

**SOHA CHEGARASIDA BUZILUVCHAN SINGULYAR
KOEFFITSIENTLI GIPERBOLIK TIPDAGI TENGLAMA UCHUN
CHEGARAVIY MASALA HAQIDA**

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Annotasiya. Ushbu maqolada soha chegarasida buziluvchan singulyar koeffitsientli giperbolik tipdagi tenglama uchun to‘liq berilmagan Gursa shartli masalaning qo‘yilishi, yechimning yagonaligi o‘rganib chiqilgan. Yechimning yagonaligi haqidagi teorema tahlil qilingan. Murakkab hisoblash ishlari tushunarli bo‘lishi uchun kengaytirib yozilgan.

Kalit so‘zlar: xarakteristik uchburchak, chegaraviy masala, Frankl sharti, giperbolik tenglama, buzilish chizig’i, Gursaning to‘liqmas sharti, singulyar koeffitsient.

**ON A BOUNDARY VALUE PROBLEM FOR AN EQUATION OF HYPERBOLIC
TYPE WITH PERTURBATION SINGULAR COEFFICIENTS ON THE
BOUNDARY OF A DOMAIN**

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Annotation. In this paper, we study the formulation of the incomplete conditional Goursat problem for a hyperbolic type equation with perturbation singular coefficients on the field boundary, the uniqueness of a solution. The theorem on uniqueness of a solution is analyzed. Complex calculations are expanded to make them more understandable.

Keywords: characteristic triangle, boundary value problem, Frankl condition, hyperbolic equation, failure line, incomplete Goursat condition, singular coefficient.

Agar $y < 0$ yarim tekislikdagi Ω^- xarakterli uchburchak berilgan, AC_1 va BC_1 , xarakter nuqtalar bilan chegaralangan bo’lsin. Bu yerda $A(-1,0)$, $B(1,0)$, $C_1\left(0, -((m+2)/2)^{2/(m+2)}\right)$ bo’lsa tenglama

$$-(-y)^m u_{xx} + u_{yy} + \alpha_0 (-y)^{(m-2)/2} u_x + (\beta_0/y) u_y = 0, \quad y < 0, \quad (1)$$

ko‘rinishda bo‘ladi.

Bu yerda m, α_0, β_0 bazi doimiy shartlarni $m > 0, -m/2 < \beta_0 < 1, -(m+2)/2 < \alpha_0 < (m+2)/2$ qanoatlantiradi.

(1) tenglama uchun chegaraviy masalalar qo‘yilishining korrektligi tenglamaning kichik hadlari koeffitsientlari α_0 va β_0 sonli qatorlarga bog‘liq. α_0/β_0 parametrik

tekislikda

$$A_0^*C_0^* : \beta_0 + \alpha_0 = -m/2; \quad B_0^*C_0^* : \beta_0 - \alpha_0 = -m/2; \quad A_0^*B_0^* : \beta_0 = 1.$$

to‘g‘ri chiziqlar bilan chegaralangan $A_0^*B_0^*C_0^*$ ni qaraymiz.

Agar $P(\alpha_0, \beta_0) \in \Delta A_0^*B_0^*C_0^*$ bo‘lsa $0 < \alpha, \beta < 1, \alpha + \beta < 1$, bu yerda $\alpha = (m + 2(\alpha_0 + \beta_0))/2(m + 2)$, $\beta = (m + 2(\beta_0 - \alpha_0))/2(m + 2)$.

$E(c, 0)$ xarakter nuqtalardan chiquvchi AC_1 va BC_1 xarakter nuqtalari bilan kesishuvchi mos A_0 va B_0 nuqtalar orqali ifodalaymiz, bu yerda $c \in J = (-1, 1)$ interval $y = 0$ o‘qida [1].

Agar $p(x) = \delta - kx$, funksiya $[c, 1]$, to‘plamda bu yerda $k = \frac{1-c}{1+c}$, $\delta = \frac{2c}{1+c}$ bo‘lganida akslantiriladi ya’ni bu yerda $p(-1) = 1$, $p(c) = c$ ga teng bo‘ladi.

Gursa vazifasi AC_1 va BC_1 xarakter chegaralari chegaraviy shartlarning tashuvchisi hisoblanadi.

Ushbu ilmiy ish Ω^- masala to‘g‘risidagi (1) giperbolik tenglamada buzilish chiziq chegarasida, qachonki AC_1 chegaraviy xarakter nuqtasi Ω^- sohada AA_0 va A_0C_1 ikki bo‘lakka ajralsa va bиринчи bo‘lak $AA_0 \subset AC_1$ da kerakli funksiya qiymatlari berilsa, иккинчи bo‘lak esa $AA_0 \subset AC_1$ chegaraviy shartdan ozod bo‘lib va ushbu Gursaning yetishmovchilik sharti Frankl shartining analogiga AB buzilish chizig‘ida almashtirilishini o‘rganib chiqishga bag‘ishlangan [2-4].

A masala. Quyidagi shartlarni qanoatlantiradigan $u(x, y) \in C(\bar{\Omega}^-)$ sohada Ω^- funktsiyani topish talab qilinadi:

1) $u(x, y) R_1$ sinfdagi (1) tenglamaning umumlashgan yechimi

$$2) \quad u(x, y)|_{BC_1} = \psi_1(x), \quad 0 \leq x \leq 1, \quad (2)$$

$$3) \quad u(x, y)|_{AA_0} = \psi_2(x), \quad -1 \leq x \leq \frac{c-1}{2}, \quad (3)$$

$$4) \quad u(x, 0) - \mu u(p(x), 0) = f(x), \quad -1 \leq x \leq c, \quad (4)$$

Bu yerda $\mu = const$, $\psi_1(x) \in C[0, 1] \cap C^2(0, 1)$, $\psi_2(x) \in C\left[-1, \frac{c-1}{2}\right] \cap C^2\left(-1, \frac{c-1}{2}\right)$, $f(x) \in C[-1, c] \cap C^2(-1, c)$, bundan tashqari $\psi_1(1) = 0$, $\psi_2(-1) = 0$, $f(c) = 0$ larga teng bo‘ladi.

(3) shart Gursaning to‘liqmas sharti hisoblanadi, chunki AC_1 xarakteristikating faqat AA_0 qismida beriladi.

(4) shart AA_0 buzilish chizig‘ida Frankl shartining analogi hisoblanadi.

$u(x, 0) = \tau(x)$ ekanligini hisobga olgan holda (1.4) tenglamani quyidagicha yozib olamiz

$$\tau(x) - \mu\tau(p(x)) = f(x), \quad x \in [-1, c]. \quad (4^*)$$

Agar Ω^+ sohasi $y > 0$ yarim tekislikda yotuvchi $y = 0$ to‘g‘ri chiziqda Ω^- sohaga nisbatan simmetrik bo‘lsa va $\Omega^- \cup \Omega^+ \cup AB$ bo‘lsin. Ω^+ soha AC_2 и BC_2 xarakter nuqtalari bilan chegaralangan, bu yerda

$$C_2 \left(0, ((m+2)/2)^{2/(m+2)} \right) \text{ bo'lsa, u holda tenglama} \\ -y^m u_{xx} + u_{yy} + \alpha_0 y^{(m-2)/2} u_x + (\beta_0/y) u_y = 0, \quad y > 0. \quad (5)$$

ko'rinishda bo'ladi.

Bu yerda ko'rishimiz mumkinki $u(x, y) \quad y < 0$ yarim tekislikdagi (1) tenglamaning yechimi bo'lsa, u xolda $u(x, -y) \quad y > 0$ yarim tekislikdagi (5) tenglamaning ham yechimi bo'la oladi.

Bu xossadan (1) va (5) tenglamalar yechimini hisobga olgan xolda Ω simmetrik sohadagi A^* yordamchi masalani ko'rib chiqamiz.

A^* masalaning tuzilishi: Ω sohadagi $u(x, y) \in C(\bar{\Omega})$ funksiyani qanoatlantiruvchi shartni topish talab qilinadi:

1) Ω^- va Ω^+ sohada R_1 sinfdagi $u(x, y) -$ umumi yechim

2) $u(x, y)$ (6) tenglamadagi shartni va A masaladagi (3) va (4) sharlarni qanoatlantiradi

$$u(x, y)|_{BC_2} = \psi_1(x), \quad 0 \leq x \leq 1. \quad (6)$$

3) $y = 0$ buzilish chizig'i $-1 < x < 1$ oralig'ida ulanish sharti o'rinli bo'ladi

$$\lim_{y \rightarrow -0} (-y)^{\beta_0} \frac{\partial u}{\partial y} = \lim_{y \rightarrow +0} y^{\beta_0} \frac{\partial u}{\partial y} = v(x), \quad x \in J, \quad (7)$$

Bu yerda limitlar $x \rightarrow \pm 1$ $1 - \alpha - \beta$ kichik tartibli xususiyatga ega bo'lishi mumkin, bu yerda $\alpha + \beta = (m + 2\beta_0)/(m + 2) \in (0, 1)$.

Bu yerda ko'rishimiz mumkinki $u(x, y)|_{BC_1} = u(x, -y)|_{BC_2}$ tenglikni hisobga olgan holda $\Omega^- = \Omega \cap \{y < 0\}$ sohadagi A^* masalaning yechimi bo'ladi va Ω^- sohadagi A masalaning yechimini izlash orqali A^* masalaning yechimi topilgan [5].

A^* masala yechimining yagonaligi: Ω^-, Ω^+ soxadagi (1) tenglamaning yechimi Koshi ko'rinishi o'zgartirilgan shartini qanoatlantirsa quyidagi ko'rinishga keladi:

$$\lim_{y \rightarrow 0} u(x; y) = \tau(x), \quad x \in \bar{J}; \quad \lim_{y \rightarrow \pm 0} |y|^{\beta_0} \frac{\partial u}{\partial y} = \mp v(x), \quad x \in J, \quad (8)$$

quyidagi ko'rinishga keladi

$$u(x, y) = \gamma_1 \int_{-1}^1 \tau \left[x + \frac{2t}{m+2} |y|^{\frac{m+2}{2}} \right] (1+t)^{\beta-1} (1-t)^{\alpha-1} dt + \gamma_2 |y|^{1-\beta_0} \cdot \\ \cdot \int_{-1}^1 v \left[x + \frac{2t}{m+2} |y|^{\frac{m+2}{2}} \right] (1+t)^{-\alpha} (1-t)^{-\beta} dt, \quad (9)$$

bu yerda

$$\gamma_1 = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} 2^{1-\alpha-\beta}, \quad \gamma_2 = -\frac{\Gamma(2 - \alpha - \beta)}{(1 - \beta_0)\Gamma(1 - \alpha)\Gamma(1 - \beta)} 2^{\alpha+\beta-1}.$$

(6) chegaraviy shart (Ω^+ soxada o'rinli bo'lgan xolda (9) hisobga olingan xolda

$$\gamma_1 \left(\frac{(1-x)}{2} \right)^{1-\alpha-\beta} \int_x^1 \frac{\tau(s)(1-s)^{\alpha-1}}{(s-x)^{1-\beta}} ds - \gamma_2 \left(\frac{(m+2)}{2} \right)^{1-\alpha-\beta}.$$

$$\cdot \int_x^1 \frac{\nu(s)(1-s)^{-\beta}}{(s-x)^\alpha} ds = \psi_1(x), \quad x \in (-1,1),$$

hosil qilib olamiz yoki

$$\nu(x) = -\gamma D_{x,1}^{1-\alpha-\beta} \tau(x) + \psi_1(x), \quad x \in (-1,1), \quad (10)$$

Bu yerda $D_{x,1}^l$ –diferensiallash kasr operatori

$$\begin{aligned} \gamma &= \frac{2\Gamma(1-\beta)\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(1-\alpha-\beta)} \left(\frac{m+2}{4}\right)^{\alpha+\beta}, \\ \psi_1(x) &= -\frac{\left(\frac{2}{m+2}\right)^{1-\alpha-\beta}}{\gamma_2 \Gamma(1-\alpha)} (1-x)^\beta D_{x,1}^{1-\alpha} \psi_1\left(\frac{1+x}{2}\right). \end{aligned}$$

(3) nisbati noma'lum funksiyalar orasidagi birinchi funksional $\tau(x)$ va $\nu(x)$ Ω^+ soxadagi (-1,1) oraliqqa kiritamiz.

Bu yerda ko'rishimiz mumkinki [6-9]

$$\nu(x) = \gamma D_{-1,x}^{1-\alpha-\beta} \tau(x) + \psi_2(x), \quad x \in (-1, c),$$

Endi (3) tenglamada chegaraviy sharti (Ω^- soxada o'rinli bo'lgan) (10) tenglamani xisobga olgan holda quyidagi formulani hosil qilib olamiz.

Bu yerda

$$\psi_2(x) = \frac{\left(\frac{2}{m+2}\right)^{1-\alpha-\beta}}{\gamma_2 \Gamma(1-\beta)} (1+x)^\alpha D_{-1,x}^{1-\beta} \psi_2\left(\frac{x-1}{2}\right)$$

(1) tenglama Ω^- soxadagi (-1,c) intervalda kiritilgan $\tau(x)$ va $\nu(x)$ noma'lum funksiyalar o'rtaida ikkinchi funksional nisbati hisoblanadi.

Teorema. Ushbu berilgan shartda

$$k^{\alpha+\beta} < \mu^2$$

A* masala bittadan ko'p yechimga ega bo'lmaydi.

Izbot. 1⁰. Berilgan shartni ishlatib ($\psi_1(x) \equiv 0$) izbotlaymiz

$$J = \int_{-1}^1 \tau(x) \nu(x) dx \leq 0.$$

$$\begin{aligned} J &= -\gamma \int_{-1}^1 \tau(x) \left(D_{x,1}^{1-\alpha-\beta} \tau(x)\right) dx = \frac{\gamma}{\Gamma(\alpha+\beta)} \int_{-1}^1 \tau(x) \cdot \\ &\quad \cdot \left(\frac{d}{dx} \int_x^1 \frac{\tau(t) dt}{(t-x)^{1-\alpha-\beta}}\right) dx. \end{aligned}$$

hisoblaymiz.

Agar $\tau_1(x) \in C(\bar{J}) \cap C^2(J)$, $\tau_1(1) = \tau'_1(1) = 0$ bo'lsa u holda

$$\tau(x) = \int_x^1 \frac{\tau_1(s) ds}{(s-x)^{\alpha+\beta}}, \quad x \in (-1,1),$$

tenglik

$$J = \frac{\gamma}{\Gamma(\alpha + \beta)} \int_{-1}^1 \tau(x) \left(\frac{dt}{(t-x)^{1-\alpha-\beta}} \int_t^1 \frac{\tau_1(s)ds}{(s-t)^{\alpha+\beta}} \right) dx.$$

ushbu ko‘rinishda ega bo‘ladi.

$$\frac{d}{dx} \int_x^1 \frac{dt}{(t-x)^{1-\alpha-\beta}} \int_t^1 \frac{\tau_1(s)ds}{(s-t)^{\alpha+\beta}} = -\Gamma(\alpha + \beta)\Gamma(1 - \alpha - \beta)\tau_1(x).$$

buni isbotlash qiyin emas [4-7].

Shunday qilib (10) tenglamani hisobga olib (9) tenglamani ushbu ko‘rinishda yozamiz:

$$J = -\gamma\Gamma(1 - \alpha - \beta) \int_{-1}^1 \tau(x)\tau_1(x)dx. \quad (11)$$

Endi (8) tenglamani hisobga olgan holda (11) tenglamani ushbu ko‘rinishga keltiramiz:

$$J = -\gamma\Gamma(1 - \alpha - \beta) \int_{-1}^1 \tau_1(x)dx \int_x^1 \frac{\tau_1(s)ds}{(s-x)^{\alpha+\beta}}. \quad (12)$$

Bu yerda integrallash tartibini o‘zgartirganimizda

$$J = -\gamma\Gamma(1 - \alpha - \beta) \int_{-1}^1 \tau_1(s)ds \int_{-1}^s \frac{\tau_1(x)dx}{(s-x)^{\alpha+\beta}}. \quad (13)$$

hosil bo‘ladi.

(12) tenglamada s va x o‘zgaruvchilarni almashtirganimizda integral quyidagi ko‘rinishda bo‘ladi:

$$J = -\gamma\Gamma(1 - \alpha - \beta) \int_{-1}^1 \tau_1(s)ds \int_s^1 \frac{\tau_1(x)dx}{(x-s)^{\alpha+\beta}}. \quad (14)$$

Endi (13) va (14) tenglamalarni birlashtirganimizda ushbu ko‘rinishdagi integralni olamiz

$$J = -\frac{\gamma\Gamma(1 - \alpha - \beta)}{2} \int_{-1}^1 \int_{-1}^1 \frac{\tau_1(s)\tau_1(x)}{|s-x|^{\alpha+\beta}} dx ds. \quad (15)$$

Endi $\Gamma(z)$ funksiya uchun keyingi ma’lum formula orqali foydalanamiz:

$$\int_0^\infty t^{z-1} \cos(kt) dt = \frac{\Gamma(z)}{k^z} \cos\left(\frac{z\pi}{2}\right), k > 0, 0 < z < 1. \quad (16)$$

Agar (16) tenglamada $k = |s-x|$, $z = \alpha + \beta$ bo‘lsa, (16) tenglamadan

$$\frac{1}{|s-x|^{\alpha+\beta}} = \frac{1}{\Gamma(\alpha + \beta) \cos((\alpha + \beta)\pi/2)} \int_0^\infty \xi^{\alpha+\beta-1} \cos((s-x)\xi) d\xi. \quad (17)$$

ko‘rinishdagi tenglamaga ega bo‘lamiz

(17) tufayli (15) tenglikni ushbu ko‘rinishda yozamiz:

$$J = -\frac{\gamma \Gamma(1-\alpha-\beta)}{2\Gamma(\alpha+\beta) \cos((\alpha+\beta)\pi/2)} \int_0^\infty \xi^{\alpha+\beta-1} d\xi \int_{-1}^1 \int_{-1}^1 \tau_1(s) \cdot \\ \cdot \tau_1(x) \cos((s-x)\xi) dx ds = -\frac{\gamma \Gamma(1-\alpha-\beta)}{2\Gamma(\alpha+\beta) \cos((\alpha+\beta)\pi/2)} \int_0^\infty \xi^{\alpha+\beta-1} \cdot \\ \cdot \left\{ \left[\int_{-1}^1 \tau_1(t) \cos(t\xi) dt \right]^2 + \left[\int_{-1}^1 \tau_1(t) \sin(t\xi) dt \right]^2 \right\} d\xi. \quad (18)$$

shunday qilib, (18) orqali (6) tongsizlikga ega bo‘lamiz.

2⁰. Endi (4) ($c \psi_2(x) \equiv 0$) tenglamani va (2.1.4*) shartlarni ishlatgan holda (6) integralni nomanfiyligini, ya’ni

$$J = \int_{-1}^1 \tau(x)v(x)dx \geq 0. \quad (19)$$

isbotlaymiz.

Haqiqatdan ham ushbu

$$J = \int_{-1}^1 \tau(x)v(x)dx = \int_{-1}^c \tau(x)v(x)dx + \int_c^1 \tau(x)v(x)dx, \quad (20)$$

Bu yerda (2.20)-dagi o‘ng tomonda joylashgan ikkinchi integralni quyidagicha o‘zgartiramiz, ya’ni

$$J_0 = \int_c^1 \tau(x)v(x)dx. \quad (21)$$

(21) tenglamada $x = p(t) = \delta - kt$ integral o‘zgaruvchilarni almashtirgan holda quyidagi ko‘rinishda yozamiz:

$$J_0 = k \int_{-1}^c \tau(p(t))v(p(t))dt. \quad (22)$$

Endi $v(p(x))$ topamiz, buning uchun (3) tenglamadagi $J = (-1; 1)$ oraliqdagi $x \in (-1; 1)$ xususiy holidan foydalananamiz:

$$v(x) = -\gamma D_{x,1}^{1-\alpha-\beta} \tau(x) = \frac{\gamma}{\Gamma(\alpha+\beta)} \frac{d}{dx} \int_x^1 \frac{\tau(t)dt}{(t-x)^{1-\alpha-\beta}}, \quad x \in (c, 1).$$

Bu yerda avval qism bo‘yicha integrallash, so‘ngra differentialsallash bajarilgandan keyin

$$v(x) = \frac{\gamma}{\Gamma(\alpha+\beta)} \int_x^1 \frac{\tau'(t)dt}{(t-x)^{1-\alpha-\beta}}, \quad x \in (c, 1), \quad (23)$$

tenglikni hosil qilamiz.

(23) tenglamada $x \in (c, 1)$ o‘zgaruvchini $p(x)$ –ga o‘zgartirganda (bu yerda $p(x) \in (c, 1)$, argument esa $x \in (-1, c)$)

$$v(p(x)) = \frac{\gamma}{\Gamma(\alpha + \beta)} \int_{p(x)}^1 \frac{\tau'(t)dt}{(t - p(x))^{1-\alpha-\beta}}, \quad x \in (-1, c), \quad (24)$$

tenglikni hosil qilamiz.

Endi (24) tenglamada $t = p(s)$ integral o‘zgaruvchini, (2.1.4*) shartni hisobga olgan holda ($c f(x) \equiv 0$): $\tau(x) = \mu\tau(p(x))$, $\tau'(x) = -\mu\tau'(p(x))$ almashtirish bajarilganda:

$$\begin{aligned} v(p(x)) &= -\frac{\gamma k^{\alpha+\beta-1}}{\mu\Gamma(\alpha + \beta)} \int_{-1}^x \frac{\tau'(t)dt}{(x - t)^{1-\alpha-\beta}} = \\ &= -\frac{\gamma k^{\alpha+\beta-1}}{\mu\Gamma(\alpha + \beta)} \lim_{\varepsilon \rightarrow 0} \int_{-1}^{x-\varepsilon} \frac{d\tau(t)}{(x - t)^{1-\alpha-\beta}} = \\ &= -\frac{\gamma k^{\alpha+\beta-1}}{\mu\Gamma(\alpha + \beta)} \lim_{\varepsilon \rightarrow 0} \left[\frac{\tau(t)}{(x - t)^{1-\alpha-\beta}} \Big|_{-1}^{x-\varepsilon} - (1 - \alpha - \beta) \int_{-1}^{x-\varepsilon} \frac{\tau(t)dt}{(x - t)^{2-\alpha-\beta}} \right] = \\ &= -\frac{\gamma k^{\alpha+\beta-1}}{\mu\Gamma(\alpha + \beta)} \lim_{\varepsilon \rightarrow 0} \left[\frac{d}{dx} \int_{-1}^{x-\varepsilon} \frac{\tau(t)}{(x - t)^{1-\alpha-\beta}} \right]. \end{aligned} \quad (25)$$

hisoblab chiqamiz

Shunday qilib ushbu tenglikni (25) tenglama orqali quyidagi ko’rinishda yozib olamiz

$$J_0 = -\frac{\gamma k^{\alpha+\beta-1}}{\mu} \int_{-1}^c \tau(p(x)) D_{-1,x}^{1-\alpha-\beta} \tau(x) dx. \quad (26)$$

Endi ($c f(x) \equiv 0$): $\tau(p(x)) = \tau(x)/\mu$ shartni hisobga olgan holda va (26) tenglik orqali (20) tenglama quyidagi ko’rinishga ega bo’ladi

$$\begin{aligned} J &= \int_{-1}^1 \tau(x)v(x)dx = \gamma \left(1 - \frac{k^{\alpha+\beta}}{\mu^2}\right) \int_{-1}^c \tau(x) \left(D_{-1,x}^{1-\alpha-\beta} \tau(x)\right) dx = \\ &= \frac{\gamma \Gamma(1 - \alpha - \beta) a^{2-(\alpha+\beta)}}{2\Gamma(\alpha + \beta) \cos((\alpha + \beta)\pi/2)} \left(1 - \frac{k^{\alpha+\beta}}{\mu^2}\right) \int_0^\infty \xi^{\alpha+\beta-1} \cdot \\ &\cdot \left\{ \left[\int_{-1}^1 \tau_2(at - b) \cos(t\xi) dt \right]^2 + \left[\int_{-1}^1 \tau_2(at - b) \sin(t\xi) dt \right]^2 \right\} d\xi. \end{aligned} \quad (27)$$

Bu yerda

$$\tau(x) = \int_{-1}^x \frac{\tau_2(s)ds}{(x - s)^{\alpha+\beta}}, \quad x \in (-1, c),$$

$\tau_2(x) \in C[-1, c] \cap C^2(-1, c)$, $\tau_2(-1) = \tau'_2(-1) = 0$, (2.5)-ni hisobga olgan holda (27) orqali (19) kelib chiqadi.

Bu orqali (6) va (19) tengsizliklarni hisobga olgan holda

$$J = \int_{-1}^1 \tau(x)v(x)dx = 0. \quad (28)$$

ko‘rinishga ega bo‘lamiz.

Shunday qilib (18) tenglamaning o‘ng qismi 0-ga teng bo‘lsa, lekin (18)-dagi integrallashgan ifodaning ikkala qo‘shiluvchisi ham nomanfiy. Bundan kelib chiqadiki bu qo‘shiluvchilar ham nolga teng bo‘ladi:

$$\int_{-1}^1 \tau_1(t) \cos(t\xi) dt \equiv 0, \quad \int_{-1}^1 \tau_2(t) \sin(t\xi) dt, \quad (29)$$

$\xi \in [0, +\infty]$ –ning barcha qiymatlarida va xususiy hollarida ya’ni $\xi = k\pi$, $k = 0, 1, 2, \dots$, bo‘lganida, $\cos(t\xi)$ va $\sin(t\xi)$ trigonometrik funksiya sistemasi $L_2[-1,1]$ -da ortogonal ko‘rinishda shakllanadi. (29) tenglamadan kelib chiqadiki $[-1,1]$ oraliqda deyarli hamma joyda $\tau_1(x) \equiv 0$ bo‘ladi, lekin $[-1,1]$ oraliqda $\tau_1(x)$ uzuksiz funksiya bo‘lib, bu yerdan kelib chiqadiki $\forall x \in [-1,1]$ –da doim $\tau_1(t) \equiv 0$, demak bundan hulosa qilishimiz mumkinki (8) tenglama $\forall x \in [-1,1]$ –da $\tau(x) \equiv 0$ bo‘ladi. Bu yerdan (3) ($c\psi_1(x) \equiv 0$) tenglamadan kelib chiqqan holda $\forall x \in (-1,1)$ –da $v(x) \equiv 0$ bo‘ladi.

Teorema isbotlandi.

Yuqorida tahlil qilingan masalalar o‘zbek va xorijlik olimlar tomonidan o‘rganilgan bo‘lib, nashr qilingan maqolada juda qisqa ma'lumotlar berilgan. Maqoladagi deyarli barcha qiyin hisoblashlar kengaytirilgan va izohlar berilgan holda tishuntirishga harakat qilinib, kelgusida ilmiy qilish maqsadida shu soha bilan shug‘ullanmoqchi bo‘lgan o‘quvchilarga maqolani o‘rganish osonlashtirildi. Shu kabi masalalar [9-18] larda ham chuqur o‘rganilgan.

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