KO'P O'ZGARUVCHILI FUNKSIYALARNI DIFFERENSIALLASH

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Annotasiya. Tezisda kop o'zgaruvchili funksiyalarni differensiallash usullari haqida ma'lumotlar keltirilgan va unga oid misollar yordamida tushintirib o'tilgan. Ko'p o'zgaruvchili funksiyalarini differensialashdirish, funksiyadagi o'zgarishni o'zgaruvchilar bo'yicha baholash imkoniyatini beradi. Ko'p o'zgaruvchili funksiyaning differensiali esa, barcha o'zgaruvchilar bo'yicha o'zgarish tezligini topishga yordam beradi .

Kalit so'zlar: Ko'p o'zgaruvchili funksiya, funksiya differensial, differensialning asosiy xossasi, nuqta orttirmasi, funksiya orttirmasi.

Faraz qilaylik, f(x) = $(x_1, x_2, \dots, x_m)$ funksiya E ⊂ R^m da berilgan bo'lib, x^0=$(x_1^0, x_2^0, \dots, x_m^0)$ ∈ E

nuqtada differensiallanuvchi bo'lsin. U holda ta'rifga ko'ra funksiyaning x^0 nuqtadagi to'liq orttirmasi

$$\Delta f(x^0) = \frac{\partial f(x^0)}{\partial x_1}\Delta x_1 + \frac{\partial f(x^0)}{\partial x_2}\Delta x_2 + \dots + \frac{\partial f(x^0)}{\partial x_m}\Delta x_m + o\,(p) \qquad (1)$$

bo'ladi. Bu munosabatda

$$P = \sqrt{x_1^2, x_1^2, \dots, x_m^2}$$

bo'lib, $\Delta x_1 \to 0$, $\Delta x_2 \to 0$, … , $\Delta x_m \to 0$, da p → 0.

1-ta'rif. f(x) funksiyaning Δf(x^0)orttirmasidagi

$$\frac{\partial f(x^0)}{\partial x_1}\Delta x_1 + \frac{\partial f(x^0)}{\partial x_2}\Delta x_2 + \dots + \frac{\partial f(x^0)}{\partial x_m}\Delta x_m$$

ifoda f(x) funksiyaning x^0 nuqtadagi differensiali (to'liq differensiali) deyiladi va

$df\,(x^0)$ yoki $df\,(x_1^0, x_2^0, \dots, x_m^0)$

kabi belgilanadi:

1-misol:

Funksiya differensialini hisoblaymiz:

$$f\,(x, y) = \left(\frac{x}{y}\right)^2 = e^{x\ln(\frac{x}{y})}$$

Funksiya differensialini quyidagi formula yordamida hisoblaymiz.

$d\,f\,(x, y) = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$

Murakkab funksiyaning har bir o'zgaruvchisi bo'yicha xususiy hosilalarini olamiz

$\frac{\partial f}{\partial x} = (e^{x}ln(\frac{x}{y})(\,ln(\frac{x}{y}) + \frac{xy}{x}))dx = (e^{x}ln(\frac{x}{y})(\,ln(\frac{x}{y}) + y))dx$

$\frac{\partial f}{\partial y} = (e^{xln(\frac{x}{y})}(\frac{xy}{x}))dy = (e^{xln(\frac{x}{y})}\ y)dy$

Xususiy hosilalarni mos ravishda formulaga qo'yamiz

Natija: $d\,f\,(x, y) = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = (e^{x}ln(\frac{x}{y})(\,ln(\frac{x}{y}) + y))dx + (e^{xln(\frac{x}{y})}\ y)dy.$

Differensialning asosiy xossasi agar $y = f(M)$ funksiya M_0 nuqtada differensiallanuvchi bo'lsa, u holda cheksiz kichik $\Delta x_1 \Delta x_2 \dots \Delta x_n$ lar uchun

$$\Delta f(M_0) \approx df(M_0)$$

Bajariladi, ya'ni

$$\Delta f(M_0) \approx \frac{\partial f(M_0)}{\partial x_1}\cdot \Delta x_1 + \frac{\partial f(M_0)}{\partial x_2}\cdot \Delta x_2 + \cdots + \frac{\partial f(M_0)}{\partial x_n}\cdot \Delta x_n$$

Bir necha o'zgaruvchili funksiya uchun taqribiy hisoblash fo'rmulasi quidagi ko'rinishga ega:

$$f(M_0) \approx f(M_0) + \frac{\partial f(M_0)}{\partial x_1}\cdot \Delta x_1 + \frac{\partial f(M_0)}{\partial x_2}\cdot \Delta x_2 + \cdots + \frac{\partial f(M_0)}{\partial x_n}\cdot \Delta x_n$$

2-misol:

Funksiya differensialini hisoblaymiz:

$f\,(x, y) = \arctan(\frac{x}{y}) + \arctan(\frac{y}{x})$

Funksiya differensialini quyidagi formula yordamida hisoblaymiz.

$d\,f\,(x, y) = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$

Murakkab funksiyaning har bir o'zgaruvchisi bo'yicha xususiy hosilalarini olamiz.

$\frac{\partial f}{\partial x} = \left(\frac{\frac{y}{x^2}}{\sqrt{1+\frac{y^2}{x^2}}} + \frac{\frac{1}{y}}{\sqrt{1+\frac{x^2}{y^2}}}\right)dx$

$\frac{\partial f}{\partial y} = \left(\frac{\frac{1}{x}}{\sqrt{1+\frac{y^2}{x^2}}} - \frac{\frac{x}{y^2}}{\sqrt{1+\frac{x^2}{y^2}}}\right)dy$

Xususiy hosilalarni mos ravishda formulaga qo'yamiz

Natija: $d\,f\,(x, y) = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \left(\frac{\frac{y}{x^2}}{\sqrt{1+\frac{y^2}{x^2}}} + \frac{\frac{1}{y}}{\sqrt{1+\frac{x^2}{y^2}}}\right)dx + \left(\frac{\frac{1}{x}}{\sqrt{1+\frac{y^2}{x^2}}} - \frac{\frac{x}{y^2}}{\sqrt{1+\frac{x^2}{y^2}}}\right)dy$

3-misol:

Funksiya differensialini hisoblaymiz: Bizga quyidagi murakkab funksiyalar berilgan bo'lsin.

P = f (u, v, w), u = $x^2 + y^2 + z^2$, v = x + y + z, w = xyz;

Funksiya differensialini quyidagi formula yordamida hisoblaymiz.

$$df = \left(\frac{\partial f}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x} + \frac{\partial f}{\partial w}\frac{\partial w}{\partial x}\right)dx + \left(\frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y} + \frac{\partial f}{\partial w}\frac{\partial w}{\partial y}\right)dy + \left(\frac{\partial f}{\partial u}\frac{\partial u}{\partial z} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial z} + \frac{\partial f}{\partial w}\frac{\partial w}{\partial z}\right)dz$$

Murakkab funksiyaning har bir o'zgaruvchisi bo'yicha xususiy hosilalarini olamiz.

$$\frac{\partial u}{\partial x} = 2x,\quad \frac{\partial v}{\partial x} = 2y,\quad \frac{\partial w}{\partial x} = 2z;$$
$$\frac{\partial u}{\partial y} = 1,\quad \frac{\partial v}{\partial y} = 1,\quad \frac{\partial w}{\partial y} = 1;$$
$$\frac{\partial u}{\partial z} = yz,\quad \frac{\partial v}{\partial z} = xz,\quad \frac{\partial w}{\partial z} = xy;$$

Xususiy hosilalarni mos ravishda formulaga qo'yamiz

Natija: $df = \left(\frac{\partial f}{\partial u}\, 2x + \frac{\partial f}{\partial v}\, 1 + \frac{\partial f}{\partial w}\, yz\right)dx + \left(\frac{\partial f}{\partial u}\, 2y + \frac{\partial f}{\partial v}\, 1 + \frac{\partial f}{\partial w}\, xz\right)dy + \left(\frac{\partial f}{\partial u}\, 2z + \frac{\partial f}{\partial v}\, 1 + \frac{\partial f}{\partial w}\, xy\right)dz.$

Agar $f(M_0)$ funksiya M_0 nuqtada differensiallanuvchi bo'lsa, M_0 nuqtada $f(M_0)$ funksiya to'la orttirmasining bosh chiziqli qismiga M_0 nuqtada uning differensiali deyiladi va $df(M_0)$ kabi belgilanadi, ya'ni

$$df(M_0) = \frac{\partial f(M_0)}{\partial x_1}\cdot\Delta x_1 + \frac{\partial f(M_0)}{\partial x_2}\cdot\Delta x_2 + \cdots + \frac{\partial f(M_0)}{\partial x_n}\cdot\Delta x_n$$

Bu yerda $dx_1 = \Delta x_1, dx_2 = \Delta x_2, \ldots, dx_n = \Delta x_n$ deb olish mumkin. U holda

$$df(M_0) = \frac{\partial f(M_0)}{\partial x_1}\cdot\Delta x_1 + \frac{\partial f(M_0)}{\partial x_2}\cdot\Delta x_2 + \cdots + \frac{\partial f(M_0)}{\partial x_n}\cdot\Delta x_n$$

ko'rinishida bo'ladi

4 - misol $f(M) = x_1^3x_2 + x_2^2x_3 + x_3$ funksiyaning $M_0(2; 1; -3)$ nuqtadagi differensialini toping.

Yechish. $f(M)$ differensiali quyidagich

$$df(M_0) = \frac{\partial f(M_0)}{\partial x_1}\cdot\Delta x_1 + \frac{\partial f(M_0)}{\partial x_2}\cdot\Delta x_2 + \frac{\partial f(M_0)}{\partial x_3}\cdot\Delta x_3$$

ko`rinishda bo`ladi. Bundan

$\frac{\partial f}{\partial x_1} = 3x_1^2 x_2,$ $\frac{\partial f}{\partial x_2} = x_1^3 + 2x_2 x_3,$ $\frac{\partial f}{\partial x_3} = x_2^2 + 1$ va

$\frac{\partial f(M_0)}{\partial x_1} = 12$ $\frac{\partial f(M_0)}{\partial x_2} = 2$ $\frac{\partial f(M_0)}{\partial x_3} = 2$ bo'lgani uchun $df(M_0) = 12dx_1 + 2dx_2 + 2dx_3$ bo'ladi.

Xulosa: Xulosa qilib shuni aytish mumkinki, Differensiallashning asosiy maqsadi funksiyalarning keng qo'llaniladigan ko'rsatkichlarni topish va ulardan foydalanib matematik modellarini yaratishdir. Talabalar murakkab funksiyalarni differensiyallash jarayonini soddalashtirish va bu usul bo'yicha ko'plab misol va masalalar yechimini topish bo'yicha bilim va ko'nikmalar hosil qilishadi.

Foydalanilgan adabiyotlar:

1. Sadullayev.Matematik analiz kursidan misol va masalalar toplami.1-qism.
2. G.Xudayberganov, A.K.Vorisov, X.T.Mansurov,B.A.Shoimqulov.Matematik analizdan ma'ruzalar, 2-qism Toshkent-2010.”Voris-nashriyoti”, 80-bet.
3. https://arxiv.uz/uz/documents/referatlar/algebra/ko-p-o-zgaruvchili-funksiyaning-differensial-hisobi-aniq-integral
4. https://www.google.com/search?q=KO%E2%80%99P+O%E2%80%99ZGARUVCHILI+FUNKSIYALARNI+DIFFERANSIALLASH&oq=KO%E2%80%99P++O%E2%80%99ZGARUVCHILI++FUNKSIYALARNI++DIFFERANSIALLASH&aqs=chrome..69i57j33i10i160.2083j0j7&sourceid=chrome&ie=UTF-8