

KVADRATIK STOXASTIK OPERATORLARNING YANA BIR TADBIQI HAQIDA

Mamurov Boboxon Jo'rayevich,

Buxoro davlat universiteti

«Matematik analiz» kafedrasida dotsenti

Ro'ziyev Adham,

Buxoro davlat universiteti 2-bosqich talabasi

Annotatsiya: Bu maqolada kvadratik stoxastik operatorlarning yana bir tatbiqi, statistik mexanikadagi Boltsman modeliga tatbiqi qaraladi. Bu model ham kvadratik stoxastik operator orqali tasvirlab berilgan. Mazkur modeldan statistik mexanika va kvadratik stoxastik operatorlar nazariyasidan o'qiladigan ma'ruzalarda ularning tatbiqlari sifatida ham foydalanish mumkin.

Kalit so'zlar: Gaz, molekula, Boltsman modeli, kvadratik stoxastik operator.
About another application of quadratic stochastic operators

ABOUT ANOTHER APPLICATION OF QUADRATIC STOCHASTIC OPERATORS

Mamurov Bobokhon Jurayevich

Associate Professor of the Department

of Mathematical Analysis, Bukhara State University

Roziyev Adham

2nd year student of Bukhara State University

Annotation: This article considers another application of quadratic stochastic operators, its application to the Boltzmann model in statistical mechanics. This model is also described by a quadratic stochastic operator. This model can be used as their application in lectures on the theory of statistical mechanics and quadratic stochastic operators.

Keywords: Gas, molecule, Boltzmann model, quadratic stochastic operator.

Kvadratik operatorlar biologiya, ximiya, mexanikaning ba'zi modellarini o'rganishda muhim rol o'ynaydi.

$E = \{1, 2, \dots, n\}$ bo'lsin.

1-ta'rif. $S^{n-1} = \{x = (x_1, \dots, x_n) \in R^n : x_i \geq 0, \sum_{i=1}^n x_i = 1\}$ to'plamga (n-1) o'lchamli simpleks deb aytiladi.

Bunda har bir $x \in S^{n-1}$ element E to'plamda ehtimollik o'lchovi bo'lib, uni n ta elementdan iborat qandaydir biologik (fizik) tizim kabi talqin qilish mumkin.

2-ta`rif. Kvadratik stoxastik operator deb, $V: S^{n-1} \rightarrow S^{n-1}$

$$V: x_k' = \sum_{i,j=1}^n p_{ij,k} x_i x_j,$$

ko`rinishdagi operatorga aytiladi, bunda

$$p_{ij,k} \geq 0, p_{ij,k} = p_{ji,k}, \sum_{k=1}^n p_{ij,k} = 1.$$

Biz bu maqolada kvadratik stoxastik operatorlarning statistik mexanikaga tatbiqlaridan birini qarab chiqamiz.

Masalan, statistik mexanikadagi Boltsman modelini ham kvadratik stoxastik operatorlar orqali tasvirlab berish mumkin.

Qattiq mutlaqo elastik devorlar orasiga o`ralgan gazni qaraymiz, uning molekulari radiuslari va massalari bir xil bo`lgan qat`iy absolyut elastik sharlardan iborat bo`lsin. Molekulalarning tezlik vektorlari to`plamini R^3 ning elementlari sifatida qaraymiz.

R^3 ni n ta E_1, E_2, \dots, E_n sohalarga shunday bo`lamizki, $E_i \cap E_j = \emptyset$ ($i \neq j$) va $\bigcup_{i=1}^n E_i = R^3$ bo`lsin.

$i = 1, 2, \dots, n$ lar uchun

$$x_i(t) = \left\{ \begin{array}{l} t \text{ momentda tezligi } E_i \text{ sohada} \\ \text{yotadigan molekularning ulushi} \end{array} \right\}$$

deb belgilab olamiz.

Bir qator tabiiy fizik farazlarda

$$\left\{ \begin{array}{l} \text{birlik vaqt oraliq'ida} \\ l - m \text{ to'qnashuvlar} \\ \text{soni} \end{array} \right\} = \mu_{lm} x_l x_m, \quad \mu_{lm} > 0.$$

p_{lm}^i bilan E_l dagi molekularning E_m dagi molekular bilan to`qnashganda E_i ga tarqaladigan qismini belgilaymiz.

$\sum_{i=1}^n p_{lm}^i = 1$ bo`lishligiga osongina ishonch hosil qilish mumkin. Bu holda

$$\left\{ \begin{array}{l} E_i \text{ dagi molekularning birlik} \\ \text{vaqt oraliq'ida } l - m \text{ to'qnashuv} \\ \text{natijasida } E_i \text{ ga tarqaladiganlari} \\ \text{soni} \end{array} \right\} = p_{lm}^i \mu_{lm} x_l x_m,$$

qaysiki,

$$\begin{aligned} x_i(t+1) - x_i(t) &= \\ \sum_{lm} p_{lm}^i \mu_{lm} x_l x_m - \sum_{lm} p_{lm}^l \mu_{lm} x_l x_m &= \\ &= \sum_{l,m} a_{lm}^i x_l x_m, \end{aligned}$$

bu yerda $a_{lm}^i = \frac{1}{2} [(p_{ml}^i + p_{ml}^i) - (\delta_{il} + \delta_{im})] \cdot \mu_{lm}$

va

$$\delta_{ij} \text{- Kroneker belgisi: } \delta_{ij} = \begin{cases} i = j \text{ bo'lsa, } 1 \\ i \neq j \text{ bo'lsa, } 0 \end{cases}$$

Oxirgi tenglikdan

$$x_i(t + 1) = \sum_{l,m=1}^n p_{ml,i} x_m(t) x_l(t), \quad i=1,2, \dots, n,$$

bunda $p_{ml,i} = a_{lm}^i + \delta_{ml,i}$ va

$$\delta_{ml,i} = \left\{ \begin{array}{l} 1, \text{ agar } m = l = i \\ \frac{1}{2}, \text{ agar } m \neq l \text{ va } m = i \text{ yoki } l = i \\ 0, \text{ qolgan hollarda} \end{array} \right\}.$$

Qilingan farazlar va belgilashlarga asoslanib, $p_{ml,i}$ koeffitsentlarning quyidagi shartlarni qanoatlantirishiga ishonch hosil qilish mumkin:

- 1) $p_{ml,i} \geq 0$;
- 2) $p_{ml,i} = p_{lm,i}$;
- 3) $\sum_{i=1}^n p_{ml,i} = 1$.

Yuqoridagilardan ko`rinadiki, statistik mexanikadagi Boltsman modelini kvadratik stoxostik operatorlar orqali ta`riflash mumkin ekan.

Biz [1] da populyatsiya evolyutsiyasini tasodifiy jarayon deb qarab, avtomat populyatsiyaning ozod populyatsiya holida populyatsiyaning evolyutsion operatori kvadratik akslantirish bo`lishiga ishonch hosil qilgan edik. Shuni ta`kidlash lozimki, [1-7] va [8-13] larda qaralgan biologik va tibbiy modellarda vaqt diskret edi. Bu maqolada qaralgan statistik mexanikadagi Boltsman modelida esa vaqt uzluksiz bo`ladi.

XULOSA

Yuqorida hamda [1-13] maqolalarda kvadratik va kubik stoxastik operatorlarga doir nazariy masalalar va ular orqali ta`riflash mumkin bo`lgan turli modellar o`rganilgan. Keltirilgan modellardan statistik mexanika, biologiya, differentsial tenglamalar va kvadratik stoxastik operatorlar nazariyasidan o`qiladigan ma`ruzalarda, ularning tadbirlari sifatida foydalanish mumkin. Bu esa, o`tilgan nazariy materialni chuqurroq tushunush imkonini beradi.

FOYDALANILGAN ADABIYOTLAR

1. Mamurov B.J., Rozikov U.A. On cubic stochastic operators and processes. Journal of Physics: Conference Series. **697** (2016), 012017.
2. Mamurov B.J., Rozikov U.A., Xudayarov S.S. Quadratic stochastic processes of type (σ/μ) . arXiv: 2004.01702 . Pp. 1-14. math.D.S
3. Mamurov B.J., Rozikov U.A. and Xudayarov S.S. Quadratic Stochastic Processes of Type (σ/μ) . Markov Processes Relat.Fields 26, 915-933 (2020).
4. Мамуров Б.Ж. О кубических стохастических процессов. Тезисы докладов межн. конфер. CODS-2009. С.72.
5. Мамуров Б.Ж. О решения эволюционных уравнений для кубических стохастических процессов. Сборник материалов международной конференции КРОМШ-2019. 305-307 стр.

6. Мамуров Б.Ж., Шарипова М. Об одном квадратичном стохастическом операторе в S^2 . "Scientific Progress". Int. Scientific-Pract. conf. Tashkent. 2021, March 15. Стр. 121-122.
7. Mamurov B.J. A central limit theorem for quadratic chains with finite enotypes. Scientific reports of Bukhara State University. 1:5, 2018. Pp. 18-21.
8. Мамуров Б.Ж., Сохибов Д.Б. О неподвижных точках одного квадратичного стохастического оператора. Наука, техника и образование. 2021. №2 (77). Часть 2. Стр. 10-15.
9. Мамуров Б.Ж. Эволюционные уравнения для конечномерных однородных кубических стохастических процессов. Bulletin of Institute of Mathematics 2019. №6, pp. 35-39.
10. Мамуров Б.Ж. О решения эволюционных уравнений для кубических стохастических процессов. Сборник материалов международной конференции КРОМШ-2019. 305-307 стр.
11. Mamurov B.Zh. The convex combinations of quadratic operators on S^2 . Abstracts of the VII inter.conf. Modern prob. of applied mat. inf. tex. Al-Khwar. 21. pp. 87.
12. Mamurov B.J., Bazorova D. Biologiya va tibbiyotdagi ba'zi matematik modellar haqida. Science and education. Volume 1, issue 8 UIF-2022:8.2. 418-426.
13. Mamurov B.J., Bazorova D. Kvadratik stoxostik operatorlarga olib kelinadigan ba'zi modellar haqida. Science and education. Volume 4, issue 3. March 2023. 41-48.