

ANIQ INTEGRALNING TATBIQLARI. TAQRIBIY HISOBLASH USULLARI

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"Tabiiy va aniq" fanlarga ixtisoslashtirilgan

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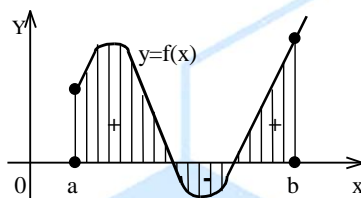
oliy toifali matematika fani o'qituvchisi

Annotatsiya: Ushbu maqolada aniq Integralning tatbiqlari va hisoblash usullari haqida ma'lumotlar berilgan.

Kalit so'zlar: Integral, dekart koordinatalar sistemasi, kesma, funksiya Dekart koordinatalar tekisligida yuzalarni hisoblash

Avvalgi bobdan ma'lumki, agar $[a, b]$ kesmada funksiya $f(x) \geq 0$ bo'lsa, u holda $y=f(x)$ egri chiziq, OX o'qi va $x=a$ hamda $x=b$ to'g'ri chiziqlar bilan chegaralangan egri chizikli trapetsiyaning yuzi
$$S = \int_a^b f(x) dx \quad (1)$$

ga teng edi. Agar $[a, b]$ kesmada $f(x) \leq 0$ bo'lsa, u holda aniq integral $\int_a^b f(x) dx \leq 0$ bo'ladi. Absolyut qiymatiga ko'ra bu integralning qiymati ham tegishli egri chizikli trapetsiyaning yuziga teng:
$$S = \left| \int_a^b f(x) dx \right| \quad (1')$$



Agar $f(x)$ funksiya $[a, b]$ kesmada ishorasini chekli son marta o'zgartirsa, u holda integralni butun $[a, b]$ kesmada qisman kesmachalar bo'yicha integrallar yig'indisiga ajratamiz. $f(x) > 0$ bo'lgan kesmalarda integral musbat, $f(x) < 0$ bo'lgan kesmalarda integral manfiy bo'ladi. Butun kesma bo'yicha olingan integral OX o'qidan yuqorida va pastda yotuvchi shakllar yuzining tegishli algebraik yig'indisini beradi (99-rasm). Yuzalar yig'indisini odatdagi ma'noda hosil qilish uchun yuqorida ko'rsatilgan kesmalar bo'yicha olingan integrallar absolyut qiymatlari yig'indisini topish yoki

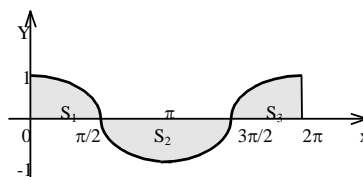
$$S = \int_a^b |f(x)| dx \quad (1'')$$
 integralni hisoblash kerak.

Agar $y_1=f_1(x)$ va $y_2=f_2(x)$ egri chiziqlar hamda $x=a$ va $x=b$ to'g'ri chiziqlar bilan chegaralangan shaklning yuzini hisoblash kerak bo'lsa, u holda $f_1(x) \geq f_2(x)$ shart

bajarilgan shaklning yuzi quyidagiga teng: $S = \int_a^b (f_1(x) - f_2(x)) dx$.

(2)

1-misol. $y = \cos x$, $y = 0$ chiziqlar bilan chegaralangan shaklning yuzi hisoblansin,



bunda $x \in [0, 2\pi]$.

Yechish. $x \in [0, \pi/2]$ va $x \in [3\pi/2, 2\pi]$ da $\cos x \geq 0$ hamda $x \in [\pi/2, 3\pi/2]$ da $\cos x \leq 0$ bo'lgani uchun

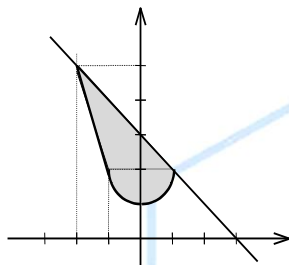
$$S = \int_0^{2\pi} |\cos x| dx = \int_0^{\pi/2} \cos x dx + \left| \int_{\pi/2}^{3\pi/2} (-\cos x) dx \right| + \int_{3\pi/2}^{2\pi} \cos x dx = \sin x \Big|_0^{\pi/2} + \left| \sin x \Big|_{\pi/2}^{3\pi/2} \right| + \sin x \Big|_{3\pi/2}^{2\pi} = \sin \frac{\pi}{2} - \sin 0 + \left| \sin \frac{3\pi}{2} - \sin \frac{\pi}{2} \right| + \sin 2\pi - \sin \frac{3\pi}{2} = 1 + |-1 - 1| - (-1) = 4$$

Demak, $s=4$ (kv. birlik) ekan.

2-misol. $y = x^2 + 1$ va $y = 3 - x$ chiziqlar bilan chegaralangan sohaning yuzini hisoblang.

Yechish. Shaklni yasash uchun avval ushbu $\begin{cases} y = x^2 + 1 \\ y = 3 - x \end{cases}$ sistemani yechib, chiziqlarning

kesishish nuqtalarini topamiz.



Bu chiziqlar $A(-2; 5)$ va $V(1; 2)$ nuqtalarda kesishadi. U holda

$$S = \int_{-2}^1 (3 - x) dx - \int_{-2}^1 (x^2 + 1) dx = \int_{-2}^1 (2 - x - x^2) dx = \left(2x - \frac{x^2}{2} - \frac{x^3}{3} \right) \Big|_{-2}^1 = \left(2 - \frac{1}{2} - \frac{1}{3} \right) - \left(-4 - \frac{4}{2} + \frac{8}{3} \right) = \frac{9}{2} \text{ (kv. birlik)}$$

Endi, tenglamasi $x = \varphi(t)$, $y = \psi(t)$ parametrik ko'rinishda berilgan chiziq bilan chegaralangan egri chizikli trapetsiyaning yuzasini hisoblaymiz. Faraz qilaylik, bu tenglamalar $[a, b]$ kesmada biror $u = f(x)$ funktsiyani aniqlasin, bunda $t \in [\alpha, \beta]$ va $\varphi(\alpha) = a$, $\varphi(\beta) = b$.

U holda egri chizikli trapetsiyaning yuzini $S = \int_a^b y dx$ formula bo'yicha hisoblanish

mumkin. Bu integralda o'zgaruvchini almashtiramiz: $x = \varphi(t)$, $dx = \varphi'(t) dt$, $y = f(x) = f(\varphi(t)) = \psi(t)$.

Demak,

$$S = \int_{\alpha}^{\beta} \psi(t)\varphi'(t)dt \tag{3}$$

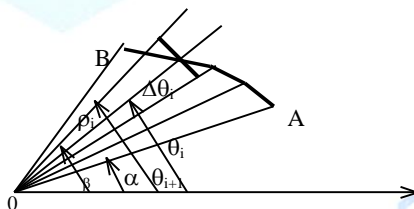
3-misol. $x=acost, y=bsint$ ellips bilan chegaralangan sohaning yuzi hisoblansin.

Yechish. Ellipsning yuqori yarim yuzini hisoblab, uni 2 ga ko'paytiramiz. $-a \leq x \leq +a$ uchun $-a=acost, cost=-1, t=\pi; a=acost, cost=1, t=0$

$$S = 2 \int_{\pi}^0 b \sin t (-a \sin t dt) = -2ab \int_{\pi}^0 \sin^2 t dt = \pi ab.$$

1.2. Tekis shakllar yuzini qutb koordinatalarda hisoblash

AB egri chiziq qutb koordinatalarida $\rho=f(\theta)$ formula bilan berilgan va $f(\theta)$ funktsiya $[\alpha, \beta]$ kesmada uzluksiz bo'lsin.



Ushbu $\rho=f(\theta)$ egri chiziq va qutb o'qlari hamda α va β burchak hosil qiluvchi ikkita $\varphi=\alpha, \varphi=\beta$ nurlar bilan chegaralangan egri chizikli sektorning yuzini aniqlaymiz. Buning uchun berilgan yuzani $\alpha=\theta_0, \theta=\theta_1, \dots, \theta=\theta_i, \dots, \theta_n=\beta$ nurlar bilan n ta ixtiyoriy qismga bo'lamiz. O'tkazilgan nurlar orasidagi burchaklarni $\Delta\theta_1, \Delta\theta_2, \dots, \Delta\theta_n$ bilan belgilaymiz. θ_{i-1} bilan θ_i orasidagi biror $\bar{\theta}_i$ burchakka mos nurning uzunligini $\bar{\rho}_i$ orqali belgilaymiz. Radiusi $\bar{\rho}_i$ va markaziy burchagi $\Delta\theta_i$ bo'lgan doiraviy sektorni qaraymiz.

Uning yuzi $\Delta S_i = \frac{1}{2} \bar{\rho}_i^2 \Delta\theta_i$ bo'ladi.

Ushbu yig'indi

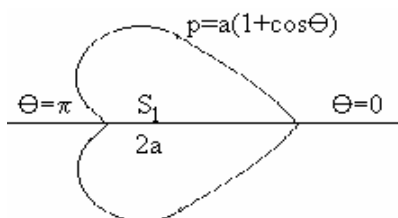
$$S_n = \frac{1}{2} \sum_{i=1}^n \bar{\rho}_i^2 \Delta\theta_i = \frac{1}{2} \sum_{i=1}^n [f(\bar{\theta}_i)]^2 \Delta\theta_i$$

zinapoyasimon sektorning yuzini beradi.

Bu yig'indi $\alpha \leq \theta \leq \beta$ kesmada $\rho^2 = [f(\theta)]^2$ funktsiyaning integral yig'indisi bo'lgani sababli uning limiti $\max \Delta\theta_i \rightarrow 0$ da $\frac{1}{2} \int_{\alpha}^{\beta} \rho^2 d\theta$ aniq integralga teng. Bu $\Delta\theta_i$ burchak ichida qanday ρ_i nur olishimizga bog'liq emas. Demak, OAV sektorning yuzi:

$$S = \frac{1}{2} \int_{\alpha}^{\beta} \rho^2 d\theta = \frac{1}{2} \int_{\alpha}^{\beta} [f(\theta)]^2 d\theta. \tag{4}$$

4-misol. $\rho=a(1+\cos\theta), a>0$ kardioida bilan chegaralangan figuraning yuzini hisoblang.



$$S = 2S_1 = 2 \int_{\alpha}^{\beta} \frac{1}{2} \rho^2 d\theta = \int_{\alpha}^{\beta} \rho^2 d\theta,$$

$$S = \int_0^{\pi} a^2 (1 + \cos\theta)^2 d\theta = a^2 \int_0^{\pi} (1 + 2\cos\theta + \cos^2\theta) d\theta =$$

$$= a^2 \int_0^{\pi} \left(\frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos 2\theta \right) d\theta = a^2 \left(\frac{3}{2}\theta + 2\sin\theta + \right.$$

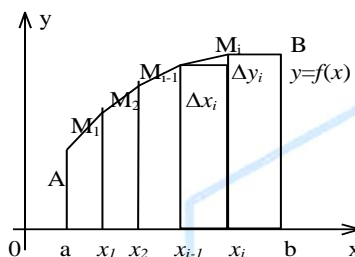
$$\left. + \frac{1}{4}\sin 4\theta \right) \Big|_0^{\pi} = \frac{3}{2}\pi a^2; S = \frac{3}{2}\pi a^2 \text{ (kv.birl.)}$$

2-§. Egri chiziq yoyining uzunligi

2.1. Yoy uzunligini dekart koordinatalar sistemasida hisoblash

To'g'ri burchakli dekart koordinatalar tekisligida egri chiziq $y=f(x)$ tenglama bilan berilgan bo'lsin.

Bu egri chiziqning $x=a$ va $x=b$ vertikal to'g'ri chiziqlar orasidagi AB yoyining uzunligini topamiz.



104-rasm.

AB yoyda abstsissalari $a=x_0, x_1, x_2, \dots, x_i, \dots, x_n=b$ bo'lgan $A, M_1, M_2, \dots, M_i, \dots, B$ nuqtalarni olamiz va $AM_1, M_1M_2, \dots, M_{n-1}B$ vatarlarni o'tkazamiz, ularning uzunliklarini mos ravishda $\Delta S_1, \Delta S_2, \dots, \Delta S_n$ bilan belgilaymiz.

AB yoy ichiga chizilgan sinq chiziqning uzunligi $s_n = \sum_{i=1}^n \Delta S_i$ bo'lgani uchun AB yoyning uzunligi

$$S = \lim_{\max \Delta S_i \rightarrow 0} \sum_{i=1}^n \Delta S_i \quad (1)$$

Faraz qilaylik, $f(x)$ funktsiya va uning $f'(x)$ hosilasi $[a, b]$ kesmada uzluksiz bo'lsin. U holda

$$\Delta S_i = \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} = \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2} \cdot \Delta x_i$$

yoki Lagranj teoremasiga asosan $\frac{\Delta y_i}{\Delta x_i} = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}} = f'(\xi_i)$,

bunda $x_{i-1} < \xi_i < x_i$ bo'lgani uchun $\Delta S_i = \sqrt{1 + (f'(\xi_i))^2} \Delta x_i$

bo'ladi. Ichki chizilgan siniq chiziqning uzunligi esa $S_n = \sum_{i=1}^n \sqrt{1 + (f'(\xi_i))^2} \Delta x_i$.

Shartga ko'ra, $f(x)$ funktsiya uzluksiz, demak, $\sqrt{1 + (f'(x))^2}$ funktsiya ham uzluksizdir. Shuning uchun integral yig'indining limiti mavjud va u quyidagi aniq integralga teng:

$$S = \lim_{\max \Delta x_i \rightarrow 0} \sum_{i=1}^n \sqrt{1 + (f'(\xi_i))^2} \Delta x_i = \int_a^b \sqrt{1 + (f'(x))^2} dx.$$

Demak, yoy uzunligini hisoblash formulasi: $S = \int_a^b \sqrt{1 + (f'(x))^2} dx = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$ (2)

ekan.

Endi egri chiziq tenglamasi

$$x = \varphi(t), y = \psi(t), \alpha \leq t \leq \beta \quad (3)$$

parametrik ko'rinishda berilgan bo'lsin, bunda $\varphi(t)$ va $\psi(t)$ uzluksiz hosilali uzluksiz funktsiyalar va $\varphi'(t)$ berilgan oraliqda nolga aylanmaydi.

Bu holda (3) tenglama biror $u=f(x)$ funktsiyani aniqlaydi. Bu funktsiya uzluksiz bo'lib, $\frac{dy}{dx} = \frac{\psi'(t)}{\varphi'(t)}$ uzluksiz hosilaga ega. $a=\varphi(\alpha)$, $b=\varphi(\beta)$ bo'lsin. (2) integ-ralda

$x=\varphi(t)$, $dx=\varphi'(t) dt$ almashtirish bajaramiz. U holda

$$S = \int_a^b \sqrt{1 + \left(\frac{\psi'(t)}{\varphi'(t)}\right)^2} \cdot \varphi'(t) dt \quad \text{ёки} \quad S = \int_a^b \sqrt{(\varphi'(t))^2 + (\psi'(t))^2} dt. \quad (4)$$

Agar egri chiziq fazoda $x = \varphi(t)$, $y = \psi(t)$, $z = \chi(t)$ (5)

parametrik tenglamalar bilan berilgan va $\varphi(t)$, $\psi(t)$, $\chi(t)$ funktsiyalar $[\alpha, \beta]$ kesmada uzluksiz hamda uzluksiz hosilalarga ega bo'lsa, egri chiziq aniq limitlarga ega bo'ladi va u

$$S = \int_a^b \sqrt{(\varphi'(t))^2 + (\psi'(t))^2 + (\chi'(t))^2} dt \quad (6) \quad \text{formula bilan aniqlanadi.}$$

2.2. Yoy uzunligini qutb koordinatalar sistemasida hisoblash

Qutb koordinatalar sistemasida egri chiziqning tenglamasi

$$\rho = f(\theta) \quad (7)$$

bo'lsin. Qutb koordinatalaridan Dekart koordinatalariga o'tish formulasi: $x = \rho \cos \theta$, $y = \rho \sin \theta$ yoki (7) dan foydalansak:

$$x = f(\theta) \cos \theta, y = f(\theta) \sin \theta.$$

Bu tenglamalarga egri chiziqning parametrik tenglamalari deb qarab, yoy uzunligini hisoblash uchun (4) formulani tatbiq qilamiz:

$$\frac{dx}{d\theta} = f'(\theta)\cos\theta - f(\theta)\sin\theta, \quad \frac{dy}{d\theta} = f'(\theta)\sin\theta + f(\theta)\cos\theta.$$

U holda $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = (f'(\theta))^2 + (f(\theta))^2 = \rho'^2 + \rho^2.$

Demak, $S = \int_{\theta_0}^{\theta_1} \sqrt{\rho'^2 + \rho^2} d\theta.$ (8)

1-misol. $x^2 + u^2 = r^2$ aylana uzunligi hisoblansin.

Yechish. Dastlab aylananing 1-chorakda yotgan to'rtidan bir qismining uzunligini hisoblaymiz. U holda AV yoyning tenglamasi

$$y = \sqrt{r^2 - x^2}, \quad \frac{dy}{dx} = -\frac{x}{\sqrt{r^2 - x^2}};$$

$$\frac{1}{4}S = \int_0^r \sqrt{1 + \frac{x^2}{r^2 - x^2}} dx = \int_0^r \frac{r}{\sqrt{r^2 - x^2}} dx = r \cdot \arcsin \frac{x}{r} \Big|_0^r = r \cdot \frac{\pi}{2};$$

Butun aylananing uzunligi: $S = 2\pi r;$

2-misol. $\rho = a(1 + \cos\theta)$ kardioidaning uzunligi topilsin. Kardioida qutb o'qiga nisbatan simmetrikdir. θ qutb burchagini 0 dan π gacha o'zgartirib, izlanayotgan uzunlikning yarmini topamiz (103-rasm). (8) formuladan foydalanamiz, bunda

$$\rho' = -a\sin\theta$$

$$\begin{aligned} S &= 2 \cdot \int_0^\pi \sqrt{a^2(1 + \cos\theta)^2 + a^2 \sin^2 \theta} d\theta = 2a \int_0^\pi \sqrt{2 + 2\cos\theta} d\theta = \\ &= 4a \cdot \int_0^\pi \cos \frac{\theta}{2} d\theta = 8a \cdot \sin \frac{\theta}{2} \Big|_0^\pi = 8a \cdot 1 = 8a. \end{aligned}$$

3-misol. $x = a\cos t, y = b\sin t, 0 \leq t \leq 2\pi,$ ellipsning uzunligi hisoblansin, bunda $a > b.$

Yechish. (4) formuladan foydalanamiz. Avval yoy uzunligining 1/4 qismini hisoblaymiz.

$$\begin{aligned} \frac{S}{4} &= \int_0^{\pi/2} \sqrt{a^2 \sin^2 t + b^2 \cos^2 t} dt = \int_0^{\pi/2} \sqrt{a^2(1 - \cos^2 t) + b^2 \cos^2 t} dt = \\ &= \int_0^{\pi/2} \sqrt{a^2 - (a^2 - b^2)\cos^2 t} dt = a \int_0^{\pi/2} \sqrt{1 - \frac{a^2 - b^2}{a^2} \cos^2 t} dt = \\ &= a \int_0^{\pi/2} \sqrt{1 - k^2 \cos^2 t} dt \end{aligned}$$

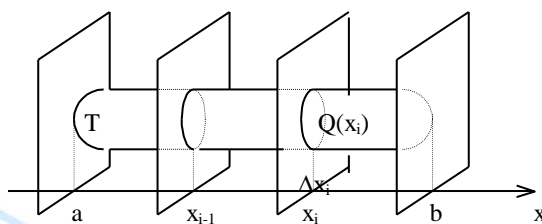
bunda $k = \frac{\sqrt{a^2 - b^2}}{a} < 1.$ Demak, $S = 4a \int_0^{\pi/2} \sqrt{1 - k^2 \cos^2 t} dt.$

3-§. Aniq integralning jism hajmlarini hisoblashga qo'llanilishi

3.1. Jism hajmini parallel kesimlar yuzalari bo'yicha hisoblash

Biror T jism berilgan bo'lsin. Bu jismni OX o'qqa perpendikulyar tekislik bilan kesishdan hosil bo'lgan har qanday kesimning yuzi ma'lum, deb faraz qilamiz. Bu holda yuza kesuvchi tekislikning vaziyatiga bog'liq, ya'ni x ning funktsiyasi bo'ladi:

$$Q = Q(x)$$



105-rasm.

$Q(x)$ ni uzluksiz funktsiya, deb faraz qilib, berilgan jism hajmini aniqlaymiz.

$X=x_0=a, x=x_1, x=x_2, \dots, x=x_n=b$ tekisliklarni o'tkazamiz. Har bir $x_{i-1} \leq x \leq x_i$ qisman oraliqda ixtiyoriy ξ_i nuqta tanlab olamiz va I ning har bir qiymati uchun yasovchisi x o'qiga parallel bo'lib, yo'naltiruvchisi T jismni $x=\xi_i$ tekislik bilan kesishdan hosil bo'lgan kesimning konturidan iborat bo'lgan tsilindrik jism yasaymiz. Asosining yuzi $Q(\xi_i)$ va balandligi Δx_i bo'lgan bunday elementar tsilindrning hajmi $Q(\xi_i)\Delta x_i$ ga teng.

Hamma tsilindrlarning hajmi $v_n = \sum_{i=1}^n Q(\xi_i)\Delta x_i$ bo'ladi.

Bu yig'indining $\max \Delta x_i \rightarrow 0$ dagi limiti berilgan jismning hajmi, deyiladi:

$$V = \lim_{\max \Delta x_i \rightarrow 0} \sum_{i=1}^n Q(\xi_i)\Delta x_i.$$

V_n miqdor $[a, b]$ kesmada uzluksiz $Q(x)$ funktsiyaning integral yig'indisidir, shuning uchun limit mavjud va u $V = \int_a^b Q(x)dx$ (1)

aniq integral bilan ifodalanadi.

Misol. $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ellipsoidning hajmi hisoblansin.

Yechish. Ellipsoidni OYZ tekislikka parallel bo'lib undan x masofa uzoqlikdan o'tgan tekislik bilan kesganda yarim o'qlari

$b_1 = b\sqrt{1 - \frac{x^2}{a^2}}, c_1 = c\sqrt{1 - \frac{x^2}{a^2}}$ bo'lgan $\frac{y^2}{(b\sqrt{1 - \frac{x^2}{a^2}})^2} + \frac{z^2}{(c\sqrt{1 - \frac{x^2}{a^2}})^2} = 1$ ellips hosil bo'ladi. Bu ellipsning

yuzi: $Q(x) = \pi b_1 c_1 = \pi bc (1 - x^2/a^2)$.

Ellipsoidning hajmi: $v = \pi bc \int_{-a}^a (1 - \frac{x^2}{a^2}) dx = \pi bc (x - \frac{x^3}{3a^2}) \Big|_{-a}^a = \frac{4}{3} \pi abc$ (kub.birl.).

3.2. Aylanma jismning hajmi

$y=f(x)$ egri chiziq Ox o'q va $x=a, x=b$ to'g'ri chiziqlar bilan chegaralangan egri chizikli trapetsiyaning OX o'qi atrofida aylanishidan hosil bo'lgan jismni qaraylik. Bu jismni abstsissalar o'qiga perpendikulyar tekislik bilan kesishdan hosil bo'lgan ixtiyoriy kesim doira bo'ladi. Uning yuzi $Q = \pi y^2 = \pi(f(x))^2$.

Hajmni hisoblashning (1) umumiy formulasini tatbiq etib, aylanma jismning hajmini hisoblash formulasini hosil qilamiz:
$$v = \pi \int_a^b y^2 dx = \pi \int_a^b (f(x))^2 dx. \quad (2)$$

Misol. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellipsni OX va OY o'qlari atrofida aylantirish natijasida hosil qilingan jismlarning hajmlarini hisoblang.

Yechish. Ellips tenglamasidan: $y^2 = \frac{b^2}{a^2}(a^2 - x^2)$; $x^2 = \frac{a^2}{b^2}(b^2 - y^2)$

Ellipsni OX o'qi atrofida aylantirishdan hosil bo'lgan jismning hajmi:

$$\begin{aligned} V = 2V_1 &= 2\pi \int_0^a y^2 dx = 2\pi \frac{b^2}{a^2} \int_0^a (a^2 - x^2) dx = 2\pi \frac{b^2}{a^2} (a^2 x - \frac{x^3}{3}) \Big|_0^a = \\ &= 2\pi \frac{b^2}{a^2} (a^3 - \frac{a^3}{3}) = \frac{4}{3} \pi ab^2; \quad V = \frac{4}{3} \pi ab^2 (\text{kub.birl.}). \end{aligned}$$

Ellipsni OY o'qi atrofida aylantirishdan hosil bo'lgan jismning hajmi:

$$\begin{aligned} V = 2V_1 &= 2\pi \int_0^b x^2 dy = 2\pi \frac{a^2}{b^2} \int_0^b (b^2 - y^2) dy = 2\pi \frac{a^2}{b^2} (b^2 y - \frac{y^3}{3}) \Big|_0^b = \\ &= 2\pi \frac{a^2}{b^2} (b^3 - \frac{b^3}{3}) = \frac{4}{3} \pi a^2 b; \quad V = \frac{4}{3} \pi a^2 b (\text{kub.birl.}). \end{aligned}$$

Aniq integral. Test-1

1. $\int_0^1 x^{90} dx$ nechaga teng?
A) 1 B) 91 C) $\frac{1}{89}$ D) $\frac{1}{90}$ E) $\frac{1}{91}$
2. $\int_2^5 x^2 dx$ nechaga teng?
A) 35 B) 36 C) 37 D) 38 E) 39
3. $\int_0^1 (x^2 + e^x) dx$ nechaga teng?
A) $\frac{2}{3}+e$ B) $\frac{1}{3}+e$ C) $1+e$
D) $-\frac{1}{3}+e$ E) $-\frac{2}{3}+e$
4. $\int_0^1 (x^2 + 4x)^3 (x + 2) dx$ nechaga teng?
A) $\frac{25}{2}$ B) $\frac{125}{16}$ C) $\frac{125}{4}$ D) $\frac{625}{8}$ E) $\frac{25}{4}$
5. $\int_0^2 (x^2 - x)^4 \cdot (2x - 1) dx$ nechaga teng?
A) $2x^3 - 5y^2$ B) $5x^3 + y^2$ C) $x^3 + 21y^2$
D) $x^3 - y^2$ E) $3x^3 - 4y^2$
18. $\int_2^{e+1} \frac{x+2}{x-1} dx$ nechaga teng?
A) e B) e-1 C) e-2 D) e+2 E) e+1
19. $\frac{m}{n} = e\sqrt{e}$ bo`lsa, $\int_m^n \frac{1}{x} dx$ nechaga teng?
A) $\frac{3}{2}$ B) $-\frac{3}{2}$ C) $\frac{2}{3}$ D) $-\frac{2}{3}$ E) $\frac{1}{3}$
20. $\int_3^9 f(x) dx = 24$ bo`lsa, $\int_1^3 f(3x) dx$ nechaga teng?
A) 8 B) 24 C) 36 D) 48 E) 72
21. $\int_0^1 (2x + 1) \cdot e^{x^2+x+1} dx$ nechaga teng?
A) e^3 B) $e^3 - e$ C) $e^2 + 1$ D) $e^2 - e$ E) $e^3 - 1$
22. $\int_1^e \frac{dx}{x(1+\ln^2 x)}$ nechaga teng?
A) 0 B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{6}$ E) $\frac{2\pi}{3}$
23. $\int_4^a \frac{x dx}{\sqrt{x^2-15}} = 6$ bo`lsa, a nechaga teng?
A) 1 B) 2 C) 6 D) 8 E) 10
24. $\int_{-1}^0 \sqrt{1+t^4} t^3 dt$ nechaga teng?
A) $\frac{\sqrt{2}}{6}$ B) $1-2\sqrt{2}$ C) $\frac{1+\sqrt{2}}{6}$ D) $\frac{1}{6}$ E) $\frac{1-2\sqrt{2}}{6}$
25. $\int_e^{e^2} \frac{dx}{x(\ln x)^2}$ nechaga teng?
A) $\frac{3}{2}$ B) $\frac{2}{3}$ C) $\frac{1}{2}$ D) $-\frac{1}{2}$ E) $\frac{3}{4}$

Foydalanilgan adabiyotlar ro'yxati:

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