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FAZOSI KO'PHADDAN IBORAT TEBRANUVCHI INTEGRALLARNI BAHOLASH

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Anotatsiya: Ushbu maqolamizda so'ndirivchi tebranivchi intigralni ba'zi parametrlar bo'yicha baholangan. Bu yerda $K(x)$- x$\in S$ nuqtasidagi giper sirt sathining Gauss egriligi va $y(x)$ sirt o'lchovi , $\varphi \in C_0^\infty(S)$ silliq manfiy bo'lmagan funksiya.

Kalit so'zlar: Maksimal operatorlar, Gauss egriligi, skalyar ko'paytmasi, kordinata boshi, giper sirt, silliq manfiy bo'lmagan funksiya, $detHess\Phi(x_1, x_2), a \in C_0^\infty(V)$ *va* $\xi \neq 0$, *bunda* C q *ga bog'liq doimiy*. Minimal.

Soggy va I.M. Steinni [1] da yuqori sirt $S \subset \mathbb{R}^{n+1}$ bilan bog'liq bo'lgan maksimal operatorlar uchun chegaralanganlik muomosi bilan quyidagi so'ndiruvchili tebranuvchan integralni kiritdilar

$$\hat{\mu}_q(\xi) := \int_S e^{i(\xi,x)} |K(x)|^q \varphi(x) dy(x), \qquad (3.3.1)$$

Bu yerda $K(x)$- x$\in S$ nuqtasidagi giper sirt sathining Gauss egriligi va $y(x)$ sirt o'lchovi , $\varphi \in C_0^\infty(S)$ silliq manfiy bo'lmagan funksiya, (ξ, x)- ξ va x ni skalyar ko'paytmasi. Ular agar $q \geq 2n$ bo'lsa, yuqoridagi (1.1) integralni

$O(|\xi|^{-\frac{n}{2}})$ $\quad$ $(|\xi| \to +\infty)$ kabi kamayishi isbotlangan.

$S \subset \mathbb{R}^n$ silliq giper sirt bo'lsin.Shunday minimal q ni toppish kerakligi talab qilinganki quyidagi baho o'rinli

$$\left| \int_S e^{i(x,\xi)} |K(x)|^q \varphi(x) dy(x) \right| \leq A|\xi|^{-\frac{n}{2}}$$

bunda $\xi \neq 0$ uchun.

Biz C.D. Sogge va E.M. Stein muomosini uch o'lchamli Evklid fazosidagi analitik yuzalari uchun qaradik. Biz S ni $x_3 = \Phi(x_1, x_2)$ funksiyani grafigi sifatida berilgan deb qarashimiz munkin :

$$\{(x_1, x_2) \in V \subset \mathbb{R}^2 : x_3 = \Phi(x_1, x_2), \Phi(x_1, x_2) := x_1^2 + x_1 x_2^n\}$$

Bu yerda $n \geq 2$. Agar $n = 1$ bo'lsa, u holda $\widehat{\mathrm{M}}_q(\xi)$ integral har qanday q uchun bahoga ega bo'ladi, chunki $detHess\Phi(x_1, x_2) \neq 0$. Shuning uchun, $n \geq 2$ deb olamiz.

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Keyin, $detHess\Phi(x_1,x_2)$ funksiya uchun quyidagi tenglik o'rinli

$$\det Hess\Phi(x_1,x_2) = n^2x_2^{2n-2} - 2n(n-1)x_1x_2^{n-2}$$

U holda (3.2.1) integralni quyidagicha shaklda yozish munkin:

$$\hat{\mu}_q(\xi) = n^{2q}\int_{\mathbb{R}^2} e^{i\xi_3(s_1x_1+s_2x_2+x_1^2+x_1x_2^n)}|x_2|^{2q(n-1)}a(x_1,x_2)dx_1dx_2 \quad (3.3.2)$$

Bu yerda $a(x_1,x_2) = \frac{\varphi(x_1,x_2,\Phi(x_1,x_2))}{\sqrt{(1+|\nabla\Phi(x_1,x_2)|^2)^{4q-1}}}$.

Ishning asosiy natijasi quyidagi teoremadan iborat.

Teorema. Faraz qilaylik $q > \frac{1}{2}$ bo'lsin, u holda kordinata boshining shunday V atrofi mavjudki (3.2.1) integral uchun quyidagi baho o'rinli

$$|\hat{\mu}_q(\xi)| \leq \frac{C||a||_{C^3}}{|\xi|},$$

Bu yerda $a \in C_0^\infty(V)$ va $\xi \neq 0$, *bunda* C q *ga bog'liq doimiy.*

Endi, (3.2.2) ni quyidagi parametrlar uchun qarab chiqamiz (ξ_1,ξ_2,ξ_3).

Agar $max\{|\xi_1|,|\xi_2|\} \geq |\xi_3|$ bo'lsa, bizda quyidagi lemma mavjud:

Lemma. $max\{|\xi_1|,|\xi_2|\} \geq |\xi_3|$ *va* $q > 0$ *bo'lganda. U holda* V *atrof mavjudki quyidagi baho o'rinli*

$$\left|\hat{\mu}_q(\xi)\right| \leq \frac{C||a||_{C^3}}{|\xi|} \quad (3.3.3)$$

Bu yerda $a \in C_0^\infty(V)$ *va* $\xi \neq 0$, *bunda* C q *ga bog'liq doimiy. l*emmani isboti analitik ravishda [3] dagi lemma 5 dan kelib chiqadi.

Agar $max\{|\xi_1|,|\xi_2|\} \leq |\xi_3|$ bo'lsa, biz (3.2.2) integralni quyidagicha yozishimiz munkin:

$$\hat{\mu}_q(\xi) := n^{2q}\int_{\mathbb{R}^2} e^{i\xi_3(s_1x_1+s_2x_2+x_1^2+x_1x_2^n)}|x_2|^{2q(n-1)}a(x_1,x_2)dx_1dx_2 \quad (3.3.4)$$

bu yerda $s_1 = \frac{\xi_1}{\xi_3}$ va $s_2 = \frac{\xi_2}{\xi_3}$.

Fubini toremasini qo'llasak

$$\hat{\mu}_q(\xi) := n^{2q}\int_R \widehat{M}_q(\xi,x_2)dx_1$$

Bu yerda

$$\hat{\mu}_q(\xi) := n^{2q}\int_{\mathbb{R}^2} e^{i\xi_3(s_1x_1+s_2x_2+x_1^2+x_1x_2^n)}|x_2|^{2q(n-1)}\left|nx_2^n - 2(n-1)x_1\right|a(x_1,x_2)dx_2$$

Bunda $s_2 = \frac{\xi_2}{\xi_3}$.

quydagicha almashtirish olamiz

$$t = nx_2^n - 2(n-1)x_1$$

(3.3.5)

ISSN 2181-3187

$$x_1=\frac{nx_2^n-t}{2(n-1)}=\frac{nx_2^n}{2(n-1)}-\frac{t}{2(n-1)}$$

$$dx_1=-\frac{1}{2(n-1)}dt$$

Darajani xisoblasak x_1 o'rniga olib borib qo'ysak

$$\hat{\mu}_q(\xi):=n^{2q}\int_{\mathbb{R}^1} e^{i\xi_3 nx_2^n}|x_2|^{2q(n-1)}\left|nx_2^n-2(n-1)x_1\right|a(x_1,x_2)dx_2$$

$$\hat{\mu}_q(\xi):=n^{2q}\int_R e^{i\xi_3\frac{nx_2^n}{2(n-1)}\left(s_1+x_2^n\frac{(n+1)}{2(n-1)}\right)}e^{-o_3\frac{x_2^n t}{4(n-1)^2}}|x_2|^{q(n-2)}\left|t^q\left(-\frac{1}{2(n-1)}\right)dt\right.$$

$$\hat{\mu}_q(\xi):=n^{2q}|x_2|^{q(n-2)}\left|e^{i\xi_3\frac{nx_2^n}{2(n-1)}\left(s_1+x_2^n\frac{(n+1)}{2(n-1)}\right)}\left(-\frac{1}{2(n-1)}\right)\int e^{-o_3\frac{x_2^n t}{4(n-1)^2}}t^q dt\right.$$

Agarda $-i\xi_3\frac{nx_2^n}{4((n-1)^2)}=K$ deb belgilash olsak

$$I_1=\int e^{Kt}t^q dt$$

1 marta bo'laklab intigrallasak

$$\int e^{Kt}t^q dt=\frac{e^{Kt}t^q}{K}-\int\frac{e^{Kt}t^q}{K}qt^{q-1}dt$$

(3.3.6)

q marta intigrallash kerak

$t=X(x_1;x_2)$

$t_1(\varepsilon_1;x_1)\leq t\leq t_1(\varepsilon_2;x_2)$; $0\leq x_1\leq\varepsilon$

bo'lsa $t_1=|nx_2^n|$

$t_2=|nx_2^n-2(n-1)\varepsilon_1|$

$$\int e^{Kt}t^q dt=\frac{e^{K(nx_2^n-2(n-1)\varepsilon}}{K}|nx_2^n-2(n-1)x_1|^q-\frac{e^{Knx_2^n}}{K}(nx_2)^{nq}$$

Endi dx_2 bo'yicha intigrallaymiz.

$$\hat{\mu}_q(\xi)=$$

$$\int_0^\varepsilon\frac{n^{2q}|x_2|^{q(n-2)-n}\left|e^{i\xi_3\frac{nx_2^n}{2(n-1)}\left(s_1+x_2^n\frac{(n+1)}{2(n-1)}-1\right)}\left((nx_2^n-2(n-1)\varepsilon)^q e^{-2(n-1)\varepsilon}-(nx_2)^{nq}\right)\right.}{io_3 n}dx_2$$

Quydagi almashtirish olamiz

$$P(x_2)=\frac{\left((nx_2^n-2(n-1)\varepsilon)^q e^{-2(n-1)\varepsilon}-(nx_2)^{nq}\right)|x_2|^{q(n-2)-n}|}{io_3 n}$$

$$Q(x_2)=e^{i\xi_3\frac{nx_2^n}{2(n-1)}\left(s_1+x_2^n\frac{(n+1)}{2(n-1)}-1\right)}$$

$$\hat{\mu}_q(\xi)=\int_0^\varepsilon e^{q(x_2)}p(x_2)dx_2$$

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Puassonga kura;

$$\hat{\mu}_q(\xi) = \frac{e^{q(x_2)}p(x_2)}{q'(x_2)} - \frac{1}{q(x_2)}\int e^{q(x_2)}\, d\,\frac{p(x_2)}{q(x_2)}$$

1-xadini hisoblasak quydagi natejaga erishamiz

$$\hat{\mu}_q(\xi) = \frac{e^{q(\varepsilon)}\left(n\varepsilon_2^n - 2(n-1)\varepsilon\right)^q e^{-2(n-1)\varepsilon} - (n\varepsilon)^{nq}\right)\left|\varepsilon\right|^{q(n-2)+1}\Big|}{\frac{i\xi_3 n^3}{2(n-1)^2}\left(s_1(n-1) + 2n^2\varepsilon^n - n^2\right)}$$

Agarda $C_q = \frac{e^{q(\varepsilon)}\left(n\varepsilon_2^n - 2(n-1)\varepsilon\right)^q e^{-2(n-1)\varepsilon} - (n\varepsilon)^{nq}\right)\left|\varepsilon\right|^{q(n-2)+1}\Big|}{\frac{in^3}{2(n-1)^2}\left(s_1(n-1)+2n^2\varepsilon^n-n^2\right)}$ belgilash olsak quydagi natejaga erishamiz.

$$|\hat{\mu}_q(\xi)| \leq \frac{C_q}{|\xi|}$$

Ana endi 2-xadini qarab chiqamiz

$$\widehat{\mathrm{M}}_q(\xi) = \frac{e^{q(x_2)}p(x_2)}{q'(x_2)} - \frac{1}{q(x_2)}\int e^{q(x_2)}\, d\,\frac{p(x_2)}{q(x_2)} = \frac{e^{q(x_2)}p(x_2)}{q'(x_2)} - \frac{e^{q(x_2)}p'(x_2)}{q'(x)^2} - \int(\dots)dx$$

$$\frac{e^{q(x_2)}p'(x_2)}{q'(x)^2} =$$

$$\frac{e^{q(\varepsilon)}p'(x)}{\left(\left(\frac{i\xi_3 n^3}{2(n-1)^2}\right)^2 x^{(n-1)}\left(ns_1 + \frac{x_2^n(n+1)}{2(n-1)} - 1\right) + \frac{x_2^n n(n+1)}{2(n-1)}\right)^2}$$

Quydagicha belgilash olsak;

$$C_q = \frac{e^{q(\varepsilon)}p'(x)}{\left(\left(\frac{i\xi_3 n^3}{2(n-1)^2}\right)^2 x^{(n-1)}\left(ns_1 + \frac{x_2^n(n+1)}{2(n-1)} - 1\right) + \frac{x_2^n n(n+1)}{2(n-1)}\right)^2}$$

Quydagi baholashga erishamiz.

$$|\hat{\mu}_q(\xi)| \leq \frac{C_q}{|\xi|}$$

kurinishga keladi.

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Foydalanilgan adabiyotlar

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