MATEMATIKA DARSLARIDA LIMIT TEOREMA

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Annotatsiya: Ishda ma'lum davrda uzluksiz tarmoqlanish jarayonidan boshlangan tarmoqlanish jarayon uchun limit teorema isbotlangan.

Tayanch so'zlar: Tarmoqlanish jarayon, tasodifiy sondan boshlanadigan jarayon, hosil qiluvchi funksiya, xarakteristik funksiya, ommaviy xizmat nazariyasi, demografik jarayon, energiya miqdori.

Tarmoqlanish jarayoni uchun refarativ xarakterdagi ma'lumotlar [1] da keltirilgan.

Robbins ishida [2] yoritilgan masalani uzluksiz tarmoqlanish jarayoniga ko'chiramiz. Bu masala diskret tarmoqlanish jarayoni uchun [3] da ko'rilgan.

μ_t bilan t vaqtdagi uzluksiz tarmoqlanish jarayonini belgilaymiz va μ_t uchun quyidagi shartlarni kiritamiz.

$P_k(\Delta t)$ bilan bitta zarracha Δt vaqt ichida k ta zarrachaga aylanish ehtimolligi,

$$P_k = \lim_{\Delta t\to 0}\frac{P_k(\Delta t)}{\Delta t}, k \neq 1, P_1 = \lim_{\Delta t\to 0}\frac{P_k(\Delta t)-1}{\Delta t},$$

$$\sum_{k=0}^{\infty} P_k = 0, \qquad f(s) = \sum_{k=0}^{\infty} P_k S^k, a = f'(1), \quad f''(1) = b,$$

$$F(t,s) = MS^{\mu_t} = \sum_{k=0}^{\infty} P(\mu_t = k)S^k, |S| \leq 1, \mu_t^{(i)}, i = \overline{1, x_t}$$

$va\ x_t$ uzluksiz tarmorlanish jarayonlar bog'liqsiz, $\mu_t^{(i)}$ i bo'yicha bog'liqsiz va μ_t bilan bir xil taqsimlangan jarayon bo'lsa quyidagi yig'indini qaraymiz: $Z_{t,x_t}(\mu_t)=\mu_t^{(1)}+\mu_t^{(2)}+\mu_t^{(3)}+ \dots +\mu_t^{(x_t)}$, P($x_t$=0)=0

x_t uchun quyidagi talablarni kiritamiz:

$$q_k = \lim_{\Delta t\to 0}\frac{q_k(\Delta t)}{\Delta t}, k \neq 1, q_1 = \lim_{\Delta t\to 0}\frac{q_1(\Delta t)-1}{\Delta t}, \sum_{k=0}^{\infty} q_k = 0,$$

bu yerda $q_k(\Delta t)$ bitta zarrachani Δt vaqt ichida k ta zarrachada aylanish ehtimolligi, hosil qiluvchi funksiyalarni kiritamiz:

$$H_t(s) = Ms^{x_t}, N(s) = \sum_{k=0}^{\infty} q_k\, s^k, \quad a_1 = N'(1), \quad b_1 = N''(1).$$

$$MS^{z_{t,x_t}(\mu_t)} = \sum_{k=1}^{\infty} P(x_t = k)F^k(t,s) = H_t(F(t,s))$$

ligiga ishonch hosil qilish mumkin. Oxirgi ifodani **s** bo'yicha 1- va 2-tartibli hosilalarini hisoblab topamiz.

$$(MS^{z_{t,x_t}(\mu_t)})'_{s=1} = (H_t(F(t,s)))'_{s=1} = (F'(t,s))'_{s=1} e^{at} e^{a_1 t},$$

$$(MS^{z_{t,x_t}(\mu_t)})''_{s=1} = (H_t(F(t,s)))''_{s=1}((F'(t,s))'_{s=1})^2_+(H_t(F(t,s)))'_{s=1} \cdot (F''(t,s))_{s=1}$$

Natijada faktorial momentlar

$$MZ_{t,x_t}^{(\mu_1)} = e^{at} e^{a_1 t}$$

$$\mathrm{M}Z_{t,x_t}(\mu_t)\big(Z_{t,x_t}(\mu_t) - 1\big) = \frac{b}{a} e^{at}(e^{at} - 1)e^{a_1 t} + \frac{b_1}{a_1} e^{a_1 t}(e^{a_1 t} - 1) \cdot e^{2at}$$

Endi quyidagi normallashtirilgan va markazlashgan jarayonni qaraymiz:

$$\eta_{\,t,x_t}(\mu_t) = \frac{\mu_t^{(1)} + \mu_t^{(2)} + \mu_t^{(3)} + \dots + \mu_t^{(x_t)} - MZ_{t,x_t}(\mu_t)}{\sigma_t} \sqrt{k_t},$$

bu yerda $k_t = 1 + \frac{b_1}{a_1} \cdot \frac{a}{b} e^{a_1 t + at}$, yuqoridagi shartlarda quyidagi teorema o'rinli:

Teorema. Agar $a_1 > 0$, $a < 0$, $b, b_1 < +\infty$, $t \to \infty$ da x_t $e^{a_1 t}$ atrofidagi qiymatlarni qabul qilsa, u holda

$$\lim_{t\to\infty} P\big(\eta_{\,t,x_t}(\mu_t) < x\big) = \Phi(x),$$

bu yerda $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{u^2}{2}} du$

Teoremani isbotlash uchun quyidagi lemmani isbotlaymiz:

Lemma. Teorema shartlari bajarilsa, $\eta_{\,t,x_t}(\mu_t)$ ning xarakteristik funksiyasi $|s| < T, T \epsilon R$ da

$$\lim_{t\to\infty} \psi_t(s) = e^{-\frac{s^2}{2}}$$

o'rinli

Lemma isboti. Ma'lumki,

$$\psi_t(s) = \sum_{k=1}^{\infty} P(x_t = k) e^{-\frac{i e^{at} e^{a_1 t} s \sqrt{k_t}}{\sqrt{\sigma_t^2}}} \cdot \left(F\left(t, e^{\frac{is\sqrt{k_t}}{\sigma_t}}\right)\right)^k =$$

$$= \sum_{k=1}^{\infty} P(x_t = k) e^{\frac{i(x - e^{a_1 t}) e^{at} s \sqrt{k_t}}{\sigma_t}} \left(e^{\frac{-ise^{at}\sqrt{k_t}}{\sigma_t}} F\left(e^{\frac{is\sqrt{k_t}}{\sigma_t}}\right)\right)^k \qquad (2)$$

Qo'shimcha xarakteristik funksiyasini kiritamiz:

$$\bar{\psi}(s) = \sum_{k=1}^{\infty} P(x_t = k) e^{\frac{i(k-e^{a_1 t})e^{at} s\sqrt{k_t}}{\sigma_t}} \left(e^{\frac{-ise^{at}\sqrt{k_t}}{\sigma_t}} F(e^{\frac{is\sqrt{k_t}}{\sigma_t}})\right)^{e^{a_1 t}} \quad (3)$$

(2) ва (3) дан

$$|\psi_t(s) - \bar{\psi}(s)| = \sum_{k=1}^{\infty} P(x_t = k) \left| e^{\frac{i(k-e^{a_1 t})e^{at} s\sqrt{k_t}}{\sigma_t}} \right| \cdot$$

$$\left| (e^{\frac{-ise^{at}\sqrt{k_t}}{\sigma_t}} F(t, e^{\frac{is\sqrt{k_t}}{\sigma_t}})^k - \left(e^{\frac{-ise^{at}\sqrt{k_t}}{\sigma_t}} F\left(e^{\frac{is\sqrt{k_t}}{\sigma_t}}\right)\right)^{e^{a_1 t}} \right| =$$

$$= \sum_{k=1}^{\infty} P(x_t = k)|D_1(t)||D_2(t)| \quad (4)$$

[3] ga asosan t→ ∞da D_2 (t) = 0 (D_{μ_t}) ва $|D_1(t)|$→0 demak (4) dan

$$\psi_t(s) - \bar{\psi}_t(s) \to 0 . \quad (5)$$

Ikkinchi tomondan Teylor qatoriga yoyib,

$$\left[e^{\frac{-ise^{at}\sqrt{k_t}}{\sigma_t}} F\,(t, e^{\frac{is\sqrt{k_t}}{\sigma_t}})\right]^{e^{a_1 t}} = \left[1 - \frac{ise^{at}\sqrt{k_t}}{\sigma_t} - \frac{s^2 e^{2at} k_t}{2\sigma_t} + o(e^{at})(1 + \right.$$

$$\left. + \frac{ise^{at}}{\sigma_t}\sqrt{k_t} - \frac{s^2 F''(t,1)}{2\sigma_t^2} + o(e^{at}))\right]^{e^{a_1 t}} = \left(1 - \frac{s^2 K_t}{2\,\sigma_t^2}(F''(t,1) - e^{2at})\right)^{e^{a_1 t}} =$$

$$= e^{-\frac{s^2}{2}\cdot\frac{K_t}{\sigma_t^2}} (F''(t,1) - e^{2at})^{e^{a_1 t}} + \mathrm{o}\,(e^{at}) = e^{-\frac{s^2}{2}} (1+\mathrm{o}(e^{at})) \quad (6)$$

$x_t\ ni$ $e^{a_1 t}$ atrofida yig'ilganini hisobga olsak, t → ∞da

$$\sum_{k=1}^{\infty} P(x_t = k) e^{\frac{i(k-e^{a_1 t}) s\sqrt{k_t}}{\sigma_t}} \to 1 \quad (7)$$

bo'ladi. (4) - (7) lar yig'ilsa

$$\lim_{t\to\infty} \psi_t\,(s) = e^{-\frac{t^2}{2}} \quad (8)$$

kelib chiqadi.

Demak (5), (8) dan lemma isbotlandi.

Teoremani isboti lemmadan kelib chiqadi. Bu teorema ommaviy xizmat nazariyasida, demografik jarayonda, kimyo, biologiya va boshqa jarayonlarda muhim ro'l o'ynaydi.

ADABIYOTLAR

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