# PANJARADAGI KO'PI BILAN BIR FOTONLI SPIN-BOZON MODELINING SPEKTRI

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**Annotatsiya:** Maqolada panjaradagi ko'pi bilan bir fotonli spin-bozon modelining spektri "maxsus" integral chekli va cheksiz bo'lgan hollarda o'rganilgan. Xos qiymatlarining soni va joylashish o'rni ta'sirlashish parametridan bog'liq ravishda tahlil qilingan.

**Kalit so'zlar**: Bozonli Fok fazo, muhim spektr, diskret spektr, xos qiymat, Fredgolm determinanti, Veyl teoremasi

**Annotation:** In the article, the spectrum of the at most one photon spin-boson model on the lattice is studied in cases where the "special" integral is finite and infinite. The number and location of eigenvalues were analyzed as a function of exposure parameter.

**Key words:** Bosonic Fock space, essential and discrete spectra, eigenvalue, Weyl theorem, Fredholm determinant

$m = 1$ uchun $L_2((\mathbb{T}^{\mathrm{d}})^m)$ orqali $(\mathbb{T}^{\mathrm{d}})^m$ to'plamda aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatli) funksiyalarning Hilbert fazosini belgilaymiz hamda quyidagi Hilbert fazolarni kiritamiz:

$$\mathcal{F}(L_2(\mathbb{T}^{\mathrm{d}})) := \mathbb{C} \oplus L_2(\mathbb{T}^{\mathrm{d}}) \oplus L_2((\mathbb{T}^{\mathrm{d}})^2) \oplus \dots;$$

$$\mathcal{F}^{(1)}(L_2(\mathbb{T}^{\mathrm{d}})) := \mathbb{C} \oplus L_2(\mathbb{T}^{\mathrm{d}});$$

$\mathcal{F}(L_2(\mathbb{T}^{\mathrm{d}}))$ Hilbert fazosiga $L_2(\mathbb{T}^{\mathrm{d}})$ fazo yordamida qurilgan Fok fazosi deyiladi, $\mathcal{F}^{(m)}(L_2(\mathbb{T}^{\mathrm{d}}))$ Hilbert fazosiga esa Fok fazosining $m+1$ zarrachali qirqilgan qism fazosi deyiladi.

$m = 1$ sonlari uchun $\mathcal{H}^{(m)}$ Hilbert fazosida ta'sir qiluvchi quyidagi $(m+1)$-tartibli operatoli matrisani qaraymiz:

$$\mathcal{A}_1 := \begin{pmatrix} \mathrm{A}_{00} & \mathrm{A}_{01} \\ \mathrm{A}_{01}^* & \mathrm{A}_{11} \end{pmatrix},$$

Ta'kidlash joizki, $\mathcal{A}_m$, $m = 1$ operatorli matrisa $\mathcal{H}^{(m)}$ Hilbert fazosida chiziqli, chegaralangan va o'z-o'ziga qo'shma bo'ladi. Bunda

$$\mathrm{A}_{00} f_0^{(\mathrm{s})} = \mathrm{s}\varepsilon f_0^{(\mathrm{s})}, \quad \mathrm{A}_{01} f_1^{(\mathrm{s})} = \alpha \int_{\mathbb{T}^{\mathrm{d}}} v(t) f_1^{(-\mathrm{s})}(t) dt,$$

$$(\mathrm{A}_{11} f_1^{(\mathrm{s})})(k_1) = (\mathrm{s}\varepsilon + w(k_1)) f_1^{(\mathrm{s})}(k_1),$$

kabi aniqlangan. Bu yerda $\{f_0^{(\mathrm{s})}, f_1^{(\mathrm{s})}, \mathrm{s} = \pm\} \in \mathcal{H}^{(1)}$; $i < j$ bo‘lganda $\mathrm{A}_{ij}^*$ orqali $\mathrm{A}_{ij}$ operatorga qo‘shma operator belgilangan, $v(\cdot)$, $w(\cdot)$ funksiyalar $T^{\mathrm{d}}$ torda aniqlangan haqiqiy qiymatli va uzluksiz funksiyalar hamda

$$\min_{k \in T^{\mathrm{d}}} w(k) = 0,$$

$\alpha > 0$ esa ’’ta’sirlashish parametr’’. $\mathcal{A}$ operatorli matrisa $\mathcal{H}^{(1)}$ Hilbert fazosidagi chiziqli chegaralangan va o‘z-o‘ziga qo‘shma ekanligini tegishli ta’riflar va Koshi-Bunyakovskiy tengsizligidan foydalanib ko‘rsatish mumkin.

Keyingi izlanishlarda qulaylik tug‘dirish maqsadida $\mathcal{F}^{(m)}(L_2(\mathbb{T}^{\mathrm{d}}))$ Hilbert fazosida ta’sir qiluvchi quyidagi chegaralangan va o‘z-o‘ziga qo‘shma bo‘lgan hamda $\mathcal{A}_m^{(\mathrm{s})}$, $m = 1$ $\mathrm{s} = \pm$ orqali belgilangan $(m+1)$ – tartibli diskret parametrli operatorli matrisalarni qaraymiz:

$$\mathcal{A}_1^{(\mathrm{s})} := \begin{pmatrix} \widehat{\mathrm{A}}_{00}^{(\mathrm{s})} & \widehat{\mathrm{A}}_{01} \\ \widehat{\mathrm{A}}_{01}^* & \widehat{\mathrm{A}}_{11}^{(\mathrm{s})} \end{pmatrix},$$

$$\widehat{\mathrm{A}}_{00}^{(\mathrm{s})} f_0 = \mathrm{s}\varepsilon f_0, \quad \widehat{\mathrm{A}}_{01} f_1 = \alpha \int_{\mathbb{T}^{\mathrm{d}}} v(t) f_1(t) dt,$$

$$(\widehat{\mathrm{A}}_{11}^{(\mathrm{s})} f_1)(k_1) = (-\mathrm{s}\varepsilon + w(p)) f_1(p), \quad (\widehat{\mathrm{A}}_{12} f_2)(\mathrm{p}) = \alpha \int_{\mathbb{T}^{\mathrm{d}}} v(t) f_2(\mathrm{p}, t) dt,$$

$$(f_0, f_1) \in \mathcal{F}^{(1)}(L_2(\mathbb{T}^{\mathrm{d}})),$$

ko‘rinishda aniqlangan.

Qo‘shma operatorni hisoblash qoidasidan foydalanib

$$(\widehat{\mathrm{A}}_{01}^* f_0)(p) = \alpha v(p) f_0,$$

$$(f_0, f_1) \in \mathcal{F}^{(1)}(L_2(\mathbb{T}^{\mathrm{d}}))$$

tengliklarni hosil qilamiz. $m = 1$ sonlari uchun $\mathcal{A}_m$ va $\mathcal{A}_m^{(\mathrm{s})}$, $\mathrm{s} = \pm$ operatorli matrisalarning spektrlari o‘rtasidagi munosabatlarni aniqlovchi teoremani bayon qilamiz.

**Teorema 1.** Faraz qilaylik, $m = 1$ bo‘lsin. $\mathcal{A}_{\mathrm{m}}$ va $\mathcal{A}_{\mathrm{m}}^{(\mathrm{s})}$, $s = \pm$ operatorli matrisalarning spektrlari uchun $\sigma(\mathcal{A}_{\mathrm{m}}) = \sigma(\mathcal{A}_{\mathrm{m}}^{(+)}) \cup \sigma(\mathcal{A}_{\mathrm{m}}^{(-)})$ tenglik o‘rinli. Bundan tashqari, ularning muhim va nuqtali spektrlari uchun

$$\sigma_{\mathrm{ess}}(\mathcal{A}_m) = \sigma_{\mathrm{ess}}(\mathcal{A}_m^{(+)}) \cup \sigma_{\mathrm{ess}}(\mathcal{A}_m^{(-)}),$$

$$\sigma_{\mathrm{p}}(\mathcal{A}_m) = \sigma_{\mathrm{p}}(\mathcal{A}_m^{(+)}) \cup \sigma_{\mathrm{p}}(\mathcal{A}_m^{(-)})$$

tengliklar o‘rinlidir.

$\mathcal{F}^{(1)}(L_2(\mathbb{T}^{\mathrm{d}}))$ Hilbert fazosida

$$\mathcal{A}_{1,0}^{(\mathrm{s})} := \begin{pmatrix} 0 & 0 \\ 0 & \hat{\mathcal{A}}_{11}^{(\mathrm{s})} \end{pmatrix}$$

ko‘rinishda aniqlangan $\mathcal{A}_{1,0}^{(\mathrm{s})}, \mathrm{s} = \pm$ diskret parametrli operatorli matrisani qaraymiz. U holda $\mathcal{A}_{1,0}^{(\mathrm{s})}$ operatorli matrisaning $\mathcal{A}_{1}^{(\mathrm{s})} - \mathcal{A}_{1,0}^{(\mathrm{s})}$ qo‘zg‘alish operatori ikki o‘lchamli chegaralangan va o‘z-o‘ziga qo‘shma (demak, kompakt) operatorli matrisa bo‘ladi. Chekli o‘lchamli qo‘zg‘alishlarda muhim spektrning o‘zgarmasligi haqidagi mashhur G. Veyl teoremasiga ko‘ra $\mathcal{A}_{1}^{(\mathrm{s})}$ operatorli matrisaning muhim spektri $\mathcal{A}_{1,0}^{(\mathrm{s})}$ operatorli matrisaning muhim spektri bilan ustma-ust tushadi. Ma’lumki,

$$\sigma_{\mathrm{ess}}(\mathcal{A}_{1,0}^{(\mathrm{s})}) = [-s\varepsilon, -s\varepsilon + M], \quad M := \max_{k \in \mathbb{T}^{\mathrm{d}}} w(k).$$

Oxirgi ma’lumotlardan foydalanib

$$\sigma_{\mathrm{ess}}(\mathcal{A}_{1}^{(\mathrm{s})}) = [-s\varepsilon, -s\varepsilon + M]$$

ekanligini hosil qilamiz. Shunday qilib, 1-teoremaga ko‘ra $\mathcal{A}_1$ operatorli matrisaning muhim spektri uchun

$$\sigma_{\mathrm{ess}}(\mathcal{A}_1) = [-\varepsilon, -\varepsilon + M] \cup [\varepsilon, \varepsilon + M] \qquad (1)$$

tenglik o‘rinli ekan.

Ta’kidlash joizki, $\mathbb{R}^{\mathrm{d}}$ Yevklid fazosi holda [3]–[4] shu turdagi modelning muhim spektri $[-\varepsilon, \infty)$ yarim o‘qdan iborat bo‘ladi. Ko‘rinib turibdiki, agar (1)-tenglikda $\varepsilon > M/2$ bo‘lsa, u holda $\mathcal{A}_1$ operatorli matrisaning muhim spektri o‘zaro kesishmaydigan ikkita kesmalarning birlashmasidan iborat bo‘ladi. Boshqacha qilib aytganda, $\mathcal{A}_1$ operatorli matrisaning muhim spektrida $(-\varepsilon + M, \varepsilon)$ bo‘shliq (lakuna) hosil bo‘ladi.

$\mathbb{C}\backslash[-s\varepsilon, -s\varepsilon + M]$ sohada regulyar bo‘lgan ushbu

$$\Delta_1^{(s)}(z) := s\varepsilon - z - \alpha^2 \int_{\mathbb{T}^d} \frac{v^2(t)dt}{-s\varepsilon + w(t) - z}$$

funksiyani qaraymiz.

Odatda $\Delta_1^{(s)}(\cdot)$ funksiyaga $\mathcal{A}_1^{(s)}$ operatorli matrisaga mos Fredgolm determinanti deyiladi.

Quyidagi lemmada $\mathcal{A}_1^{(s)}$ operatorli matrisaning xos qiymatlari va $\Delta_1^{(s)}(\cdot)$ funksiyaning nollari o‘rtasidagi bog‘liqlik o‘rnatilgan.

**Lemma 1.** $z^{(s)} \in \mathbb{C}\backslash\sigma_{ess}(\mathcal{A}_1^{(s)})$ soni $\mathcal{A}_1^{(s)}$ operatorli matrisaning xos qiymati bo‘lishi uchun $\Delta_1^{(s)}(z^{(s)}) = 0$ bo‘lishi zarur va yetarli.

$\mathcal{A}_1^{(s)}$ operatorli matrisaning xos qiymatlarini o‘rganish maqsadida

$$\int_{\mathbb{T}^d} \frac{v^2(t)dt}{w(t)} < \infty, \quad \int_{\mathbb{T}^d} \frac{v^2(t)dt}{M - w(t)} < \infty \tag{2}$$

deb faraz qilamiz hamda

$$\alpha_1 := \sqrt{M + 2\varepsilon}\left(\int_{\mathbb{T}^d} \frac{v^2(t)dt}{M - w(t)}\right)^{-1/2}, \quad \alpha_2 := \sqrt{2\varepsilon}\left(\int_{\mathbb{T}^d} \frac{v^2(t)dt}{w(t)}\right)^{-1/2},$$

$$\alpha_3 := \sqrt{M}\left(\int_{\mathbb{T}^d} \frac{v^2(t)dt}{2\varepsilon + M - w(t)}\right)^{-1/2}$$

kabi belgilashlarni kiritamiz.

**Lemma 2.** a) $\alpha > 0$ *ta’sirlashish parametrining barcha qiymatlarida* $\mathcal{A}_1^{(-)}$ *operatorli matrisa* $-\varepsilon$ *dan chapda yotuvchi yagona oddiy xos qiymatga ega. Agar* $\alpha \in (0, \alpha_1]$ *bo‘lsa, u holda* $\mathcal{A}_1^{(-)}$ *operatorli matrisa* $M + \varepsilon$ *dan o‘ngda yotuvchi xos qiymatga ega emas.* $\alpha > \alpha_1$ *bo‘lganda esa* $\mathcal{A}_1^{(-)}$ *operatorli matrisa* $M + \varepsilon$ *dan o‘ngda joylashgan yagona oddiy xos qiymatga ega.*

b) *Agar* $\alpha \in (0, min\{\alpha_2, \alpha_3\}]$ *bo‘lsa, u holda* $\mathcal{A}_1^{(+)}$ *operatorli matrisa* $-\varepsilon$ *dan* *chapda yotuvchi va* $M+\varepsilon$ *dan o'ngda yotuvchi xos qiymatlarga ega emas. Agar* $\alpha > max\{\alpha_2, \alpha_3\}$ *bo'lsa, u holda* $\mathcal{A}_1^{(+)}$ *operatorli matrisa* $-\varepsilon$ *dan chapda va* $M+\varepsilon$ *dan o'ngda joylashgan bittadan oddiy xos qiymatlarga ega.*

**Natija 1.** $\alpha > 0$ ta'sirlashish parametri ning barcha qiymatlarida $\mathcal{A}_1$ operatorli matrisa kamida bitta va ko'pi bilan to'rtta xos qiymatlarga ega. Bundan tashqari, agar $\alpha \in (0, \alpha_{\min}]$ bo'lsa, u holda $\mathcal{A}_1$ operatorli matrisa yagona oddiy (yakkalangan) xos qiymatga ega va bu xos qiymat $-\varepsilon$ dan chapda yotadi. Agar $\alpha \in (\alpha_{\max}, +\infty)$ bo'lsa, u holda $\mathcal{A}_1$ operatorli matrisa mos ravishda $-\varepsilon$ dan chapda va $M+\varepsilon$ dan o'ngda joylashgan ikkitadan xos qiymatlarga ega bo'ladi.

Endi

$$\int_{\mathbb{T}^1} \frac{v^2(t)dt}{w(t)} < \infty$$

shart bajariladigan $v(t)$ va $\omega(t)$ funksiyalarga misollar keltiramiz.

**Misol 2.2.1** Faraz qilaylik,

$$v(t) = sint, \quad \omega(t) = 1 - cost$$

bo'lsin. U holda shunday $c_1, c_2 > 0$ $va$ $\delta > 0$ sonlari topilib, ixtiyoriy $x\epsilon(-\delta, \delta)$ elementlar uchun

$$c_1|t| \le |v(t)| \le c_2|t| \qquad (1)$$

$$c_1|t|^2 \le |\omega(t)| \le c_2|t|^2 \qquad (2)$$

baholashlar o'rinli bo'ladi. Bundan tashqari, $x\epsilon\mathbb{T}^1\backslash(-\delta, \delta)$ lar uchun

$$|\omega(t)| \ge c_1 \qquad (3)$$

tengsizlik bajariladi. Lebeg integralining additivlik xossasiga ko'ra,

$$\int_{\mathbb{T}^1} \frac{v^2(t)dt}{w(t)} = \int_{-\delta}^{\delta} \frac{v^2(t)dt}{w(t)} + \int_{\mathbb{T}^1\backslash(-\delta,\delta)} \frac{v^2(t)dt}{w(t)} \le \int_{-\delta}^{\delta} \frac{{c_2}^2|t|^2dt}{c_1|t|^2} + \frac{c}{c_1}\int_{\mathbb{T}^1(-\delta,\delta)} \mathrm{d}t$$
$$= \frac{{c_2}^2}{c_1} \cdot 2\delta + \frac{c}{c_1} \cdot 2(\pi - \delta) < \infty$$

**Adabiyotlar ro'yxati:**

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