PUASSON TENGLAMASINI SINOV FUNKSIYALAR METODIDAN FOYDALANGAN HOLDA YECHISH

Mirzayeva Bo`rixol Muxammat qizi
Termiz Davlat Unversiteti
Amaliy matematika (sohalar bo`yicha) 2- kurs magistranti

Annotatsiya: Ushbu maqolada Puasson tenglamasini sinov funksiyalar metodidan foydalangan holda yechimlarini topish usullari bayon qilingan.

Kalit so'zlar: Puasson tenglamasi, Direxli masalasi, ayirmali usullar, iteratsiya metodi, sinov funksiya, to`rtburchakli soxa, spektr to`ri.

Аннотация: В данной статье тестируются функции уравнения Пуассона описаны нахождения решений с помошью метода.

Ключевые слова: уравнение Пуассона, задача Дириле , дифференсиалъные методы , итерационный метод, тестовая функция прямоугольная область спектралъная сетка.

Abstract: The article tests functions of the Poisson equation methods of finding solutions using the method are described.

Key words: Poisson`s equation, Dirichlet`s problem, differensial methods, iteration method, test function, rectangular areas, spectral grid.

Ko`pgina amaliy masalalar ayirmali sxemalarni va iteratsiya metodlaridan foydalangan holda yechiladi. Sinov funksiyalar metodidan foydalanib biror bir funksiyani yechishda asosiy masala qo`yiladi , ixtiyoriy funksiya tanlanadi , unga boshlang`ich va zaruriy chegaraviy shartlar qo`yiladi. Tanlagan funksiyani asosiy differentsial tenglamaga qo`yib , tenglamaning ozod hadi aniqlanadi , boshlang`ich va chegaraviy shartlar bilan hisoblash ishlari amalga oshiriladi. Puasson tenglamasi uchun Dirixle masalasini qaraymiz :

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x_1^2} + \frac{\partial^2\psi}{\partial x_2^2} = -\xi\,(x_1,x_2),\quad 0 < x_\alpha < l_\alpha,\quad \alpha=1,2 \qquad (1)$$

$$\psi/g=\mu\,(x_1,x_2) \qquad (2)$$

bu yerda $\xi\,(x_1,x_2)$, $\mu\,(x_1,x_2)$ - berilgan funksiyalar $l_\alpha=1,2$ to`g`ri to`rtburchak tomonlarining uzunligi, $\psi\,(x_1,x_2)$ – no`malum funksiya.

Differensial masala (1)-(2) ni $\bar{G}=G+g=\{0\leq x_\alpha \leq l_\alpha,\ \alpha=1,2\}$samarali yechishga mo`ljallangan o`zgaruvchan yo`nalishli iterasiya metodini bayon qilamiz [3-5]. Differensial masala (1)-(2) ga mos quyidagi ayirmali masalani qaraymiz

$$\Lambda\psi = \Lambda_1\psi + \Lambda_2\psi = -\xi\,(x_1,x_2), x \in \omega_h \qquad (3)$$

$$\psi/j_h=\mu\,(x_1,x_2) \qquad (4)$$

$$\Lambda_\alpha \psi = \psi_{\ddot{x}_\alpha x_\alpha} = \frac{\psi(x_\alpha - h_\alpha) - 2\psi(x_\alpha) + \psi(x_\alpha + h_\alpha)}{h_\alpha^2}, \alpha = 1,2$$

bunda o`zgarmaydigan argument soddalik uchun keltirilmagan,

$$\acute{\omega}_h = \omega_h + j_h = \{x_i = (i_1 h_1, i_2 h_2) \in \bar{G}\}$$

Ayirmali masala (1)-(2) ni yechish uchun iterasiya metodi sifatida issiqlik o`tkazuvchanlik tenglamasi uchun o`zgaruvchan yo`nalishli iterasiya sxemasini qarash mumkin, bunda iterasiyadagi yechimlarni y orqali belgilaymiz va quyidagi sxemaga ega bo`lamiz:

$$\frac{y^{j+\frac{1}{2}} - y^j}{\tau_{j+1}^{(1)}} = \Lambda_1 y^{j+\frac{1}{2}} + \Lambda_2 y^1 + f(x), f(x) = \xi(x), x \in \omega_h, \quad (5)$$

$$y^{j+\frac{1}{2}}]_{j_k} = \mu(x),$$

$$\frac{y^{j+1} - y^{j+\frac{1}{2}}}{\tau_{j+1}^{(2)}} = \Lambda_1 y^{j+\frac{1}{2}} + \Lambda_2 y^{j+1} + f(x), \ x \in \omega_h,$$

$$y^{j+1}]_{j_k} = \mu(x),$$

bunda j=0,1,2… bo`lib, boshlang`ich yaqinlashish y^0=y(x,0) ixtiyoriy olinadi, bu yerda j-iteratsiya raqami , $y^{j+\frac{1}{2}}$ - oraliq iteratsiya (qismiy iteratsiya), $\tau_{j+1}^{(1)} > 0$ va $\tau_{j+1}^{(2)}$ iteratsiya parametrlari bo`lib, ular iteratsiyalar sonining shartidan tanlab olinadi. Bunda j iteratsiyadan (j+1) iteratsiyaga o'tish progonka metodini uch nuqtali tenglamalar uchun satrlar bo'ylab va ustunlar bo'ylab qo'llash orqali amalga oshiriladi:

$$y^{j+\frac{1}{2}} - \tau_{j+1}^{(1)} \Lambda_1 y^{j+\frac{1}{2}} = F^j, \ \text{(satrlar bo`yicha)}, \quad (6)$$

bunda $F^j = y^j + \tau_{j+1}^{(1)} \Lambda_2 y^j + \tau_{j+1}^{(1)} f$ va

$$y^{j+1} - \tau_{j+1}^{(2)} \Lambda_2 y^{j+1} = F^{j+\frac{1}{2}}, \ \text{(ustunlar bo`yicha)} \quad (7)$$

$$F^{j+\frac{1}{2}} = y^{j+\frac{1}{2}} + \tau_{j+1}^{(2)} \Lambda_1 y^{j+\frac{1}{2}} + \tau_{j+1}^{(2)} f$$

Shunday qilib, bitta iteratsiyani to'liq amalga oshirish uchun $\theta(1\ l\{h_x h_2\})$ arifmetik amal talab etiladi. Endi $\acute{\omega}_h$ to'rda berilgan va uning chegarasi j_h da nolga aylanuvchi to'r funksiyalari fazosi $H=\Omega$ da operatorlar $A_x y= -A_x y$, $A_2 y= -A_2 y$ ni kiritamiz, ixtiyoriy h,y€Ω Shu tariqa kiritilgan operatorlar quyidagi xossalarga ega bo'ladi [3]:

$$A_\alpha^* = A_\alpha, \delta_\alpha E \le \Delta_\alpha E, \delta_\alpha > 0, \alpha = 1,2$$

$$\delta_\alpha = \frac{4}{h_\alpha^2} sin^2 \frac{\pi h_\alpha}{2 l_\alpha}, \Delta_\alpha = \frac{4}{h_\alpha^2} cos^2 \frac{\pi h_\alpha}{2 l_\alpha}, \alpha = 1,2 \quad (8)$$

Sistema (6) va(7) yechish uchun optimal iteratsiya parametrlarini tanlashga to'xtalamiz. Dastlab quyidagi o'zgarmaslar hisoblab olinadi:

$$t = \sqrt{\frac{(\Delta_1 - \delta_1)(\Delta_2 - \delta_2)}{(\Delta_1 + \delta_1)(\Delta_2 + \delta_2)}}, r = \frac{1-t}{1+t}, \quad (9)$$

$$\chi = \frac{(\Delta_1-\delta_1)\Delta_2}{(\Delta_2-\delta_1)\Delta_1}, p = \frac{\chi-t}{\chi+t}$$

$$r = \frac{\Delta_1-\Delta_2+(\Delta_1+\Delta_2)p}{2\Delta_1\Delta_2}, q = r + \frac{1-p}{\Delta_1} \qquad (10)$$

bunda x>t, p>0

Agar iteratsiya jarayoni aniqligi ε>0 berilgan va Δ_a operatorning spektri chegaralari ζ_a,, Δ_a ma'lum bo'lsa, u holda formulalar (9) va (10) bo'yicha o'zgarmaslar η, *p,* q va r lar hisoblanadi. Bundan keyin iteratsiyalar soni n=n(ε) ni aniqlash mumkin, bu sondagi iteratsiyalarni amalga oshirish berilgan ε>0 aniqlikni ta'minlaydi.

Iteratsiyalar soni n(ε) uchun quyidagi taqribiy formula o'rinli

$$n(\varepsilon) \approx \frac{1}{n^2} ln\frac{4}{\varepsilon} ln\frac{4}{\eta}$$

ushbu belgilashlarni kiritib

$$\theta = \frac{1}{16}\eta^2\left(1+\frac{1}{2}\eta^2\right), \sigma = \frac{2j-1}{2n}, j = 1,2,\dots,n$$

$\acute{\omega} j$ ni hisoblash uchun

$$\omega_j = \frac{(1+2\theta)(1+\theta^\sigma)}{2\theta^{\frac{\sigma}{2}}(1+\theta^{1-\sigma}+\theta^{1+\sigma})}, \qquad j = 1,2,\dots,n(\varepsilon)$$

formulaga ega bo`lamiz.

Endi iteratsiya parametrlari τ_{j+1}^1 va τ_{j+1}^2 ni quydagi formula bo`yicha aniqlaymiz:

$$\tau_j^{(1)} = \frac{q\omega_j+r}{1+\omega_j p}, \tau_j^{(2)} = \frac{q\omega_j-r}{1-\omega_j p}$$

Ana shundan keyin ayirmali masala (6)-(7) ni yechish mumkin.

Kompyuter dasturi

Puasson tenglamasini optimal iterasiya parametrlarini qo`llab yechishni C++ Builder muhiti imkoniyatlaridan foydalanib quyidagicha ko`rinishdagi dastur

tuzishimiz mumkin:

Dastur kodi (Borland C++ Builder muhitida).

```
#include <vcl.h>
#include <math.h>
#pragma hdrstop
#include "Unit1.h"
//---------------------------------------------------------------------------
#pragma package(smart_init)
#pragma resource "*.dfm"
```

```
TForm1 *Form1;
//---------------------------------------------------------------------------
__fastcall TForm1::TForm1(TComponent* Owner)
        : TForm(Owner)
{
}
//---------------------------------------------------------------------------void
__fastcall TForm1::Button1Click(TObject *Sender)
{
const float pi = 3.14159;// pi konstanta qiymatini berish
float efselon = StrToFloat(Edit1 -> Text); //efselonni qiymatini kiritish
//kerakli o`zgaruvchilarni tavsiflash
float ldelta1, ldelta2, bdelta1, bdelta2, ldelta, bdelta, l1, l2, F1, F2, x1, x2,
p3;
float h1, h2, t, kappa, r, ro, tau1, tau2, q, etta, A, p5, p6, z, p4, t1, t2;
int k,n,i,j;
// x va y koordinatalar bo`yicha to`r qadamlarini kiritish
const int n1=10, n2=10;
// A parametr qiymatini kiritish
A = StrToFloat(Edit2->Text);
// x va y bo`yicha progonka uchun alfa va betta parametrlar qiymatlarini
saqlash uchun massivlar e'lon qilish
float alfa1[n1];
float alfa2[n2];
float beta1[n1];
float beta2[n2];
// yordamchi o`zgaruvchilarni e'lon qilish
float sigma, wj,wj1,wj2, p1,p2;
float y[n1][n2],ut[n1][n2],yc[n1][n2];
//x va y bo`yicha kesmalar uzunliklarini o`rnatish
l1 = 1;
l2 = 1;
h1 = l1/n1;
h2 = l2/n2;
// optimal iterasiya parametrlarini aniqlash
ldelta = 4*pow(sin((pi*h1)/(2*l1)),2)/(h1*h1);
bdelta = 4*pow(cos((pi*h1)/(2*l1)),2)/(h1*h1);
etta = ldelta/bdelta;
ro=0;
```

```
r=0;
q=1/bdelta;
n=int((1/(pi*pi))*log(4/efselon)*log(4/etta));
z=(pow(etta,2)*(1+etta*etta/2))/16;
// boshlang`ich va chegaraviy qiymatlarni berish
for(i=0; i<=n1; i++)
{
for(j=0; j<=n2; j++)
{
y[i][j]=1;
};
};
for(i=0; i<=n1; i++)
{
y[i][0]=exp(A*i*h1);
y[i][n2]=exp(A*(i*h1+1));
}
for(j=0; j<=n2; j++)
{
y[0][j] = exp(A*j*h2);
y[n1][j] = exp(A*(j*h2+1));
}
// haqiqiy yechim qiymatlarini berish
for(i=0; i<=n1; i++)
{
for(j=0; j<=n2; j++)
{
ut[i][j]=exp(A*(i*h1+j*h2));
}
}
//Optimal iterasiya soniga bog`liq siklning boshlanishi
for(k=1; k<=n; k++)
{
sigma=(2*k-1)/(2*n);
wj1=((1+2*z)*(1+pow(z,sigma)));
wj2=(2*pow(z,sigma/2)*(1+pow(z,1-sigma)+pow(z,1+sigma)));
wj=wj1/wj2;
tau1=(q*wj+r)/(1+wj*ro);
tau2=(q*wj-r)/(1-wj*ro);
```

```
p5=tau2/(h2*h2);
p6=tau2/(h1*h1);
p1=tau1/(h1*h1);
p2=tau1/(h2*h2);
// birinchi yo`nalishda taqribiy yechim qiymatlarini aniqlash tashqi sikli
for(j=1; j<=n2-1; j++)
{
alfa1[1]=0;
beta1[1]=exp(A*j*h2);
yc[n1][j]=exp(A*(1+j*h2));
// progonkaning to`g`ri yo`li
for(i=1; i<=n1-1; i++)
{
x1=tau1*(-2*A*A*exp(A*(i*h1+j*h2)));
F1=p2*y[i][j-1]+(1-2*p2)*y[i][j]+p2*y[i][j+1]+x1;
p3=((1+2*p1)-alfa1[i]*p1);
alfa1[i+1]=p1/p3;
beta1[i+1]=(p1*beta1[i]+F1)/p3;
}
// progonkaning teskari yo`li
for (i=n1-1; i>=0; i--)
{
yc[i][j]=alfa1[i+1]*yc[i+1][j]+beta1[i+1];
}
}
// ikkinchi yo`nalishda taqribiy yechim qiymatlarini aniqlash tashqi sikli
for(i=1; i<=n1-1; i++)
{
alfa2[1]=0;
beta2[1]=exp(A*i*h1);
y[i][n2]=exp(A*(1+i*h1));
// progonkaning to`g`ri yo`li
for(j=1; j<=n2-1; j++)
{
x2=tau2*(-2*A*A*exp(A*(i*h1+j*h2)));
F2=p6*yc[i-1][j]+(1-2*p6)*yc[i][j]+p6*yc[i+1][j]+x2;
p4=((1+2*p5)-alfa2[j]*p5);
alfa2[j+1]=p5/p4;
beta2[j+1]=(p5*beta2[j]+F2)/p4;
```

```
}
// progonkaning teskari yo`li
for (j=n2-1; j>=0; j--)
230
{
y[i][j]=alfa2[j+1]*y[i][j+1]+beta2[j+1];
};
};
};
// Haqiqiy va taqribiy yechimlarni chiqarish uchun sikkllar
for(i=1; i<=n1-1; i++)
for(j=1; j<=n2-1; j++)
{
//haqiqiy yechimni Memo1 ga chiqarish
Memo1->Lines->Add(FloatToStr(ut[i][j]));
//taqribiy yechimni Memo2 ga chiqarish
Memo2->Lines->Add(FloatToStr(y[i][j]));
}
}
//--------------------------------------------------------------------------
```

Hisoblash natijalari va ularning taxlili

Dasturdan olingan natija quyidagi jadvalda keltirilgan.

i	j	Taqribiy yechim (n(ε)=17 iterasiyadagi qiymatlar)	Aniq yechim ($\psi=e^{0.5(x+y)}$)	Xatolik
1	5	1.0101	1.0101	0
2	5	1.0174	1.0126	0.0048
3	5	1.0148	1.0151	3.3946e-04
4	5	1.0168	1.0177	8,2181e-04
5	5	1.0200	1.0202	2.3111e-04
6	5	1.0231	1.0228	3.0006e-04
7	5	1.0259	1.0253	5.6285e-04
8	5	1.0285	1.0279	6.2548e-04
9	5	1.0310	1.0305	5.9530e-04
10	5	1.0336	1.0330	5.5061e-04
11	5	1.0362	1.0356	5.3336e-04
12	5	1.0388	1.0382	5.5625e-04
13	5	1.0414	1.0408	6.0622e-04
14	5	1.0441	1.0434	6.3988e-04
15	5	1.0466	1.0460	5.7564e-04

16	5	1.0489	1.0486	3.0110e-04
17	5	1.0510	1.0513	2.5520e-04
18	5	1.0530	1.0539	8.7361e-04
19	5	1.0562	1.0565	3.6356e-04
20	5	1.0643	1.0592	0.0051
21	5	1.0618	1.0618	0
1	10	1.0228	1.0228	0
2	10	1.0252	1.0253	1.4018e-04
3	10	1.0282	1.0279	3.3473e-04
4	10	1.0309	1.0305	4.8721e-04
5	10	1.0336	1.0330	5.5061e-04
6	10	1.0362	1.0356	5.9927e-04
7	10	1.0389	1.0382	6.4814e-04
8	10	1.0415	1.0408	6.9415e-04
9	10	1.0441	1.0434	7.3142e-04
10	10	1.0468	1.0460	7.5563e-04
11	10	1.0494	1.0486	7.6449e-04
12	10	1.0520	1.0513	7.5729e-04
13	10	1.0546	1.0539	7.3464e-04
14	10	1.0572	1.0565	6.9882e-04
15	10	1.0598	1.0592	6.5424e-04
16	10	1.0624	1.0618	6.0698e-04
17	10	1.0651	1.0692	5.6017e-04
18	10	1.0677	1.0645	4.9805e-04
19	10	1.0702	1.0698	3.4240e-04
20	10	1.0724	1.0725	1.5213e-04
21	10	1.0752	1.0752	0
1	20	1.0486	1.0486	0
2	20	1.0117	1.0513	0.0396
3	20	1.0586	1.0539	0.0047
4	20	1.0660	1.0565	0.0095
5	20	1.0643	1.0592	0.0051
6	20	1.0629	1.0618	0.0010
7	20	1.0637	1.0645	8.1400e-04
8	20	1.0661	1.0672	0.0011
9	20	1.0692	1.0698	6.3964e-04
10	20	1.0724	1.0725	1.5213e-04
11	20	1.0752	1.0752	3.4103e-04
12	20	1.0777	1.0779	1.8703e-04
13	20	1.0799	1.0806	7.0700e-04
14	20	1.0821	1.0833	0.0012
15	20	1.0851	1.0860	8.7773e-04
16	20	1.0898	1.0887	0.0011
17	20	1.0968	1.0914	0.0053

18	20	1.1041	1.0942	0.0100
19	20	1.1019	1.0969	0.0050
20	20	1.0580	1.0997	0.0416
21	20	1.1024	1.1024	0

Dastur natijasidan ko`rinib turibdiki, Puasson tenglamasini o`zgaruvchan yo`nalishli iteratsion sxema bilan yechish taqribiy yechimni yuqori aniqlikda topishga imkon beradi. C++ Builder muhitida yaratilgan dasturiy vosita yordamida ε ning va A parametrning turli qiymatlarida berilgan sinov funksiyaga nisbatan Puasson tenglamasining taqribiy yechimlari aniq yechimga qanchalik yaqinlashishini tahlil qilish qulay hisoblanadi.

FOYDALANILGAN ADABIYOTLAR RO'YXATI:

1. Normurodov Ch.B. Ayirmali sxemalar nazariyasi 2021
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