EMPIRICAL ANALYSIS OF ECONOMIC GROWTH THROUGH SOLOW-SWAN GROWTH MODEL

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Introduction

Economic growth is considered to be one of the most important topic in the field of economics, since it has been facilitating many discrepancies among economist and researchers. Consequently, this divergence of ideas initiated the emergence of new branch in macroeconomics denoted as economic growth theory. Therefore, the main purpose of this assignment is to analyze economic data and investigate the relationships of empirical analysis describing Solow-Swan growth model. Regarding the structure of the report, we will initially present the empirical literature review of academic papers, moving to the second part, data description and its collection methodology, analysis of priori and OLS estimates will be provided by supporting with the outcomes of statistic software. Following this, interpretation of empirical results and explanation of OLS assumptions will be given. Then we will summarize all the points in conclusion part.

Literature review

Throughout the years, the issue of economic growth has been one of the main aspect of economics which has caused many controversial among researchers and policymakers. As sustainable economic growth is directly associated with high standards of living, there is no vital issue than understanding the determinants which affect economic growth. Therefore, the Solow-Swan model was developed by Robert Solow and Trevor Swan to analyze the effects of economic variables, particularly capital stock, increase in labor force, advancements in technology and population, on growth of economy.

One of the main sources of economic growth is human capital which is measured in terms of health and the level of education. As Gallup et al., (1998) concludes, there is positive relationship between GDP per capita and human capital. Moreover, he emphasizes that well-educated labor force is expected to produce more products from a limited resource, than less skilled labor. Additionally, Romer (1990) claims that well-educated labor force creates new products and ideas that that will boost the technological progress in the economy and the country with high index of human capital is likely to introduce and adapt new technologies rapidly which will increase the economy faster.

According to L.Guerrini (2005), capital stock is also considered to be one of the important factors of economic growth. However, he mentions that capital stock can change over time, since some portion of it depreciates or wears out and can no longer be utilized for production. Moreover, the author conducts empirical tests and concludes that per capita capital of a particular county with labor growth will eventually tend to stabilize the steady-state of the country. Some small variations in the initial capital does not have large influence on economic growth. In a country with constant initial per capita capital but with high labour force growth rate will have less per capita consumption. However, as the author highlights, both variables will be stabilized in the long run.

Regarding population growth, Meier (1995) explains that there are many arguments due to positive and negatives sides of population growth on increase of national economy. As he states, population growth increases labor force and consumption thus providing large domestic market for a country`s economy. Furthermore, demographic improvements encourage competition, which causes advancement in technology and innovation. On the other hand, food problems, decrease in savings and scarcity of human resources are also associated with large population growth. Additionally, Tsen and Furuoka (2005), in their article “The Relationship between Population and Economic Growth in Asian Economies”, summarizes that there is no exact relationship between population and economic growth and this link depends on the level of human capital brought into economy.

Data description and its collection methodology

The purpose of this methodology is to analyze the effects of each economic variable in the given Solow-Swan model on the overall economic growth by conducting several statistic tests. In order to analyze this influence, we are required to select countries from provided data. Hence, information of several countries namely Australia, Canada, Switzerland, Germany, France, Spain, Italy, Japan, Netherlands and Portugal has been selected for the period of 1984 and 2014.

It is important to distinguish two econometric models such as linear regression model and log-linear regression model for further empirical analysis. For this reason, we can use MWD test to select the most appropriate one.

Following two models are under consideration:

Linear model: $Y_t = \alpha_0 + \alpha_1 L_t + \alpha_2 K_t + a_3 X_t + u_t$

Log-linear model: $\ln Y_t = \beta_0 + \beta_1 \ln L_t + \beta_2 \ln K_t + \beta_3 \ln X_t + u_t$

where Y is the Gross Domestic Product per capita, L is the human capital– proxy for labor productivity, K is a gross fixed capital formation (as % of GDP) – proxy for capital, X is a number of population – proxy for labor and u_t is a residual. Regarding the priori, α_1 and β_1 are expected to be positive, since human capital is truly associated with economic growth. As we mentioned earlier, high index of human capital leads to an increase in productivity of labor force and thus causing a growth in economy.

Therefore, human capital has positive relationship with GDP per capita. Moreover, α_2 and β_2 are expected to be positive as well, because increase in stock of capital such as machinery and equipment purchases, construction of roads, railways schools etc., will cause a positive change on national output and GDP per capita. Finally, α_3 and β_3, as coefficients of population growth variable, causes some disagreements to decide whether it is positively or negatively related with economic growth. As we mentioned in review, population growth increases labor force and consumption thus providing large domestic market for a country`s economy. Furthermore, demographic improvements encourage competition, which causes advancement in technology and innovation. However, food problems, decrease in savings, scarcity of human resources and poverty are also associated with demographic changes (Meier, 1995). Nevertheless, since we are dealing with GDP per capita, which formulates GDP divided by labor force, we can assume that there is a negative relationship between population and GDP per capita.

MWD test:

Now, we will estimate the linear and log-linear models and obtain the estimated Y and lnY values respectively. The regression results are presented as follows:

$$\widehat{Y}_t = -34288.79 + 22646.6 L_t + 336.1183 K_t - 0.0001072\, X_t$$

t = (-8.76) (25.63) (2.61) (-9.19)

$F = 232.08 \quad R^2 = 0.6947$

$$\ln \widehat{Y}_t = 10.00661 + 1.86336 L_t + 0.021764 K_t - 0.0953614\, X_t$$

t = (33.98) (34.95) (2.33) (-9.38)

$F = 415.74 \quad R^2 = 0.8030$

According to results, both linear model and log linear model apparently fit the data well. All the parameters show expected signs. Although R^2 of the log-linear model is higher, we cannot decide strongly between models without conducting MWD test. Firstly, we test by hypotizing that the true model is linear. As a result, we got the following regression:

$$\widehat{Y}_t = -36878.79 + 23140.3L_t + 445.2449K_t - 0.0001282\,X_t - 42388.58Z_1$$

t = (-41.98) (116.69) (15.41) (-48.75) (-76.0)

$F = 4902.81 \quad R^2 = 0.9847$

Z_1 in this case represent the difference between $\ln\widehat{Y}_t$ and $\ln Y$. Since the p-value (0.0001) is less than α (0.05), Z_1 is statistically significant, which means we can reject null hypothesis that the true model is linear. Now, let us turn the positions and then true model is log-linear. Following steps of MWD test, we estimate the following results:

$$\ln\widehat{Y}_t = 9.490952 + 1.836188\ln L_t + 0.0549545\ln K_t - 0.0697773\ln X_t + 0.0000203Z_2$$

t = (88.89) (95.48) (2.30) (-18.83) (45.27)

$F = 2911.87 \quad R^2 = 0.9745$

Since the coefficient of Z_2 is statistically significant with p-value equal to 0.0001, we can reject the hypothesis which is log-linear model.

As we observed, the MWD test showed that both linear and log-linear models are statistically significant. Therefore, we can select the most appropriate one by looking at R^2. Thus, log- linear model is considered to be the most suitable model with R^2 equal to 0.8030.

Cook-Weisberg Heteroscedasticity test:

In order to check the assumption of OLS about the variance of residuals and classify it as homoscedastic or heteroscedastic, we are going to analyze the data by examining properties of homoscedasticity and heteroscedasticity. The graphical representation does not display constant variances of residuals among each variable, which indicates heteroscedasticity. Furthermore, Breusch-Pagan and Cook-Weisberg test for heteroscedasticity has been conducted, which concludes that there is sufficient evidence to reject null hypothesis, since p-value (0.0040) is less than significance level (0.05). Therefore, our data violates fourth assumption of OLS and describes heteroscedastic variance of residuals.

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Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: fitted values of newGDPpc

         chi2(1)      =     8.26
         Prob > chi2  =   0.0040
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Multi-collinearity test:

One of the important assumptions of multiple regression model is multi-collinearity of variables which examines whether independent variables are collinear or not.

Variable	VIF	1/VIF
newPopnumber	1.07	0.934578
newCapshare	1.04	0.957111
newHC	1.03	0.968858
Mean VIF	1.05	

Looking at the results, we can conclude that our variables are not multi-collinear, since our 1/VIF values demonstrate higher rate compared to 0.10.

Specification error test:

Following this, the specification test has been conducted to examine the necessity of adding additional variables to the model. Regression is conducted with GDP per capita (Y) against Y-hat and Y-hat-squared, the null hypothesis being there is no specification error.

newGDPpc	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
_hat	2.652688	1.69794	1.56	0.119	-.6883848	5.99376
_hatsq	-.0793908	.0815533	-0.97	0.331	-.239865	.0810834
_cons	-8.592835	8.831873	-0.97	0.331	-25.9715	8.78583

As results suggest, the p-value of hat-square (0.331) is higher than significance level (0.05), which means that we can reject null hypothesis and conclude that there is no specification error. Thus, model is noticeably specified.

Jarque-Bera test of normality:

The next issue of the analysis is the normality assumption and it is very important to check data for normality distribution. Therefore, the Jarque-Bera (JB) test of normality is solution for this problem. Firstly, the skewness and kurtosis of OLS residuals will be computed and the following test statistic is used:

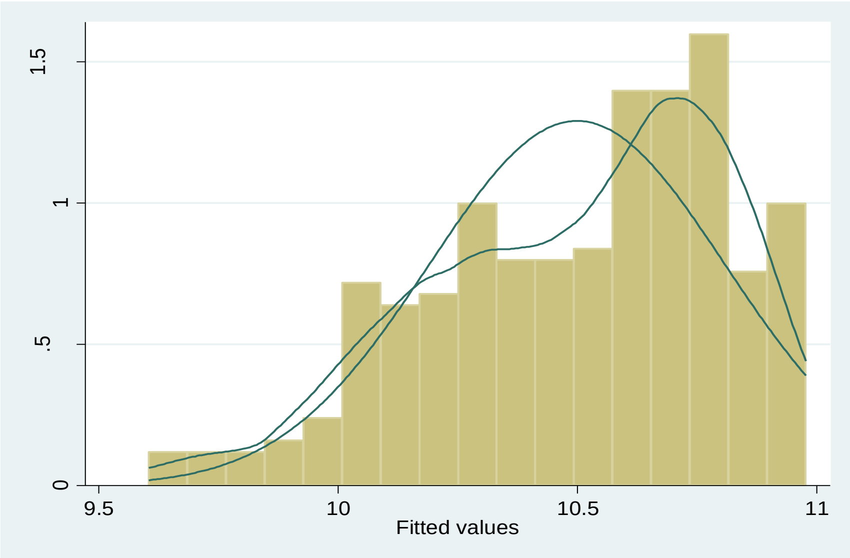
$$JB = n\left[\frac{S^2}{6} + \frac{(K-3)^2}{24}\right]$$

Results of the test using Stata software are presented as follows:

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
resid	310	0.0001	0.1099	16.04	0.0003

According to results, the p-value of the JB statistic is sufficiently low (0.0003). Moreover, the value of JB statistic (107.88) is by far higher than 0. Therefore, we can reject null hypothesis that the residuals are normally distributed and conclude that our residuals are not normally distributed. We can simply observe it looking at the graph as well.



Interpretation of empirical results

As we investigated before, log-linear model is considered to be most appropriate model in our empirical analysis and interpretation of the results will also be described using this model as follows:

$$\ln \widehat{Y}_t = 10.00661 + 1.86336\, L_t + 0.021764\, K_t - 0.0953614\, X_t + u_t$$

In fact, this log-linear model describes the effects of independent variables, such as human capital (L_t), gross fixed capital (K_t) and population growth (X_t), on dependent variable – GDP per capita. When we analyze in details, it is clear that human capital demonstrated positive relationship with GDP per capita as it was expected in priori. According to results, 1% change in human capital causes 1.8633% growth in GDP per capita, holding ceteris-paribus assumption. Moreover, the next variable – gross fixed capital formation is also described positive relationship with GDP per capita, as it was expected by theory. This can be supported by results of the regression, which states that 1% change in the variable leads to 0.022% increase in dependent variable. In contrast, population growth indicated negative correlation with regressand, as it was defined and assumed by the theory. The results confirm the theory, estimating that 1% growth in population causes 0.095% decrease in GDP per capita.

Source	SS	df	MS
Model	29.5053154	3	9.83510515
Residual	7.23906605	306	.023657079
Total	36.7443815	309	.118913856

Number of obs	310
F(3, 306)	415.74
Prob > F	0.0000
R-squared	0.8030
Adj R-squared	0.8011
Root MSE	.15381

newGDPpc	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
newHC	1.86336	.0533224	34.95	0.000	1.758435	1.968285
newCapshare	.021764	.066166	2.33	0.002	-.1084339	.1519619
newPopnumber	-.0953614	.0101621	-9.38	0.000	-.1153579	-.0753649
_cons	10.00661	.2944861	33.98	0.000	9.427135	10.58608

Regarding to the significance of variables, we can conclude that all the variables are statistically significant, since p-values of each variable is less than in any level of significance. Additionally, t-values of each variable is also less than critical value 1,96. Moreover, overall significance of the model is also subject of analysis. As results are given, p-value of F-test (0.0000…) is extremely lower than any significance level. Consequently, we can reject null hypothesis and summarize that overall this model is statistically significant.

R^2 in this analysis can be interpreted as 80.3% of variations in GDP per capita can be explained by variables in this model, while 19.7% belongs to other factors such as inflation, political stability and etc, which may influence GDP per capita as well.

In conclusion, we can summarize our empirical results and highlight that the government of selected countries are recommended to focus on improvement of human capital, since it contributes the largest share (1.86%) to GDP per capita than any other economic variables in the module. Although, the share of GFCF on GDP per capita is quite small, but it is also positive, which means countries can benefit by increasing the stock of capital. Additionally, government is highly suggested to implement effective policies to control rapid population growth, since it is negatively associated with GDP per capita.

Explanation of OLS assumptions

The assumptions underlying the method of least squares make it easier for us to understand econometric models. Hence, explanation of OLS assumptions for the above-mentioned regression model is as follows. The first assumption of OLS, which states that the regression model is linear in parameters, is hold for our model, since the degree of the parameters in this model is equal to 1. The second assumption of OLS about the fixedness of X-values and independence of them from error terms is also valid for the model above, as all the independent variables, particularly human capital, gross fixed capital formation and population growth, are independent of the error term. The validity of third assumption is confirmed for this model, since expected value of residuals is zero and there is no auto correlation between the disturbances. Regarding the fourth assumption of OLS, our model violated the condition of homoscedasticity and it was checked by testing for heteroscedasticity. The results confirmed that, our data describes not constant but heteroscedastic variance of residuals. Moreover, the number of observations in the module is 310, which is sufficiently greater than 4 parameters. Following this, multi-collinearity test has been generated to check the eighth assumption of OLS which has been concluded that there are no perfect relationships among independent variables. Moving to the next assumption, the specification test has been conducted earlier and concluded that there is no specification bias, which means our model is correctly specified. Lastly, the assumption of normal distribution of residuals is under investigation. Therefore, JB test has been conducted and failed to reject null hypothesis, which means the residuals in the model are not normally distributed.

Conclusion

So far, we have analyzed the data for selected 10 countries from the period of 1984 till 2014 and attempted to investigate the topic of economic growth using Solow-Swan model. Moreover, we have evaluated the determinants of the model namely human capital, gross fixed capital formation, population growth and their effects to GDP per capita by examining them using several statistic tests. To conclude, although the model has violated some assumptions of OLS, it demonstrated both significance of variables and overall significance of the model as well.

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