# MATEMATIKA FANIDA EHTIMOLLARNI QO‘SHISH FORMULALARI VA TEOREMALAR ASOSIDA YECHISH

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**Annotatsiya:** Mazkur maqolada matematika fanida dolzarb ahamiyatga ega bo’lgan ehtimollar nazariyasi masalasi muhokama qilingan. Maqolada ehtimollarni qo‘shish formulasining ayrim umumlashmalarining formulalar va teoremalar asosida yechimi berilgan.

**Kalit so’zlar:** matematika, ehtimollar nazariyasi, ehtimollarni qo‘shish formulasi, teorema, gipergeometrik taqsimot.

**Аннотация:** В данной статье рассматривается вопрос теории вероятностей, имеющий актуальное значение в математике. В статье дается решение некоторых обобщений формулы сложения вероятностей на основе формул и теорем.

**Ключевые слова:** математика, теория вероятностей, формула сложения вероятностей, теорема, гипергеометрическое распределение.

**Abstract:** This article discusses the issue of probability theory, which is of current importance in mathematics. The article gives a solution to some generalizations of the probability addition formula based on formulas and theorems.

**Keywords:** mathematics, probability theory, probability addition formula, theorem, hypergeometric distribution.

## KIRISH

Matematika fanida dolzarb ahamiyatga ega bo’lgan ehtimollar nazariyasi masalasi muhokama qilingan. Ehtimollarni qo‘shish formulasining ayrim umumlashmalarining formulalar va teoremalar asosida yechimni berish mumkin. Kuzatilayotgan yoki ustida tajriba o’tkazilayotgan hodisa bir nechta hodisalarning natijasi, ya’ni bir nechta hodisalardan hech bo’lmaganda bittasining ro’y berishidan yoki bir nechta hodisalarning hammasi bir paytda ro’y berishidan va hakozolardan, iborat bo’lishi mumkin, bu esa kuzatilayotgan hodisani bilish uchun hodisalar ustida qo’shish yoki ko’paytirish amallarini bajarish demakdir.

Hodisalarning quyidagi sistemasini qaraymiz:

$$A_1^{(1)}, A_2^{(1)}, \dots, A_N^{(1)}, \qquad (1^1)$$

$$\ldots\ldots$$
$$A_1^{(k-1)}, A_2^{(k-1)}, \ldots, A_N^{(k-1)}, \qquad (1^{k-1}) \qquad (1)$$

Faraz qilaylik, biror tajribada ushbu hodisalardan ko'pi bilan N tasi ro'y berishi mumkin, yani agar

$$\xi_i\left(i=\overline{1,k-1}\right)-(1^i)$$ tipdagi

hodisalar ro'y berishlar soni bo'lsa, u holda ushbu tengsizliklar o'rinli:

$$0 \le \xi_i \le N,\ i=\overline{1,k-1},$$
$$0 \le \xi_1+\xi_2+\cdots+\xi_{k-1} \le N.$$

## MUHOKAMA VA NATIJALAR

Teorema. Tajribada $(1^1)$ hodisalardan $m_1$ tasining va h.k $(1^{k-1})$ hodisalardan $m_{k-1}$ tasining ro'y berish ehtimoli ushbu formula bilan aniqlanadi:

$$p_N(m_1,m_2,\ldots,m_{k-1})=\sum_{r=0}^{m_k}(-1)^r\sum_{i_1=0}^{r}\sum_{i_2=0}^{r-i_1}\cdots\sum_{i_{k-2}=0}^{r-i_1-\cdots-i_{k-3}}S_r(m_1,\ldots,m_{k-1},i_1,\ldots,$$
$$i_1,\ldots,i_{k-2})\cdot\prod_{i=1}^{k-2}C_{m_j+i_j}^{m_j}\cdot C_{m_{k-1}+r-i_1-\cdots-i_{k-2}}^{m_{k-1}},$$

(2)

bu yerda

$$m_k=N-m_1-\cdots-m_{k-1},$$
$$S_r(m_1,\ldots,m_{k-1},i_1,\ldots,i_{k-1})=$$
$$=\sum_{j^{(1)},\ldots,j^{(k-1)}}^{N}P\left(A_{j_1^{(1)}}\cdot\ldots\cdot A_{j_{m_1+i_1}^{(1)}}\cdot\ldots\cdot A_{j_1^{(k-1)}}\cdot\ldots\cdot A_{j_{m_{k-1}+r-i_1-\cdots-i_{k-2}}^{(k-1)}}\right).$$

$k=2$ da (2) tenglikdan [2] ishda kelt irilgan natija, $k=3$ da esa [3] ishdagi natija kelib chiqadi.

Agar $A_1^{(1)},\ldots,A_N^{(k-1)}$ hodisalar to 'plamda bog 'liq emas va

$$A_r^{(i)}\cdot A_r^{(j)}=\emptyset\,;i\neq j;\ i,j=\overline{1,k-1};\ r=\overline{1,k-1},$$
$$P\left(A_1^{(i)}\right)=P\left(A_2^{(i)}\right)=\cdots=P\left(A_N^{(i)}\right)=P_i, i=\overline{1,k-1}$$

deb faraz qilinsa, (2) formuladan polinomial taqsimot formulasi kelib chiqadi.

Teoremaning isboti. Teoremani qo'shish va ajratib olish metodi yordamida isbot qilamiz.

Faraz qilaylik, $\Omega=(\omega)$ elementar hodisalar fazosi va

$$G=(\xi_1=m_1,\xi_2=m_2,\ldots,\xi_{k-1}=m_{k-1})$$

bo'lsin, u holda ushbu tenglik o'rinli:

$$P_N(m_1,\dots,m_{k-1}) = P(G) = \sum_{\omega=G} P(\omega),$$

yani agar (2) tenglikning o'ng tomoni elementar hodisalar bo'yicha yoyilsa, $G$ ga tegishli ixtiyoriy $\omega$ elementar hodisaning ehtimolining koeffitsenti birga teng bo'ladi. Demak, teoremani isbotlash uchun $G$ ga tegishli ixtiyoriy elementar hodisa (2) ning o'ng tomoniga birga teng koeffitsent bilan kirishini ko'rsatish etarli.

Aytaylik, $\omega_0 = G\,(1^i)$ hodisalardan $n_i$ tasining $(i = \overline{1,k-1})$ tarkibiga kirsin, $a(n,m)$ orqali $P(\omega_0)$ ehtimolning koeffitsentini belgilaylik, bu yerda

$$n = n_1 + n_2 + \dots + n_{k-1}$$
$$m = m_1 + m_2 + \dots + m_{k-1}$$

U holda (2) formuladan ushbu tenglikni hosil qilamiz:

$$a(n,m) = \sum_{r=0}^{n-m} (-1.)^r \sum_{i_1=0}^{r} \dots \sum_{i_{k-2}=0}^{r-i_1-\dots-i_{k-3}} \prod_{j=1}^{k-2} C_{m_j+i_j}^{m_j} \cdot C_{n_j}^{m_j+i_j} \cdot$$
$$C_{m_{i-1}+r-i_1-\dots-i_{k-2}} \cdot C_{n_{k-1}}^{m_{k-1}+r-i_1-\dots-i_{k-2}} \qquad (3)$$

Quyidagi tenglik o'rinli bo'lishini ko'rsatish qiyin emas:

$$C_{m_j+i_j}^{m_j} \cdot C_{n_j}^{m_j+i_j} = C_{n_j}^{m_j} \cdot C_{n_j+m_j}^{i_j}.$$

Ushbu tenglikga ko'ra (3) quyidagi ko'rinishga keladi:

$$a(n,m) = \prod_{j=1}^{k-2} C_{n_j}^{m_j} \cdot C_{n_{k-1}}^{m_{k-1}} \cdot \sum_{r=0}^{n-m} (-1)^r \sum_{i_1=0}^{r} \dots$$
$$\dots \sum_{i_{k-r}=0}^{r-i_1-\dots-i_{k-3}} \prod_{j=1}^{k-1} C_{n_j-m_j}^{i_j} \cdot C_{n_{k-1}-m_{k-1}}^{r-i_1-\dots-i_{k-2}}.$$

**XULOSA**

Ma'lumki,

$$\left(\prod_{j=1}^{k-2} C_{n_j-m_j}^{i_j} \cdot C_{n_{k-1}-m_{k-1}}^{r-i_1-\dots-i_{k-2}}\right)\left(C_{n-m}^{r}\right)^{-1}$$

ifoda gipergeometrik taqsimotning umumlashmasi bo'ladi va taqsimot qonunining xossasiga ko'ra o'zgaruvchilarning barcha qiymatlari bo'yicha yig'indi birga teng.

Bunga ko’ra ushbu tenglikga ega bo’lamiz:

$$\sum_{i_1=0}^{r}\cdots\sum_{i_{k-2}=0}^{r-i_1-\cdots-i_{k-3}}\prod_{j=1}^{k-2}C_{n_j-m_j}^{i_j}\cdot C_{n_{k-1}-m_{k-1}}^{r-i_1-\cdots-i_{k-2}}=C_{n-m}^{r}$$

Shunday qilib,

$$a(n,m)=\prod_{j=1}^{k-2}C_{n_j}^{m_j}C_{n_{k-1}}^{m_{k-1}}\sum_{r=0}^{n-m}(-1)^r C_{n-m}^{r},$$

tenglikga ega bo’lamiz. Ushbu tenglikdan ko ‘rinadiki, $n_1=m_1,\ldots,n_{k-1}=m_{k-1}$ da $a(n,m)=1$ va boshqa hollarda $a(n,m)=0$. Teorema isbot bo ‘ldi.

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