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UMUMLASHGAN HARDI TENGSIZLIGIGA OID BA'ZI NATIJALAR

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Annotatsiya. Ushbu maqolada umumlashgan Hardi tengsizligining yadrosi $\{K(n,m)\}$ to'rtinchi darajali ko'phad korinishida bo'lgan holda tengsizlik o'rinli bo'ladigan ekvivalent shartlar ko'rsatilgan va tengsizlikning eng yaxshi konstantasi uchun quyi va yuqori baholar olingan.

Kalit so'zlar: Umumlashgan Hardi tengsizligi, yadro, ketma-ketlik, normaning uchburchak tengsizligi

НЕКОТОРЫЕ РЕЗУЛЬТАТЫ ОТНОСИТЕЛЬНО ОБОБЩЕННОГО НЕРАВЕНСТВА ХАРДИ

Аннотация. В этой статье показаны эквивалентные условия,при которых неравенство будет уместным, если ядро обобщенного неравенства Харди $\{K(n,m)\}$ представлено в виде многочлена четвёртой степени и получены нижние и высшие оценки для лучшей константы неравенства.

Ключевые слова: Обобщенное неравенство Харди, зеленое ядро, последовательность, треугольное неравенство нормы

SOME RESULTS REGARDING GENERALIZED HARDY INEQUALITY

Annotation. This paper shows equivalent conditions under which the inequality will be appropriate if the kernel of the generalized Hardy inequality $\{K(n,m)\}$ is represented as a polynomial of the fourth degree and lower and higher bounds for the best inequality constant are obtained.

Key words: Generalized Hardy inequality, green kernel, sequence, triangular norm inequality

1-§. Kirish

1925 yilda Buyuk Britaniya olimi G.Hardi ushbu ko'rinishdagi

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}\sum_{m=1}^{n}x_m\right)^p \le \left(\frac{p}{p-1}\right)^p \sum_{n=1}^{\infty} x_n^p$$

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tengsizlik ixtiyoriy no‘manfiy $\{x_m\}_{m=1}^{\infty}$ ketma-ketlik uchun o‘rinli ekanligini ko‘rsatdi. Bu yerda $p>1$ haqiqiy son, $\left(\frac{p}{p-1}\right)^p$ son esa tengsizlikning eng yaxshi konstantasi, ya’ni tengsizlik o‘rinli bo‘ladigan eng kichik konstanta. Yuqoridagi tengsizlik tengsizliklar nazariyasi oid kitoblarda diskret Hardi tengsizligi deb yuritiladi.

O‘tgan asrning o‘rtalariga kelib bu tengsizlikning turli umulashmalari paydo bo‘ldi. Bunga oid natijalar, masalan A.Kufner, L.Maligranda va L-E Personlarning [2] kitobida ham batafsil keltirilgan. Barcha umulashgan tengsizliklarni quyidagi ko‘rinishda tasvirlash mumkin:

$$\left(\sum_{n=1}^{\infty}\left|\sum_{m=1}^{n}K(n,m)\,x_m\right|^q u_n\right)^{\frac{1}{q}} \le C\cdot\left(\sum_{n=1}^{\infty}|x_n|^p v_n\right)^{\frac{1}{p}}. \qquad (1.1)$$

(1.1) tengsizlik Umumlashgan Hardi tengsizligi deyiladi, bunda $\{K(n,m)\}$ tengsizlikning yadrosi, hamda $\{u_n\},\{v_n\}$lar vazn ketma-ketliklari deyiladi, ya’ni nomanfiy ketma-ketlik. Shu vaqtgacha, yuqoridagi tengsizliklar o‘rganilgan ishlarda asosan tengsizlik o‘rinli bo‘lishini taminlaydigan shartlar topishga katta etibor qaratilgan. Bunda, tengsizlik yadrosi ma’lum bir shartlarni qanoatlantirishi talab etilgan, masalan yadro Oynarov yadrosi bo‘lishi va shunga oxshash. Umuman olganda agar yadro oddiy ko‘phad ko‘rinishda bo‘ganda ham yuqoridagi natijalardan foydalanishning iloji yuq.

Ushbu ishda yadro to‘rtinchi darajali ko‘phad bo‘lgan holda (1.1) tengsizlik o‘rinli bo‘lishi uchun zarur va yetarli shartlar olingan. Ishning ikkinchi paragrafida asosiy natijalar keltirilgan, bu natijalarning isboti keyongi paragrafda keltirilgan.

2-§. Asosiy natijalar

Ma’lumki, $\{K(n,m)\}$ ikki o‘garuvchili ko‘phadning umumiy ko‘rinishi quyidagicha bo‘ladi:

$$K(n,m)=\sum_{i=0}^{l}\sum_{j=0}^{r}a_{ij}n^i m^j,$$

bunda l va r lar no‘manfiy butun sonlardir. Hususiy holda:

Agar bu yerda $l=r=0$ bo‘lsa, u holda yadro $\mathrm{K(n,m)=a_{00}}$ bo‘ladi. Bu holda (1.1) tengsizlik ushbu ko‘rinishga ega bo‘lib

$$\left(\sum_{n=1}^{\infty}\left|\sum_{m=1}^{n}x_m\right|^p u_n\right)^{\frac{1}{p}} \le \frac{C}{|a_{00}|}\cdot\left(\sum_{n=1}^{\infty}|x_n|^p v_n\right)^{\frac{1}{p}}. \qquad (1.2)$$

Bu tengsizlik oldindan yaxshi o‘rganilgan bo‘lib uning bajarilishi uchun quyidagi shartning bajarilishi yetarli va zarurdir [4], [2]:

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$$A := \sup_{n\in N}\left(\sum_{m=1}^{n} v_m^{1-p'}\right)^{p-1}\cdot\left(\sum_{m=n}^{\infty} u_m\right) < \infty.$$

Bundan tashqari tengsizlik konstantasi uchun

$$A \le \frac{C}{|a_{00}|} \le p^{\frac{1}{p}}(p')^{\frac{1}{p'}}A.$$

Biz bu ishda $l = 1$ va $r = 3$ bo‘lgan holni, ya’ni yadroning ko‘rinishi
$K(n,m) = a_{00} + a_{10}m + a_{02}m^2 + a_{03}m^3 + a_{10}n + a_{11}nm + a_{12}nm^2 + a_{13}nm^3$
bo‘lganda (1.1) tengsizlik o‘rganamiz. Bunday holda (1.1) ushbu ko‘rinishga ega bo‘ladi:

$$\left(\sum_{n=1}^{\infty}\left|\sum_{m=1}^{n}(a_{00} + a_{10}m + a_{02}m^2 + a_{03}m^3 + a_{10}n + a_{11}nm + a_{12}nm^2 + a_{13}m^3)\, x_m\right|^p u_n\right)^{\frac{1}{p}}$$

$$\le C\cdot\left(\sum_{n=1}^{\infty}|x_n|^p v_n\right)^{\frac{1}{p}}. \qquad (1.3)$$

Quyida bizlar shu tengsizlikni yadroni turli hollariga ajratib o‘rganamiz:

1-hol. Faraz qilaylik yadro quyidagi xossaga ega bo‘lsin, ya’ni shunday A_0, A_1, B_0, B_1, B_2 sonlar mavjudk bo‘lsinki ushbu tenglik

$$a_{00} + a_{10}m + a_{02}m^2 + a_{03}m^3 + a_{10}n + a_{11}nm + a_{12}nm^2 + a_{13}m^3$$
$$= (A_0 + A_1 n)(B_0 + B_1 m + B_2 m^2 + B_3 m^3)$$

barcha n va $m$larda o‘rinli bo‘lsin. Bundan ushbu bog’lanishni hosil qilamiz:

$$\begin{cases} A_0B_0 = a_{00} \\ A_0B_1 = a_{01} \\ A_0B_2 = a_{02} \\ A_0B_3 = a_{03} \\ A_1B_0 = a_{10}' \\ A_1B_1 = a_{11} \\ A_1B_2 = a_{12} \\ A_1B_3 = a_{13} \end{cases}$$

ya’ni

$$\frac{a_{10}}{a_{00}} = \frac{a_{11}}{a_{01}} = \frac{a_{12}}{a_{02}} = \frac{a_{13}}{a_{03}}.$$

U holda (1.3) tengsizlik ushbu ko‘rinishga ega bo‘ladi

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$$\left(\sum_{n=1}^{\infty}\left|\sum_{m=1}^{n}(A_0+A_1n)(B_0+B_1m+B_2m^2+B_3m^3)x_m\right|^p u_n\right)^{\frac{1}{p}}$$

$$\leq C\cdot\left(\sum_{n=1}^{\infty}|x_n|^p v_n\right)^{\frac{1}{p}}. \quad (1.4)$$

Endi bu yerda $y_n=(B_0+B_1m+B_2m^2+B_3m^3)\cdot x_m$, $\bar{u}_n=(A_0+A_1n)\cdot v_n$ va
$\bar{v}_n=v_n\cdot|B_0+B_1m+B_2m^2+B_3m^3|^{-p}$ deb belgilashlar kiritsak (1.4) tengsizlik quyidagi koʻrinishni oladi:

$$\left(\sum_{n=1}^{\infty}\left|\sum_{m=1}^{n}y_m\right|^p\bar{u}_n\right)^{\frac{1}{p}}\leq C\cdot\left(\sum_{n=1}^{\infty}|y_n|^p\bar{v}_n\right)^{\frac{1}{p}}. \quad (1.5)$$

Demak bu holda (1.3) tengsizligimiz (1.2) koʻrinishga oʻtar ekan.

Shunday qilib, quyidagi natijani hosil qildik:

1. 1-*teorema.* *Faraz qilaylik (1.2) tengsizlikdagi yadro uchun*

$$\frac{a_{10}}{a_{00}}=\frac{a_{11}}{a_{01}}=\frac{a_{12}}{a_{02}}=\frac{a_{13}}{a_{03}}$$

shart qanoatlantirsin. U holda bu tengsizlik oʻrinli boʻlishi uchun

$$A:=\sup_n\left(\sum_{m=1}^{n}|B_0+B_1m+B_2m^2+B_3m^3|^{p'}v_m^{1-p}\right)^{p-1}\cdot\left(\sum_{m=n}^{\infty}|A_0+A_1m|\cdot u_m\right)$$
$$<\infty$$

shartning bajarilishi zarur va yetarlidir. Bundan tashqari, tengsizlikning eng yaxshi konstantasi quyidagicha baholanadi

$$A\leq C\leq p^{\frac{1}{p}}(p')^{\frac{1}{p'}}A.$$

Endi quyida bizlar, (1.3) dagi yadro koeffisientlari ixtiyoriy boʻlgan holda qaraymiz.

1. 2-teorema. *(1.3) tengsizlik oʻrinli boʻlishi uchun*

$$A=\sup_{n\in N}\left(\sum_{m=1}^{n}|a_{10}+a_{11}m+a_{12}m^2+a_{13}m^3|^{p'}v_m^{1-p'}\right)^{p-1}\cdot\left(\sum_{m=n}^{\infty}m^p u_m\right)<\infty$$

shartning bajarilishi zarur va yetarlidir.

3-§. Natijalarning isbotlari

Isbot. [7] (Yetarliligi) (1.3)ning chap tomonini normaning uchburchak tengsizligidan foydalanib quyidagicha

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$$\left(\sum_{n=1}^{\infty}\left|\sum_{m=1}^{n}(a_{00}+a_{01}m+a_{02}m^2+a_{03}m^3+a_{10}n+a_{11}nm+a_{12}nm^2\right.\right.$$

$$\left.\left.+a_{13}nm^3)\,x_m\right|^p u_n\right)^{\frac{1}{p}}$$

$$\leq\left(\sum_{n=1}^{\infty}\left|\sum_{m=1}^{n}(a_{00}+a_{01}m+a_{02}m^2+a_{03}m^3)\,x_m\right|^p u_n\right)^{\frac{1}{p}}$$

$$+\left(\sum_{n=1}^{\infty}\left|\sum_{m=1}^{n}n(a_{10}+a_{11}m+a_{12}m^2+a_{13}m^3)\,x_m\right|^p u_n\right)^{\frac{1}{p}}$$

baholash mumkin. Bu ko‘rsatadiki tengsizlikning o‘rinli bo‘lishi uchun ushbu tengsizliklarning har biri o‘rinli bo‘lishi yetarli ekan

$$\left(\sum_{n=1}^{\infty}\left|\sum_{m=1}^{n}(a_{00}+a_{01}m+a_{02}m^2+a_{03}m^3)\,x_m\right|^p u_n\right)^{\frac{1}{p}}$$

$$\leq C\cdot\left(\sum_{n=1}^{\infty}|x_n|^p v_n\right)^{\frac{1}{p}} \qquad (1.6)$$

$$\left(\sum_{n=1}^{\infty}\left|\sum_{m=1}^{n}n(a_{10}+a_{11}m+a_{12}m^2+a_{13}m^3)\,x_m\right|^p u_n\right)^{\frac{1}{p}}$$

$$\leq C\cdot\left(\sum_{n=1}^{\infty}|x_n|^p v_n\right)^{\frac{1}{p}} \qquad (1.7)$$

(1.6) va (1.7) tengsizliklarda mos ravishda $y_m=(a_{00}+a_{11}m+a_{02}m^2)x_m$ va $z_m=(a_{10}+a_{11}m+a_{12}m^2)x_m$ almashtirishlar olsak u holda quyidagi tengsizliklarni hosil qilamiz

$$\left(\sum_{n=1}^{\infty}\left|\sum_{m=1}^{n}y_m\right|^p u_n\right)^{\frac{1}{p}}$$

$$\leq C\cdot\left(\sum_{n=1}^{\infty}|y_n|^p\frac{v_n}{|a_{00}+a_{01}m+a_{02}m^2+a_{03}m^3|^p}\right)^{\frac{1}{p}} \qquad (1.8)$$

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$$\left(\sum_{n=1}^{\infty}\left|\sum_{m=1}^{n} z_m\right|^p n^p u_n\right)^{\frac{1}{p}}$$

$$\leq C\cdot\left(\sum_{n=1}^{\infty}|z_n|^p \frac{v_n}{|a_{10}+a_{11}m+a_{12}m^2+a_{13}m^3|^p}\right)^{\frac{1}{p}} \qquad (1.9)$$

(1.8) va (1.9) lar Hardining klassik tengsizligi koʻrinishida boʻlib bu tengsizlik uchun yetarlishart Muckenhoupt tomonidan topilgan ([3-4] qarang). Oʻsha shartlardan foydalanadigan boʻlsak (1.6) va (1.7) ular uchun yetarlishartlar mos ravishda quyidagi koʻrinishni oladi

$$A^1 = \sup_{n\in N}\left(\sum_{m=1}^{n}|a_{00}+a_{01}m+a_{02}m^2+a_{03}m^3|^{p'} v_m^{1-p'}\right)^{p-1}\cdot\left(\sum_{m=n}^{\infty} u_m\right)<\infty$$

$$A^2 = \sup_{n\in N}\left(\sum_{m=1}^{n}|a_{10}+a_{11}m+a_{12}m^2+a_{13}m^3|^{p'} v_m^{1-p'}\right)^{p-1}\cdot\left(\sum_{m=n}^{\infty} m^p u_m\right)<\infty.$$

Bu yerda koʻrinadiki, agar A^2 chekli boʻlsa u holda A^1 ham chekli boʻladi, sababi, A^2 ning ikkinchi yigʻindi qatnashgan ifodasida qoʻshimcha m^p had uchraydi, bu esa m ning yetarlicha katta qiymatlarida yigʻindi A^1 uchraydigan huddi shunday yigʻindidan ancha katta boʻlishini koʻrsatadi. Demak, (2.3) tengsizlik bajarilishi uchun $A^2<\infty$ shartning bajarilishi yetarli ekan. Bundan osongina teorema shartining yetarli ekanligi kelib chiqadi.

Zaruriyligi [7] Endi bizlar teorema shartining zarur ekanligini koʻrsatamiz. Faraz qilaylik, (1.3) tengsizlik bajarilgan boʻlsin. Umumiylikka zarar keltirmasdan $a_{12}>0$ deyishimiz mumkin. U holda shunday $n_0\in N$ son mavjudki

$$a_{00}+a_{01}m+a_{02}m^2+a_{03}m^3+a_{10}n+a_{11}nm+a_{12}nm^2+a_{13}m^3\geq\frac{a_{13}nm^3}{2}$$

baho barcha $n\geq n_0$ va $1\leq m\leq n$ larda oʻrinli boʻladi. (3.1.3) ning oʻrinli ekanligidan ushbu tengsizlikning ham bajarilishi kelib chiqadi

$$\left(\sum_{n=n_0}^{\infty}\left|\sum_{m=1}^{n}(a_{00}+a_{01}m+a_{02}m^2+a_{03}m^3+a_{10}n+a_{11}nm+a_{12}nm^2\right.\right.$$

$$\left.\left.+a_{13}m^3)\,x_m\right|^p u_n\right)^{\frac{1}{p}}\leq C\cdot\left(\sum_{n=1}^{\infty}|x_n|^p v_n\right)^{\frac{1}{p}}.$$

Faraz qilaylik $\{x_m\}_{m=0}^{\infty}$ ketma-ketlikning boshlangʻich n_0 ta hadi nolga teng boʻlsin. U holda yuqoridagi tengsizlik quyidagi koʻrinishni oladi

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$$\left(\sum_{n=n_0}^{\infty}\left|\sum_{m=n_0}^{n}(a_{00}+a_{01}m+a_{02}m^2+a_{03}m^3+a_{10}n+a_{11}nm+a_{12}nm^2\right.\right.$$

$$\left.\left.+a_{13}m^3)\,x_m\right|^p u_n\right)^{\frac{1}{p}} \le C\cdot\left(\sum_{n=n_0}^{\infty}|x_n|^p v_n\right)^{\frac{1}{p}}.$$

Demak, (1.3) tengsizlik o‘rinli bo‘lishidan shunday n_0 son mavjudligi va yuqoridagi tengsizlikning o‘rinli bo‘lishi kelib chiqar ekan. Endi tengsizlikdagi ketma-ketlikni xususiy holda musbat deb qaraylik. U holda yuqoridagi tengsizlikning chap tomonini quyidagicha baholash mumkin

$$\left(\sum_{n=n_0}^{\infty}\left|\sum_{m=n_0}^{n}(a_{00}+a_{01}m+a_{02}m^2+a_{03}m^3+a_{10}n+a_{11}nm+a_{12}nm^2\right.\right.$$

$$\left.\left.+a_{13}m^3)\,x_m\right|^p u_n\right)^{\frac{1}{p}} \ge \frac{a_{13}}{2}\left(\sum_{n=n_0}^{\infty}\left(\sum_{m=n_0}^{n}nm^3\,x_m\right)^p u_n\right)^{\frac{1}{p}}.$$

Demak, (3.1.3) o‘rinli bo‘lsa u holda

$$\left(\sum_{n=n_0}^{\infty}\left(\sum_{m=n_0}^{n}nm^3\,x_m\right)^p u_n\right)^{\frac{1}{p}} \le \frac{2C}{a_{13}}\cdot\left(\sum_{n=n_0}^{\infty}x_n^p v_n\right)^{\frac{1}{p}}$$

ham musbat ketma-ketliklar uchun o‘rinli bo‘lishi kelib chiqar ekan. Bu yerda $y_m = m^2 x_m$ deb almashtirish olsak, u holda

$$\left(\sum_{n=n_0}^{\infty}\left(\sum_{m=n_0}^{n}y_m\right)^p n^p u_n\right)^{\frac{1}{p}} \le \frac{2C}{a_{13}}\cdot\left(\sum_{n=n_0}^{\infty}y_n^p\frac{v_n}{n^p}\right)^{\frac{1}{p}}$$

tengsizlikka ega bo‘lamiz. Bunda, $y_n = \chi_{[n_0,n_1]}(n)n^{p'}v_n^{1-p'}$ deb tanlab yuqoridagi tengsizlikning chap va o‘ng tomonlariga qo‘ysak ushbularni hosil qilamiz

$$\frac{2C}{a_{13}}\cdot\left(\sum_{n=n_0}^{\infty}y_n^p\frac{v_n}{n^p}\right)^{\frac{1}{p}} = \frac{2C}{a_{13}}\cdot\left(\sum_{n=n_0}^{n_1}\left(n^{p'}v_n^{1-p'}\right)^p\frac{v_n}{n^p}\right)^{\frac{1}{p}} = \frac{2C}{a_{13}}\cdot\left(\sum_{n=n_0}^{n_1}n^{p'}v_n^{1-p'}\right)^{\frac{1}{p}},$$

$$\left(\sum_{n=n_0}^{\infty}\left(\sum_{m=n_0}^{n}y_m\right)^p n^p u_n\right)^{\frac{1}{p}} = \left(\sum_{n=n_0}^{\infty}\left(\sum_{m=n_0}^{n}\chi_{[n_0,n_1]}(m)m^{p'}v_m^{1-p'}\right)^p n^p u_n\right)^{\frac{1}{p}} \ge$$

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$$\geq\left(\sum_{n=n_1}^{\infty}\left(\sum_{m=n_0}^{n}\chi_{[n_0,n_1]}(m)m^{p'}v_m^{1-p'}\right)^{p}n^{p}u_n\right)^{\frac{1}{p}}\geq$$

$$\geq\left(\sum_{n=n_1}^{\infty}\left(\sum_{m=n_0}^{n_1}m^{p'}v_m^{1-p'}\right)^{p}n^{p}u_n\right)^{\frac{1}{p}}$$

$$=\left(\sum_{n=n_1}^{\infty}n^{p}u_n\right)^{\frac{1}{p}}\left(\sum_{m=n_0}^{n_1}m^{p'}v_m^{1-p'}\right).$$

Demak, yuqoridagi tengsizlikdan quyidagi baho kelib chiqdi

$$\left(\sum_{n=n_1}^{\infty}n^{p}u_n\right)^{\frac{1}{p}}\left(\sum_{m=n_0}^{n_1}m^{p'}v_m^{1-p'}\right)\leq\frac{2C}{a_{13}}\cdot\left(\sum_{n=n_0}^{n_1}n^{p'}v_n^{1-p'}\right)^{\frac{1}{p}}$$

va

$$\left(\sum_{n=n_1}^{\infty}n^{p}u_n\right)^{\frac{1}{p}}\left(\sum_{m=n_0}^{n_1}m^{p'}v_m^{1-p'}\right)^{\frac{1}{p'}}\leq\frac{2C}{a_{13}}.$$

Bunda n_1 ning ixtiyoriyligidan

$$\mathrm{A}^3=\sup_{\mathrm{n}\geq\mathrm{n}_0}\left(\sum_{\mathrm{m}=\mathrm{n}_0}^{\mathrm{n}}\mathrm{m}^{\mathrm{p}'}\mathrm{v}_\mathrm{m}^{1-\mathrm{p}'}\right)^{\mathrm{p}-1}\cdot\left(\sum_{\mathrm{m}=\mathrm{n}}^{\infty}\mathrm{m}^{\mathrm{p}}\mathrm{u}_\mathrm{m}\right)<\infty$$

ni hosil qilamiz. Endi $A\leq C_1+C_2A^3$ ekanligini ko‘rsatsak zaruriylik isbot bo‘ladi. $A^3<\infty$ ekanligidan, ixtiyoriy $n\in N$ uchun

$$\sum_{m=n}^{\infty}m^{p}u_m<\infty$$

bo‘lishi kelib chiqadi. Bundan tashqari Ani ushbu ko‘rinishda yozamiz

$$A^2=\sup_{n\geq1}\left(\sum_{m=1}^{n}m^{p'}v_m^{1-p'}\right)^{p-1}\cdot\left(\sum_{m=n}^{\infty}m^{p}u_m\right)\leq$$

$$\leq\sup_{n\geq n_0}\left(\sum_{m=1}^{n}m^{p'}v_m^{1-p'}\right)^{p-1}\cdot\left(\sum_{m=n}^{\infty}m^{p}u_m\right)$$

$$+\sup_{n_0\geq n\geq1}\left(\sum_{m=1}^{n}m^{p'}v_m^{1-p'}\right)^{p-1}\cdot\left(\sum_{m=n}^{\infty}m^{p}u_m\right).$$

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Bulardan ko‘rinadiki ikkinchi yig‘indi chekli ekan, chunki ikkinchi yig‘indidagi birinchi qavslarda cheklita hadlarning yig‘indisidan iborat. Shuning uchun birinchi yig‘indi bilan bahoni davom ettiramiz:

$$\sup_{n\geq n_0}\left(\sum_{m=1}^{n} m^{p'} v_m^{1-p'}\right)^{p-1}\cdot\left(\sum_{m=n}^{\infty} m^p u_m\right)\leq$$

$$\leq \sup_{n\geq n_0}\left(\sum_{m=1}^{n_0} \mathrm{m}^{p'} v_m^{1-p'}+\sum_{m=n_0}^{n} \mathrm{m}^{p'} v_m^{1-p'}\right)^{p-1}\cdot\left(\sum_{m=n}^{\infty} m^p u_m\right)$$

$$\leq 2^{p-1}\sup_{n\geq n_0}\left(\left(\sum_{m=1}^{n_0} \mathrm{m}^{p'} v_m^{1-p'}\right)^{p-1}+\left(\sum_{m=n_0}^{n} \mathrm{m}^{p'} v_m^{1-p'}\right)^{p-1}\right)\cdot\left(\sum_{m=n}^{\infty} m^p u_m\right)$$

$$\leq 2^{p-1}\left(\sup_{n\geq n_0}\left(\sum_{m=1}^{n_0} \mathrm{m}^{p'} v_m^{1-p'}\right)^{p-1}\cdot\left(\sum_{m=n}^{\infty} m^p u_m\right)+\sup_{n\geq n_0}\left(\sum_{m=n_0}^{n} \mathrm{m}^{p'} v_m^{1-p'}\right)^{p-1}\right.$$

$$\left.\cdot\left(\sum_{m=n}^{\infty} m^p u_m\right)\right).$$

Ohirgi ifodalarning ikkalasi ham chekli, sababi, bunda ham birinchi yig‘indining birinchi qavsdagi hadlar cheklita, ikkinchi yig‘indining chekliligi ko‘rsatilgan edi. Teorema to‘liq isbot bo‘ldi.

FOYDALANILGAN ADABIYOTLAR:

1. G.H.Hardy, J.E.Littlewood and G.Polya, Inequalities, 2nd ed., Cambridge University Press, Cambridge, 1967.

2. A.Kufner and L.-E. Persson, Weighted Inequalities of Hardy Type, World Scientific, Singapore, 2003.

3. B. Opic and A. Kufner, Hardy-Type Inequalities, Pitman Research Notes in Mathematics, no. 219, Long man Scientific & Technical, Harlow, 1990.

4. A.Kufner, L.Maligranda, L-E.Persson. The *Hardy inequality about its history*. World Publishing House. Pilsen 2007, 162 p.

5.K.Kuliyev, J.Yaxshilikov. Umumlashgan Hardi tengsizligiga oid ba’zi natijalar. SamDU, Magistrantlarning XVIII-ilmiy konferensiyasi materiallari. 2018 y, 30-31b.

6. K.Kuliyev, J.Yaxshilikov. O.Qurbonov.Diskret Hardi tengsizligi. SamDU, Magistrantlarning XVIII-ilmiy konferensiyasi materiallari. 2018 y, 32-33b.

7. K.Kuliyev, J.Yaxshilikov. Umumlashgan Hardi tengsizligiga oid ba’zi natijalar. NamMTI. Ilmiy maqolalar to‘plami. (chop etishga qabul qilingan)