IKKI NOMALUMLI CHIZIQLI INTEGRAL TENGLAMALAR SISTEMASI YECHIMLARINING MAVJUDLIK SHARTLARI

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Annotatsiya. Ushbu ishda ikki noma'lumli integral tenglamalar sistemasi qaralgan bo'lib, bu integral tenglamalar sistemasi $\lambda_{ij}, i,j = 1,2$ parametrlar va $v_{ij}(x), i,j = 1,2$ parametr funksiyalarga bog'liq. Maqolada bu tenglamalar sistemasining yechimlarining mavjud bo'lish shartlari tahlil qilingan.

Kalit so'zlar. Integral tenglamalar sistemasi, tenglama yechimi, tenglamalar sistemasi.

CONDITIONS FOR THE EXISTENCE OF SOLUTIONS OF SYSTEMS OF LINEAR INTEGRAL EQUATIONS WITH TWO UNKNOWNS

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Abstract. In this work, the system of two unknown integral equations is considered, and this system of integral equations depends on the parameters $\lambda_{ij}, i,j = 1,2$ and functions $v_{ij}(x), i,j = 1,2$ parameters. The article analyzes the conditions for the existence of the solutions of this system of equations.

Key words. System of integral equations, equation solution, system of equations.

Chiziqli operatorlarning spectral nazariyasini tadqiq qilish jarayonida ko'p hollarda chiziqli integral tenglamalarning yechimlarini tahlil qilishga keltiriladi. Xususan, [1-22] ishlarda operatorli matritsalar uchun Fredgolm determinantini qurish hamda operatorli matritsaning spektrini tahlil qilish maqsadida unga mos Faddeev tenglamasini qurish jarayonida chiziqli integral tenglamalarning yechimlarini tahlil qilish masalasi qaraladi.

Ushbu maqolada ikki noma'lumli integral tenglamalar sistemasi qaralib, bu tenglamaning yechimlari mavjud bo'lish shartlari keltirilgan.

$C_2[-\pi,\pi]$ fazoda element normasi va skalyar ko'paytma quyidagicha aniqlangan bo'lsin:

$$\|f\| = \sqrt{\int_{-\pi}^{\pi} |f(t)|^2 \, dt}, \qquad f \in C_2[-\pi,\pi]$$

$$(f,g) = \int_{-\pi}^{\pi} f(t)g(t)dt \qquad f,g \in C_2[-\pi,\pi]$$

Quyidagi integral tenglamalar sistemasini qaraymiz:

$$\begin{cases} f_1(x)=\lambda_{11}v_{11}(x)\int\limits_{-\pi}^{\pi}v_{11}(t)f_1(t)dt+\lambda_{12}v_{21}(x)\int\limits_{-\pi}^{\pi}v_{12}(t)f_2(t)dt+\varphi_1(x) \\ f_2(x)=\lambda_{12}v_{12}(x)\int\limits_{-\pi}^{\pi}v_{21}(t)f_1(t)dt+\lambda_{22}v_{22}(x)\int_{-\pi}^{\pi}v_{22}(t)f_2(t)dt+\varphi_2(x) \end{cases} \quad (1)$$

Bu yerda $\lambda_{ij},\ i,j=1,2$ parametrlar haqiqiy sonlardan iborat; $v_{ij}(x),\ i,j=1,2$ va $\varphi_k(x),\ k=1,2$ funksiyalar haqiqiy qiymatli uzluksiz funksiyalar bo'lsin.

(1) integral tenglamalar sistemasida quyidagicha belgilashlarni kiritamiz:

$$C_{11}=\int_{-\pi}^{\pi}v_{11}(t)f_1(t)dt;\quad C_{12}=\int_{-\pi}^{\pi}v_{12}(t)f_2(t)dt$$

$$C_{21}=\int_{-\pi}^{\pi}v_{21}(t)f_1(t)dt;\quad C_{22}=\int_{-\pi}^{\pi}v_{22}(t)f_2(t)dt. \quad (2)$$

(2) belgilashlarni (1) tenglamalar sistemasiga qo'yib , $f_1(x)$ va $f_2(x)$ lar uchun quyidagi tengliklarni hosil qilamiz:

$$\begin{cases} C_{11}=\int_{-\pi}^{\pi}v_{11}(t)[\lambda_{11}v_{11}(t)C_{11}+\lambda_{12}v_{21}(t)C_{12}+\varphi_1(t)]dt \\ C_{12}=\int_{-\pi}^{\pi}v_{12}(t)\,[\lambda_{21}v_{12}(t)C_{21}+\lambda_{22}v_{22}(t)C_{22}+\varphi_2(t)]dt \\ C_{21}=\int_{-\pi}^{\pi}v_{21}(t)\,[\lambda_{11}v_{11}(t)C_{11}+\lambda_{12}v_{21}(t)C_{12}+\varphi_1(t)]dt \\ C_{22}=\int_{-\pi}^{\pi}v_{22}(t)\,[\lambda_{21}v_{12}(t)C_{21}+\lambda_{22}v_{22}(t)C_{22}+\varphi_2(t)]dt \end{cases}$$

Ya'ni

$$\begin{cases} C_{11}=\lambda_{11}\|v_{11}\|^2C_{11}+\lambda_{12}(v_{11},v_{21})C_{12}+(v_{11},\varphi_1) \\ C_{12}=\lambda_{21}\|v_{12}\|^2C_{21}+\lambda_{22}(v_{12},v_{22})C_{22}+(v_{12},\varphi_2) \\ C_{21}=\lambda_{12}\|v_{21}\|^2C_{12}+\lambda_{11}(v_{21},v_{11})C_{11}+(v_{21},\varphi_1) \\ C_{22}=\lambda_{22}\|v_{22}\|^2C_{22}+\lambda_{21}(v_{22},v_{12})C_{21}+(v_{22},\varphi_2) \end{cases}$$

Bundan

$$\begin{cases} (1-\lambda_{11}\|v_{11}\|^2)C_{11}-\lambda_{12}(v_{11},v_{21})C_{12}=(v_{11},\varphi_1) \\ C_{12}-\lambda_{21}\|v_{12}\|^2C_{21}-\lambda_{22}(v_{12},v_{22})C_{22}=(v_{12},\varphi_2) \\ C_{21}-\lambda_{12}\|v_{21}\|^2C_{12}-\lambda_{11}(v_{21},v_{11})C_{11}=(v_{21},\varphi_1) \\ (1-\lambda_{22}\|v_{22}\|^2)C_{22}-\lambda_{21}(v_{22},v_{12})C_{21}=(v_{22},\varphi_2) \end{cases} \quad (3)$$

Tenglamalar sistemasini hosil qilamiz. (3) tenglamalar sistemasini quyidagi matritsaviy holda yozib olamiz:

$$Ax=B$$

Bu yerda

$$A=\begin{pmatrix} 1-\lambda_{11}\|v_{11}\|^2 & -\lambda_{12}(v_{11},v_{21}) & 0 & 0 \\ 0 & 1 & -\lambda_{21}\|v_{12}\|^2 & -\lambda_{22}(v_{12},v_{22}) \\ -\lambda_{11}(v_{21},v_{11}) & -\lambda_{12}\|v_{21}\|^2 & 1 & 0 \\ 0 & 0 & -\lambda_{21}(v_{22},v_{12}) & 1-\lambda_{22}\|v_{22}\|^2 \end{pmatrix}$$

$$B=\begin{pmatrix} (v_{11},\varphi_1) \\ (v_{12},\varphi_2) \\ (v_{21},\varphi_1) \\ (v_{22},\varphi_2) \end{pmatrix} \quad \text{va} \quad x=\begin{pmatrix} C_{11} \\ C_{12} \\ C_{21} \\ C_{22} \end{pmatrix}$$

Yuqorida keltirilgan matritsaviy tenglamaning yechimga ega bo'lishiga qarab, (1) tenglamalar sitemasining yechimi haqida xulosa qilish mumkin.

1-tasdiq: $B=0$ bo'lsin.

a) Agar $rank(A)\neq rank(\frac{A}{B})$ bo'lib, $detA=0$ bo'lsa, u holda (1) tenglamalar sistemasi cheksiz ko'p yechimga ega;

b) Agar $detA\neq 0$ bo'lsa, u holda (1) tenglamalar sistemasi yagona yechimga ega.

2-tasdiq: $B\neq 0$ bo'lsin.

a) $detA\neq 0$ bo'lsa, u holda (1) tenglamalar sistemasi yagona yechimga ega.

b) $detA=0$ bo'lsin.

b_1) Agar $rank(A)=rank\left(\frac{A}{B}\right)$ bo'lsa u holda (1) tenglamalar sistemasi cheksiz ko'p yechimga ega;

b_2) Agar $rank(A)\neq rank(\frac{A}{B})$ bo'lsa , u holda (1) tenglamalar sistemasi yechimga ega emas.

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