CHEGARALANGAN TOR TEBRANISHI TENGLAMASI UCHUN CHEGARAVIY MASALANI FURYE USULI YORDAMIDA MAPLE DASTURIDA YECHISH

Komilova Zulxumor Hakimjonovna
Andijon iqtisodiyot va qurilish instituti o'qituvchisi
Telefon: +998(99)8101695
mohichexrafoziljonova@gmail.com
Ibrohimjonova Ruxshonaxon Ilhomjon qizi
Andijon iqtisodiyot va qurilish instituti 1-bosqich
Buxgalteriya hisobi va auditi yo'nalishi talabasi
Telefon: +998(99)8662967

Annotatsiya: Mazkur ishda chegaralangan torning erkin tebranishi tenglamasi uchun qo'yilgan masalaning yechimini Furye usuli yordamida Maple dasturida topish ko'rsatib o'tilgan.

Kalit so'zlar: tor tebranishi, boshlang'ich shart, chegaraviy shart, Furye usuli, Maple dasturi.

Bu maqolada chegaralangan torning erkin tebranishi tenglamasi uchun quyidagi masalani ko'rib chiqamiz.

Chegaraviy masala. Quyidagi

$$u_{tt}(t,x) = a^2 u_{xx}(t,x)$$

tor tebranishi tenglamasining

$$u(0,x) = F(x), \quad \frac{\partial}{\partial t} u(0,x) = f(x)$$

boshlang'ich shartlarni va

$$u(t,0) = 0, \quad u(t,L) = 0$$

chegaraviy shartlarni qanoatlantiruvchi yechimini Furye usuli yordamida Maple dasturida topilsin.

Buning uchun Maple dasturi oynasiga quyidagi buyruqlarni yozishimiz kerak.

```
> restart;
```

Furye usuli yordamida bir jinsli tenglamaning yechimini topamiz:

```
> PDE:=diff(u(t,x),t,t)=a^2*diff(u(t,x),x,x);
  struc:=pdsolve(PDE,HINT=T(t)*X(x));
```

$$PDE := \frac{\partial^2}{\partial t^2} u(t,x) = a^2 \left(\frac{\partial^2}{\partial x^2} u(t,x) \right),$$

$$struc := (u(t,x) = T(t)\,X(x))\ \&where \left[\left\{\frac{\partial^2}{\partial t^2}T(t) = \_c_1\,T(t),\ \frac{\partial^2}{\partial x^2}X(x) = \frac{\_c_1\,X(x)}{a^2}\right\}\right].$$

```
> dsolve(diff(T(t),`$`(t,2)) = _c[1]*T(t));
dsolve(diff(X(x),`$`(x,2)) = _c[1]*X(x)/a^2);
```

$$T(t) = \_C1\,e^{(\sqrt{\_c_1}\,t)} + \_C2\,e^{(-\sqrt{\_c_1}\,t)},\quad X(x) = \_C1\,e^{\left(\frac{\sqrt{\_c_1}\,x}{a}\right)} + \_C2\,e^{\left(-\frac{\sqrt{\_c_1}\,x}{a}\right)}.$$

Ajratish o‘zgarmasini quyidagi ko‘rinishda almashtiramiz: $\_c_1 = -\lambda^2$.

```
> dsolve(diff(T(t),`$`(t,2)) = -lambda^2*T(t));
dsolve(diff(X(x),`$`(x,2)) = -lambda^2*X(x)/a^2);
```

$$T(t) = \_C1\sin(\lambda\,t) + \_C2\cos(\lambda\,t),\quad X(x) = \_C1\sin\left(\frac{\lambda\,x}{a}\right) + \_C2\cos\left(\frac{\lambda\,x}{a}\right).$$

Ikkinchi tenglamani yechamiz, bunda $X(0)=0$ shartni inobatga olamiz.

```
> dsolve({diff(X(x),`dollar belgisi`(x,2)) = -
lambda^2*X(x)/a^2, X(0)=0}, X(x)).
```

Endi ikkinchi chegaraviy shartni qo‘llaymiz: $X(L)=0$.

```
> _EnvAllSolutions := true:
solve(sin(lambda*L/a)=0,lambda);
```

yoki, odatiy ko‘rinishda quyidagicha bo‘ladi: `> lambda:=Pi*n*a/L.`

Shuning uchun har bir n uchun quyidagiga ega bo‘lamiz:

```
> T[n](t):=C1[n]*cos(lambda*t)+C2[n]*sin(lambda*t);
X[n](x):=sin(lambda/a*x);
```

$$T_n(t) := C1_n\cos\left(\frac{\pi\,n\,a\,t}{L}\right) + C2_n\sin\left(\frac{\pi\,n\,a\,t}{L}\right),\quad X_n(x) := \sin\left(\frac{\pi\,n\,x}{L}\right).$$

```
> u[n](t,x):=T[n](t)*X[n](x);
```

$$u_n(t,x) := \left(C1_n\cos\left(\frac{\pi\,n\,a\,t}{L}\right) + C2_n\sin\left(\frac{\pi\,n\,a\,t}{L}\right)\right)\sin\left(\frac{\pi\,n\,x}{L}\right).$$

Natijada tenglamaning umumiy yechimini hosil qilamiz:

```
> u(t,x):=Sum(u[n](t,x), n=1..infinity);
```

$$u(t,x) := \sum_{n=1}^{\infty}\left(C1_n\cos\left(\frac{\pi\,n\,a\,t}{L}\right) + C2_n\sin\left(\frac{\pi\,n\,a\,t}{L}\right)\right)\sin\left(\frac{\pi\,n\,x}{L}\right).$$

$C1_n$ va $C2_n$ koeffisientlarni topish uchun boshlang‘ich shartlardan foydalanamiz:

```
> simplify(subs(t=0,u(t,x))=F(x));
simplify(subs(t=0,diff(u(t,x),t))=f(x));
```

Bu tengliklar shuni anglatadiki, $C1\_n$ va $C2\_n$ lar $F(x)$ va $f(x)$ funksiyalarning Furye qatori bo‘yicha yoyilmasining koeffitsientlari bo‘lar ekan.

Shuning uchun u quyidagi formula bilan aniqlanadi:

```
> C1[n]:=(2/L)*int(F(x)*sin(Pi*n/L*x),x=0..L);
C2[n]:=(2/(L*lambda))*int(f(x)*cos(Pi*n/L*x),x=0..L).
```

Vanihoyat, umumiy yechimni quyidagi ko‘rinishda yozamiz:

```
> u(t,x):=
Sum((C1[n]*cos(Pi*n*a/L*t)+C2[n]*sin(Pi*n*a/L*t))*sin(Pi*
n/L*x),n = 1 .. infinity).
```

$$u(t,x) := \sum_{n=1}^{\infty} \left(\frac{2\cos\left(\frac{\pi n a t}{L}\right)}{L} \int_0^L F(x)\sin\left(\frac{\pi n x}{L}\right)dx + \frac{2\sin\left(\frac{\pi n a t}{L}\right)}{\pi n a} \int_0^L f(x)\cos\left(\frac{\pi n x}{L}\right)dx \right) \sin\left(\frac{\pi n x}{L}\right).$$

Foydalanilgan adabiyotlar ro‘yxati:

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