uchun huddi shunday ish tutib quyidagini olishimiz mumkin:

$$\max_{x\in[0,a]}P_\lambda(x,b)=P_\lambda(a,b)=a^\lambda+b^\lambda+\left(a^2+b^2\right)^{\frac{\lambda}{2}}$$

Demak $\max\limits_{x\in[0,a],y\in[0,b]}P_\lambda(x,y)=P_\lambda(0,0)=P_\lambda(a,0)=P_\lambda(a,b)=P_\lambda(0,b)=a^\lambda+b^\lambda+\left(a^2+b^2\right)^{\frac{\lambda}{2}}$ ekan.

4. V_{16} orqali koordinatalari - 1, 0, 1 qiymatlarni qabul qiluvchi va barcha noldan farqli koordinatalari soni 8 ga teng bo'lgan 16 o'lchamli vektorlar to'plami bo'lsin. O'zaro ortogonal bo'lmagan vektorlari soni 28000 dan kam bo'lmagan shunday W H V_{16} qism to'plam mavjud bo'lishini isbotlang. *(20 ball)*

Isboti: Quyidagi vektorlar to'plamlarini qaraylik:

(1) A_1 - birinchi 5 koordinatasi ichida aniq 3 ta 1 va oxirgi 11 ta koordinatasi ichida aniq 5 ta 1 bor vektorlar to'plami;

(2) A_2 - birinchi 5 ta koordinatasida ichida 4 ta 1 va oxirgi 11 ta koodinatasi ichida 4 ta 1 bor vektorlar to'plami;

(3) A_3 - birinchi 5 ta koordinatasi ichida 4 ta 1 va oxirgi 11 ta koordinatasi ichida 3 ta 1 va bitta -1 bor vektorlar to'plami;

(4) A_4 - birinchi 5 ta koordinatasi ichida 5 ta 1 bor va oxirgi 11 ta koordinatasida 3 ta 1 bor vektorlar to'plami;

(5) A_5 - birinchi 5 ta koordinatasi ichida 5 ta 1 bor va oxirgi 11 ta koordinatasi ichida 2 ta 1, 1 ta -1 bor vektorlar to'plami;

(6) A_6 - birinchi 5 ta koordinatasida 5 ta 1 bor va oxirgi 11 ta koordinatasida 1 ta 1 va 2 ta -1 bor vektorlar to'plami;

U holda

$$A=\bigcup_{i=1}^{6}A_i$$

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to'plamni qarasak ixtiyoriy ikkita vektor o'zaro ortogonal emas(chunki ularni skalyar ko'paytmasi har doim musbat) va

$$|A|=|A_1|+|A_2|+|A_3|+|A_4|+|A_5|+|A_6|=14025$$

keladi. Endi, xuddi shunday A to'plam o'rniga B to'plamni aniqlaymiz-ki, shunday $B_i,(i=1,2,...,6)$ lar ham aniqlanadi va faqat A dagi vektorlarda 1 qo'yilga koordinatalarga -1 va -1 qo'yilganiga esa 1 qo'yib chiqamiz. Ravshan-ki, $|B|=14025$ va B dagi barcha vektorlar o'zaro ortogonal emas(chunki ixtiyoriy ikkita vektorni skalyar ko'paytmasi musbat). Endi A va B lardan ixtiyoriy ikkita vektor olsak, ular ortogonal emas(chunki, ularni skalyar ko'paytmasi har doim manfiy bo'ladi). Demak W sifatida $A\cup B$ ni olsak bo'ladi. W o'zaro ortoganal bo'lmagan vektorlardan iborat va elementlari soni $28050>28000$ bo'ladi.

5. Haqiqiy $a_1,a_2,...,a_n$ sonlar berilgan bo'lsin. Har bir i $(1\le i\le n)$ uchun

$$d_i=\max_{1\le j\le i} a_j-\min_{i\le j\le n} a_j$$

belgilashni kiritamiz. $d=\max_{1\le i\le n} d_i$ bo'lsin. Har qanday $x_1\le x_2\le ...\le x_n$ haqiqiy sonlar uchun

$$\max_{1\le i\le n}|x_i-a_i|\ge\frac{d}{2}$$

tengsizlikni o'rinli bo'lishini isbotlang. *(10 ball)*

Isboti: Ma'lumki, ixtiyoriy $i\in\{1,2,...,n\}$ uchun

$$d_i=\max_{1\le j\le i} a_j-\min_{i\le j\le n} a_j=a_l-x_l+x_k-a_k+x_l-x_k\le|a_l-x_l|+|x_k-a_k|.$$

Bundan,

$$\frac{d_i}{2}\le max(|a_l-x_l|,|x_k-a_k|)$$

Demak, d uchun ham shuni qo'llasak

$$\max_{1\le i\le n}|x_i-a_i|\ge\frac{d}{2}$$

keladi.

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