ARALASH TIPDAGI TENGLAMA UCHUN GF MASALASI HAQIDA

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Annotatsiya. Ushbu maqolada buzilish chizig‘iga ega aralash tipdagi tenglama uchun GF masalasining qo‘yilishi, yechimning yagonaligi hamda mavjudligi o’rganib chiqilgan. Yechimning yagonaligi hamda mavjudligi haqidagi teoremalar tahlil qilingan. Olingan natijalarni boshqa chegaraviy masalalarga qo’llanilishi haqida fikrlar qayd qilingan.

Kalit so‘zlar: chekli bir bog’lamli soha, funlsiya, Gellerstedt masalasi, Frankel sharti, Darbu formulasi, ekstremum prinsipi, Xopf prinsipi.

ON THE GF PROBLEM FOR THE RALASH-TYPE EQUATION

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Annotation. In this paper, the formulation of the GF problem, the uniqueness of the solution, and the existence of a mixed-type equation with a fault line are studied. The theorems about the uniqueness and existence of the solution are analyzed. Opinions about the application of the obtained results to other boundary issues are noted.

Keywords: finite one-connected field, function, Gellerstedt problem, Frankel's condition, Darboux's formula, extremum principle, Hopf's principle.

Maqolada ushbu tenglamani o’rganiladi:

$$(sign y)|y|^m u_{xx} + u_{yy} + (\beta_0 / y)\, u_y = 0 \ , \qquad (1)$$

bu yerda $m>0, \quad -m/2<\beta_0<1$ o’zgarmaslar, D $z=x+iy$ kompleks tekisligining chekli bir bog’lamli $\sigma_0 : y=\sigma_0(x)=\left[(m+2)^2(1-x^2)/4\right]^{1/(m+2)}$ normal egri chiziq va $A=A(-1{,}0)$ va $B=B(1{,}0)$ kesma va (1) tenglamaning AC va BC xarakteristikalari bilan chegaralangan soha. $y>0$ va $y<0$ yarimtekisliklarida yotuvchi C_0 va C_1 mos keluvchi nuqtalar orqali $E(c,0)$ nuqtalardan chiquvchi AC va BC xarakterlarini D^+ va D^- orqali belgilab olamiz, bu yerda $y = 0$ to’g’ri chiziqdagi $c\in I=(-1{,}1)$ -interval. $q(x)\in C^1[c,1]$ $q'(x)<0,\ q(c)=c, q(1)=-1$.

Izoh. Keltirilgan funsiyalar sinfi bo’sh emas. Masalan $q(x)=\rho-kx$ chiziqli funlsiyani misol sifatida keltiramiz, bu yerda $k=(1+ñ)/(1-c)$, $\rho=2c/(1-c)$, $\rho-k=-1,\ \rho-kc=c.$

Gellerstedt masalasida D aralash sohaning giperbolik funksiyaning qiymatlari

EC_0 va $EC_1 : u\big|_{EC_0} = \psi_0(x),\ u\big|_{EC_1} = \psi_1(x)$ xarakterlarida berilgan. Maqolada masalaning to'g'riligini o'rganiladi, ya'ni bu yerda EC_0 xarakteristikasi chetki shartlardan ozod qilinadi va AB kesmada Frankel shartining tarmoqlar bo'yicha emas bilan bir vaqta Gellerstedtning yetishmovchilik sharti bilan almashtiriladi.

GF masalasi. D sohada $u(x,y)$ funksiyani qanoatlantiruvchi keying shartlar topiladi (bu masalalar o'zbek va xorijlak olimlar tomonidan o'rganlgan):

1. $u(x,y) \in C(\overline{D})$;

2. $u(x,y) \in C^2(D^+)$ va (1) tenglamani shu sohada qanoatlantiradi;

3. $u(x,y)$ $, D^- \setminus (EC_0 \cup EC_1)$; sohada R_1 sinfdagi umumiy yechim hisoblanadi;

4. Buzulish chizig'I oralig'ida ulanish sharti o'rinli bo'ladi:

$$\lim_{y\to -0} (-y)^{\beta_0} \frac{\partial u}{\partial y} = \lim_{y\to +0} y^{\beta_0} \frac{\partial u}{\partial y},\ x \in I \setminus \{c\}, \tag{2}$$

bundan tashqari limitlar $x = \pm 1,\ x = c$ bo'lganida $1-2\beta$ tartibdan kichik qiymatga ega bo'lishi mumkin, bu yerda $\beta = (m+2\beta_0)/2(m+2)$;

5. $$u(x,\sigma_0(x)) = a(x)u(x,0) + \phi(x),\ \ -1 \le x \le 1 \tag{3}$$

$$u\big|_{EC_1} = \psi(x),\quad c \le x \le (c+1)/2, \tag{4}$$

$$u(q(x),0) = u(x,0) + f(x),\ c \le x \le 1\ , \tag{5}$$

bu yerda

$f(x) \in C[c,1] \cap C^{1,\alpha}(c,1),\ f(c) = 0,\ f(1) = 0,\ \psi(x) \in C^2[c,(c+1)/2],\ \psi(c) = 0, a(x), \varphi(x) \in C^{0,\alpha}[-1,1],$

bundan tashqari

$$a(x) = \left(1-x^2\right)\tilde{a}(x),\ \varphi(x) = \left(1-x^2\right)\tilde{\varphi}(x),\ \tilde{a}(x), \tilde{\varphi}(x) \in C^{0,\alpha}[-1,1], 0 \le a(x) \le 1, a(c) < 1. \tag{6}$$

(5) sharti Frankel sharti analogi hisoblanadi, (3) sharti esa Bitsadze-Samarskiy shartning analogi hisoblanadi.

Darbu formulana hisobga olgan holda (4) chetki shartidan keying nisbatni olamiz:

$$\nu(x) = \gamma D_{c,x}^{1-2\beta} \tau(x) + \Psi(x),\quad x \in (c,1), \tag{7}$$

bu yerda $$\Psi(x) = \frac{(1+x)^{\beta} D_{c,x}^{1-2\beta} \psi\left((x+c)/2\right)}{\gamma_2 \left((m+2)/2\right)^{1-2\beta} \Gamma(1-2\beta)},\quad \gamma = \frac{2\Gamma(1-\beta)\Gamma(2\beta)}{\Gamma(\beta)\Gamma(1-2\beta)} \left(\frac{m+2}{4}\right)^{2\beta}.$$

(7) nisbat $\tau(x)$ va $\nu(x)$ noaniq funksiyalar oralig'ida birinchi funksional hisoblanib, D aralash sohadagi D^- qismida $y = 0$ giperbolik chiziqda $(c, 1)$ intervalga ko'chiriladi.

A.V. Bitsadze ekstremum prinsip keying analogi o'rinli bo'ladi. GF masalasi $|a(x)| \le 1,\ a(c) < 1,\ \varphi(x) \equiv 0,\ \ \psi(x) \equiv 0,\ \ f(x) \equiv 0$ shartlarni bajarganda D^+ sohada musbat maksimum va manfiy minimum qiymatlarni $E(c,0)$. nuqtada erishadi.

$u(x,0) = \tau(x),\ \ x \in \bar{I}$ belgilashni hisobga olib (1.5) shartni

$$\tau(q(x)) = \tau(x) + f(x), \quad x \in [c,1]. \tag{8}$$

ko'rinishida yozib olamiz.

Xopf prinsipiga binoan, $u(x,y)$ funksiya D^{+} sohaning ichki nuqtalarida o'zining musbat maksimum va manfiy minimum qiymatlariga erisha olmaydi.

Agar $u(x,y)$ funksiya $(x_0,0)$ nuqtada $(-1,c)\cup(c,1)$ interval $y=0$ chiziqda o'zining kerakli musbat maksimum va manfiy minimum qiymatiga erishadi.

Bu yerda $x_0 \in (-1,0)$ yoki $x_0 \in (c,1)$ 2-ta holini alohida qaraymiz.

1. Aytaylik, bu yerda $x_0 \in (c,1)$, u holda bu nuqtada musbat maksimum (manfiy minimum)

$$\nu(x_0) < 0 \ \ (\nu(x_0) > 0). \tag{9}$$

Bu yerda aniqki, bu nuqtada $\tau(x)$ funksiya musbat maksimum (manfiy minimum), kasr-differentsial operatorlar uchun keyingi $D^{1-2\beta}_{-1,x_0}\tau(x) > 0$ $\left(D^{1-2\beta}_{-1,x_0}\tau(x) < 0\right)$ tengsizliklar o'rinli bo'ladi, bu yerdan (7) (bu yerda $\Psi(x)\equiv 0$).

$$\nu(x_0) = \gamma D^{1-2\beta}_{-1,x_0}\tau(x) > 0\left(\nu(x_0) = \gamma D^{1-2\beta}_{-1,x_0}\tau(x) < 0\right) \tag{10}$$

ega bo'lamiz.

(9) va (10) tengsizlik (2) ulanish shartiga zud, bu yerdan kelib chiqadi $x_0 \notin (c,1)$.

2. Aytalik, bu yerda $x_0 \in (-1,c)$. Agar $x_1 \in (c,1)$, $q(x_1) = x_0$ tengalamaning yechimi bo'lsin. U holda (8) (bu yerda $f(x)\equiv 0$)-dan

$$\tau(x_0) = \tau(q(x_1)) = \tau(x_1), \tag{11}$$

-ga ega bo'lamiz.

(11) tenglik ko'rsatadiki, bu yerda x_1 nuqta $\tau(x)$ funksiya ekstremum nuqtasida $(c,1)$ intervalda bor, bu yerdan oldingi vaziyatga zud. Bundan kelib chiqqan holda $x_0 \notin (-1,c)$.

Shunday qilib $u(x,y)$ funksiya o'zining musbat maksimum va manfiy minimum qiymatlariyb $(-1,c)\cup(c,1)$ intervallar nuqtasida erisha olmaydi. (3) shartidan, bu yerda $(\varphi(x)\equiv 0, \quad a(c) < 1)$ kelib chiqgan holda σ_0-ga ham bo'lmaydi.

Shunday qilib, yuqorida aytilgan kelib chiqgan holda, GF masalasi $|a(x)| \le 1,\ a(c) < 1,\ \varphi(x)\equiv 0,\ \ \psi(x)\equiv 0,\ \ f(x)\equiv 0$ $\overline{D}^{+}$ sohadagi $E(c,0)$ nuqtada o'zining musbat maksimum va manfiy minimum qiymatiga erishadi. Ekstremum prinsipi isbotlandi. Ekstremum prisipdan quyidagi kelib chiqgan.

Natija. GF masala ko'p yechimga ega emas.

Endi yechimni mavjudligi masalasini o'rganamiz.

$\tau(x)$ va $\nu(x)$ noaniq funksiyalar D^{+} sohadan I sohaga o'tgan yaxshi nisbatga keltiramiz

$$\nu(x) = -k_2(1-\beta_0)\frac{m+2}{2}\left\{\int_{-1}^{1}\frac{(x-t)\tau'(t)dt}{|x-t|^{2-2\beta}} - (2\beta-1)\int_{-1}^{1}\frac{\tau(t)dt}{(1-xt)^{2-2\beta}}\right\} + \Phi(x),\ x\in I.$$

(13)

bu yerda

$$\Phi(x)=k_2(1-\beta_0)(1-\beta)(m+2)\left(1-x^2\right)\int_{-1}^{1}\varphi_1(t)\left(x^2-2xt+1\right)^{\beta-2}dt$$

(13) tenglamani (7) tenglamaga almashtirish orqali, Bitsadze-Samarskiy shartni hisobga olgan holda (3): $\varphi_1(t)=a(t)\tau(t)+\varphi(t)$,

$$\int_{-1}^{1}\frac{(x-t)\tau'(t)dt}{|x-t|^{2-2\beta}}-(2\beta-1)\int_{-1}^{1}\frac{\tau(t)dt}{(1-xt)^{2-2\beta}}=$$
$$=R_0[\tau]-\frac{2\gamma}{k_2(1-\beta_0)(m+2)}D_{c,x}^{1-2\beta}\tau(x)-\frac{2(\Psi(x)-\Phi_0(x))}{k_2(1-\beta_0)(m+2)},\quad x\in(c,1), \tag{14}$$

ega bo'lamiz, bu yerda

$$R_0[\tau]=2(1-\beta)\left(1-x^2\right)\int_{-1}^{1}\left(x^2-2xt+1\right)^{\beta-2}a(t)\tau(t)dt, \tag{15}$$

$$\Phi_0(x)=k_2(1-\beta_0)(1-\beta)(m+2)\left(1-x^2\right)\int_{-1}^{1}\varphi(t)\left(x^2-2xt+1\right)^{\beta-2}dt\quad \in C(\bar{I})\cap C^1(I).$$

o'rinli.

(14) tenglamani quyidagi ko'rinishga keltirib olamiz:

$$\int_{-1}^{c}\frac{(x-t)\tau'(t)dt}{(x-t)^{2-2\beta}}+\int_{c}^{1}\frac{(x-t)\tau'(t)dt}{|x-t|^{2-2\beta}}+(1-2\beta)\int_{-1}^{c}\frac{\tau(t)dt}{(1-xt)^{2-2\beta}}+(1-2\beta)\int_{c}^{1}\frac{\tau(t)dt}{(1-xt)^{2-2\beta}}=$$
$$=R_0[\tau]-\frac{2\gamma}{k_2(1-\beta_0)(m+2)}D_{c,x}^{1-2\beta}\tau(x)-\frac{2(\Psi(x)-\Phi_0(x))}{k_2(1-\beta_0)(m+2)},\quad x\in(c,1).$$

Integrallarda $(-1,c)$ chegaralari bilan $t=q(s)$ o'zgaruvchilarni almashtirib $\tau'(q(s))\cdot q'(s)=\tau'(s)+f'(s)$ hisobga olgan holda

$$-\int_{c}^{1}\frac{\tau'(s)ds}{(x-q(s))^{1-2\beta}}+\int_{c}^{1}\frac{(x-t)\tau'(t)dt}{|x-t|^{2-2\beta}}-(1-2\beta)\int_{c}^{1}\frac{\tau(s)q'(s)ds}{(1-x\ q(s))^{2-2\beta}}+$$
$$+(1-2\beta)\int_{c}^{1}\frac{\tau(t)dt}{(1-xt)^{2-2\beta}}=R_0[\tau]-\frac{2\gamma D_{c,x}^{1-2\beta}\tau(x)}{k_2(1-\beta_0)(m+2)}+F_0(x),\ \ x\in(c,1), \tag{16}$$

$$F_0(x)=-\frac{2(\Psi(x)-\Phi_0(x))}{k_2(1-\beta_0)(m+2)}+\int_{c}^{1}\frac{f'(s)ds}{(x-q(s))^{1-2\beta}}+(1-2\beta)\int_{c}^{1}\frac{f(s)q'(s)ds}{(1-xq(s))^{2-2\beta}},\in C(\bar{I})\cap C^1(I),\ \ x\in(c,1).$$

-ni hosil qilib olamiz.

(16) tenglikga $\Gamma(1-2\beta)D_{c,x}^{2\beta-1}$ operatorni qo'llagan holda keying nisbatni hosil qilib olamiz

$$-\int_c^x \frac{ds}{(x-s)^{2\beta}} \int_c^1 \frac{\tau'(t)dt}{(s-q(t))^{1-2\beta}} + \int_c^x \frac{ds}{(x-s)^{2\beta}} \int_c^1 \frac{(s-t)\tau'(t)dt}{|s-t|^{2-2\beta}} -$$
$$-(1-2\beta)\int_c^1 \frac{ds}{(x-s)^{2\beta}} \int_c^1 \frac{\tau(t)q'(t)dt}{(1-sq(t))^{2-2\beta}} + (1-2\beta)\int_c^1 \frac{ds}{(x-s)^{2\beta}} \int_c^1 \frac{\tau(t)dt}{(1-st)^{2-2\beta}} = \qquad (17)$$
$$= -\frac{2\gamma\Gamma(1-2\beta)}{k_2(1-\beta_0)(m+2)}\tau(x) + \Gamma(1-2\beta)D_{cx}^{1-2\beta}R_0[\tau] + F(x), \ \ x \in (c,1),$$

bundan quyidagi tengliklar haqiqiyligini tekshirish qiyin emas:

1. $$\int_c^x \frac{ds}{(x-s)^{2\beta}} \int_c^1 \frac{\tau'(t)dt}{(s-q(t))^{1-2\beta}} = -\int_c^1 \left(\frac{x-c}{s-q(t)}\right)^{1-2\beta} \frac{\tau'(t)q'(t)dt}{x-q(t)}, \qquad (18)$$

2. $$\int_c^x \frac{ds}{(x-s)^{2\beta}} \int_c^1 \frac{\tau'(t)dt}{(s-q(t))^{1-2\beta}} = -\int_c^1 \left(\frac{x-c}{c-q(t)}\right)^{1-2\beta} \frac{q'(t)\tau(t)dt}{x-q(t)}, \quad x \in (c,1);$$

(19)

3. $$\int_c^x \frac{ds}{(x-s)^{2\beta}} \int_c^1 \frac{\tau(t)dt}{(1-st)^{2-2\beta}} = \frac{1}{1-2\beta}\int_c^1 \left(\frac{x-c}{1-ct}\right)^{1-2\beta} \frac{\tau(t)dt}{1-xt}, \quad x \in (c,1);$$

(20)

$$\int_c^x \frac{ds}{(x-s)^{2\beta}} \int_c^1 \frac{\tau(t)q'(t)dt}{(1-sq(t))^{2-2\beta}} = \frac{1}{1-2\beta}\int_c^1 \left(\frac{x-c}{1-cq(t)}\right)^{1-2\beta} \frac{\tau(t)q'(t)dt}{1-x\,q(t)}, \ \ x \in (c,1), \qquad (21)$$

Endi (18) tenglikni isbotlaymiz.

$$A(x) = \int_c^x \frac{ds}{(x-s)^{2\beta}} \int_c^1 \frac{\tau'(t)dt}{(s-q(t))^{1-2\beta}},$$

bu yerda integrallash tartibini va ichki integralda integral o’zgaruvchilarni $s = x + (c-x)\sigma$ -ga almashtirish orqali

$$A(x) = \int_c^x \left(\frac{x-c}{x-q(t)}\right)^{1-2\beta} \tau'(t)dt \int_0^1 \sigma^{-2\beta}\left(1-\frac{x-c}{x-q(t)}\sigma\right)^{1-2\beta} dt =$$
$$\frac{1}{1-2\beta}\int_{-1}^c \left(\frac{x-c}{x-q(t)}\right)^{1-2\beta} F\left(1-2\beta, 1-2\beta, 2-2\beta; \frac{x-c}{x-q(t)}\right)\tau'(t)dt,$$

ega bo’lamiz.

Bo’laklab integrallash operatsiyasini bajarib,

$$u = \left(\frac{x-c}{x-q(t)}\right)^{1-2\beta} F\left(1-2\beta, 1-2\beta, 2-2\beta; \frac{x-c}{x-q(t)}\right)$$

$$du = (1-2\beta)\left(\frac{x-c}{x-q(t)}\right)^{1-2\beta} F\left(2-2\beta, 1-2\beta, 2-2\beta; \frac{x-c}{x-q(t)}\right)\frac{x-c}{x-q(t)^2} q'(t)dt$$

$$dv = \tau'(t)dt, \ \ v = \tau(t),$$

keyin esa

$$\frac{d}{dx}x^a F(a;b;c;x)=ax^{a-1}F(a+1;b;c;x)$$
$$F(c;b;c;x)=(1-x)^{-b}$$

formulasini ishlatamiz va

$$A(x)=-\int_c^1\left(\frac{x-c}{c-q(t)}\right)^{1-\beta}\frac{\tau(t)q'(t)dt}{x-q(t)^{1-2\beta}}$$

(18) tenglik isbotlandi.

Endi esa (19) isbotlaymiz:

$B(x)=\int_c^x\frac{ds}{(x-s)^{2\beta}}\int_c^1\frac{(s-t)\tau'(t)dt}{|s-t|^{2-2\beta}}=\int_c^x\frac{ds}{(x-s)^{2\beta}}\int_c^s\frac{\tau'(t)dt}{|s-t|^{1-2\beta}}-$
$\int_c^x\frac{ds}{(x-s)^{2\beta}}\int_s^1\frac{\tau'(t)dt}{(s-t)^{1-2\beta}}=\lim_{\varepsilon\to 0}B_{1\varepsilon}(x)-\lim_{\varepsilon\to 0}B_{2\varepsilon}(x)$
(21)

bu yerda

$$B_{1\varepsilon}=\int_c^x\frac{ds}{(x-s)^{2\beta}}\int_c^{s-\varepsilon}\frac{\tau'(t)dt}{(s-t)^{1-2\beta}}dt,$$
$$B_{2\varepsilon}=\int_c^x\frac{ds}{(x-s)^{2\beta}}\int_{s+\varepsilon}^1\frac{\tau'(t)dt}{(t-s)^{1-2\beta}}dt.$$

$B_{1\varepsilon}=\int_c^x\frac{ds}{(x-s)^{2\beta}}\int_c^{s-\varepsilon}(s-t)^{2\beta-1}d\tau(t)$ hisoblaimiz.

Bu yerda ichki integralda bo'laklab integrallashni operatsiyasini bajarganda

$$B_{1\varepsilon}=\int_c^x\frac{1}{(x-s)^{2\beta}}\left[\varepsilon^{2\beta-1}\tau(s-\varepsilon)-(s-c)^{2\beta-1}\tau(c)-(1-2\beta)\int_c^{s-\varepsilon}(s-t)^{2\beta-2}d\tau(t)\right]\quad(22)$$

tenglikni hisobga olgan holda

$$\int_c^{s-\varepsilon}(s-t)^{2\beta-2}d\tau(t)=\frac{\varepsilon^{2\beta-1}}{1-2\beta}\tau(s-\varepsilon)-\frac{1}{1-2\beta}\frac{d}{ds}\int_c^{s-\varepsilon}(s-t)^{2\beta-1}\tau(t)dt$$

(22) tenglikni quyidagi ko'rinishga yozib olamiz:

$$B_{1\varepsilon}=\int_c^x\frac{ds}{(x-s)^{2\beta}}\frac{d}{ds}\int_c^{s-\varepsilon}(s-t)^{2\beta-1}\tau(t)dt.$$

Bu yerda limit $\varepsilon\to 0$-ga o'tganda

$$\lim_{\varepsilon\to 0}B_{1\varepsilon}(x)=\Gamma(2\beta)\Gamma(1-2\beta)D_{c,x}^{2\beta-1}D_{c,x}^{1-2\beta}\tau(x)=\Gamma(2\beta)\Gamma(1-2\beta)\tau(x)\ (23)$$

ega bo'lamiz. O'xshash hisoblash orqali

$\lim_{\varepsilon\to 0}B_{2\varepsilon}(x)=-\Gamma(2\beta)\Gamma(1-2\beta)D_{c,x}^{2\beta-1}D_{c,x}^{1-2\beta}\tau(x)==-\Gamma(2\beta)\Gamma(1-$
$2\beta)\left(-\tau(x)cos2\beta\pi-\frac{sin2\beta\pi}{\pi}\int_c^1\left(\frac{x-c}{t-c}\right)^{1-2\beta}\frac{\tau(t)dt}{t-x}\right)$ (24)

hosil qilib olamiz.

Shunday qilib (23), (24)-larni hisobga olgan (21) nisbatni (19) ko'rinishiga kelterib olamiz. (19) tenglik isbotlandi.

Qolgan (20) va (4,21) aniqliklar shunga o'xshash metod bilan isbotlanadi.

(17)-dan (18)-(21) hisobga olgan holda keying singulyar integral tenglamani olamiz:

$$\tau(x)-\lambda\int_c^1\left(\frac{x-c}{t-c}\right)^{1-2\beta}\left(\frac{1}{t-x}-\frac{1}{1-xt}\right)\tau(t)=g_0(x)\quad x\in(c,1) \tag{25}$$

bu yerda

$$g_0(x)=-\lambda\int_c^1\left(\frac{x-c}{c-q(t)}\right)^{1-2\beta}\frac{q'(t)\tau(t)dt}{x-q(t)}+R[\tau]+F_1(x),\quad x\in(c,1) \tag{26}$$

$$R[\tau]=\lambda\left[\int_c^1\left(\frac{x-c}{t-c}\right)^{1-2\beta}-\left(\frac{x-c}{1-ct}\right)^{1-2\beta}\right]\frac{\tau(t)dt}{1-xt}+\lambda\int_c^1\left(\frac{x-c}{1-cq(t)}\right)^{1-2\beta}\frac{q'(t)\tau(t)dt}{1-x\,q(t)}+$$

$$+\lambda\,\Gamma(1-2\beta)D_{cx}^{1-2\beta}R_0[\tau]+x\in(c,1) \tag{27}$$

doimiy operator, $F_1(x)=\lambda F(x), \lambda=\cos\beta\pi/\pi(1+\sin\beta\pi)$.

$g_0(x)$-dagi birinchi integral operator doimiy emas, chunki integral osti ifodasi $x=c,\ t=c$-larda birinchi tartibli izolyatsiya hususiyatiga ega, shuning uchun (26)-dagi qo'shilish alohida belgilangan.

(25) Trikonomi singulyar integral tenglamasining yechimini Gyolder funksiya sinfida izlaymiz, $x=1$ bilan chegaralangan $(x-c)^{2\beta-1}\ \tau(x)$ funksiya va $x=c$ nuqtada tartibini 1-dan pasaytirishga imkon beriladi.

(25) tenglamaga Karleman-Vekua metodi qo'llagan holda

$$\tau(x)=\cos^2\alpha\pi\ g_0(x)+\frac{\sin 2\alpha\pi}{2\pi}\int_c^1\left[\frac{(1-x)^{2\alpha}}{(1-t)^{2\alpha}}\left(\frac{x-c}{t-c}\right)^{3\alpha}\cdot\left(\frac{1-ct}{1-cx}\right)^{\alpha}\left(\frac{1}{t-x}-\frac{1}{1-xt}\right)\right]g_0(t)dt, \tag{28}$$

tenglamaning yechimini olamiz, bu yerda $\alpha=(1-2\beta)/4$.

Endi esa (26)-dagi $g_0(x)$ ifoda uchun (28) yechimini

$$\tau(x)=-\lambda\cos^2\alpha\pi\int_c^1\left(\frac{x-c}{c-q(t)}\right)^{4\alpha}\frac{\tau(t)q'(t)}{x-q(t)}dt-\frac{\lambda\sin 2\alpha\pi}{2\pi}(1-x)^{2\alpha}(x-c)^{3\alpha}\int_c^1\frac{\tau(s)q'(s)ds}{(c-q(s))^{4\alpha}}\times$$

$$\times\int_c^1\frac{(t-c)^{\alpha}}{(1-t)^{2\alpha}}\left(\frac{1}{t-x}-\frac{1}{1-xt}\right)\frac{dt}{t-q(s)}+R_1[\tau]+F_2(x),\quad x\in(c,1), \tag{29}$$

ko'rinishga keltirib olamiz, bu yerda

$$R_1[\tau]=-\lambda\frac{\sin 2\alpha\pi}{2\pi}\int_c^1\tau(s)q'(s)ds\int_c^1\left(\frac{t-c}{c-q(s)}\right)^{4\alpha}\left(\frac{1-x}{1-t}\right)^{2\alpha}\left(\frac{x-c}{t-c}\right)^{3\alpha}\left[\left(\frac{1-ct}{1-cx}\right)^{\alpha}-1\right]\times$$

$$\times\left(\frac{1}{t-x}-\frac{1}{1-xt}\right)\frac{dt}{t-q(s)}+\frac{\sin 2\alpha\pi}{2\pi}\int_c^1\left(\frac{1-x}{1-t}\right)^{2\alpha}\left(\frac{x-c}{t-s}\right)^{3\alpha}\left(\frac{1-ct}{1-cx}\right)^{\alpha}\times$$

$$\times\left(\frac{1}{t-x}-\frac{1}{1-xt}\right)R[\tau]dt+\lambda\cos^2\alpha\pi\,R[\tau],\ x\in(c,1) \tag{30}$$

$$F_2(x)=\cos^2\alpha\pi F_1(x)+\frac{\sin 2\alpha\pi}{2\pi}\int_c^1\left(\frac{1-x}{1-t}\right)^{2\alpha}\left(\frac{x-c}{t-c}\right)^{3\alpha}\left(\frac{1-ct}{1-cx}\right)^{\alpha}\cdot\left(\frac{1}{t-x}-\frac{1}{1-xt}\right)F_1(t)dt\in C(\bar{I})\cap C^1(I) \tag{31}$$

Keyingi izlanishlarda taxmin qilishimiz mumkinki $q(x)=\rho-kx$, bu yerda $k=(1+c)/(1-c),\ \rho=2c/(1-c)$.

Ushbu taxmin ostida

$$A(x,s)=\int_c^1\frac{(t-c)^{\alpha}}{(1-t)^{2\alpha}}\left(\frac{1}{t-x}-\frac{1}{1-xt}\right)\frac{dt}{t-(\rho-ks)}. \tag{32}$$

integralni hisoblaymiz.

Integral ostidagi ratsional ko'paytiruvchi ifodalarni oddiy kasr ko'rinishda yozib olamiz:

$$\left(\frac{1}{t-x}-\frac{1}{t-xt}\right)\frac{1}{t-(\rho-ks)}=\frac{1}{x+(ks-q)}\left(\frac{1}{t-x}-\frac{1}{t+ks-\rho}\right)-\frac{1}{1+kxs-\rho x}\left(\frac{x}{1-xt}+\frac{1}{t+ks-\rho)}\right)$$

va yoyib olish orqali

$$A(x,s)=\frac{1}{x+ks-\rho}(I_1(x)-I_2(s))-\frac{1}{1+kxs-\rho x}(xI_3(x)-I_2(s)) \tag{33}$$

-ga ega bo'lamiz, bu yerda

$$I_1(x)=\int_c^1\frac{(t-c)^{\alpha}}{(1-t)^{2\alpha}}\frac{dt}{t-x},\quad I_2(s)=\int_c^1\frac{(t-c)^{\alpha}}{(1-t)^{2\alpha}}\frac{dt}{t+ks-\rho},\quad I_3(x)=\int_c^1\frac{(t-s)^{\alpha}}{(1-t)^{\alpha}}\frac{dt}{1-xt}.$$

Ishonch hosil qilish qiyin emaski

$$I_1(x)=-\pi\,ctg\,(2\alpha\pi)\frac{(x-c)^{\alpha}}{(1-x)^{2\alpha}}-\frac{\Gamma(1+\alpha)\Gamma(-2\alpha)}{(1-c)^{\alpha}\Gamma(1-\alpha)}F\left(\alpha,1,1+2\alpha;\ \frac{1-x}{1-c}\right); \tag{34}$$

$$I_2(s)=\frac{(1-s)^{1-\alpha}}{1+ks-\rho}\cdot\frac{\Gamma(1-2\alpha)\Gamma(1+\alpha)}{\Gamma(2-\alpha)}F\left(1-2\alpha,1,2-\alpha;\ \frac{1-c}{1+ks-\rho}\right); \tag{35}$$

$$I_3(x)=\frac{\Gamma(1+\alpha)\Gamma(1-2\alpha)}{\Gamma(2-\alpha)}\frac{(1-c)^{1-\alpha}}{(1-xc)^{1-2\alpha}}\frac{1}{(1-x)^{2\alpha}}F\left(1-2\alpha,1-\alpha,2-\alpha;\frac{x(1-c)}{1-xc}\right). \tag{36}$$

1. (34) formulasini isbotlaymiz

$$I_1(x)=\lim_{\delta\to 0}\left[-\int_c^x\frac{(t-c)^{\alpha}}{(1-t)^{2\alpha}}\frac{dt}{(x-t)^{1-\delta}}+\int_x^1\frac{(t-c)^{\alpha}}{(1-t)^{2\alpha}}\frac{dt}{(t-x)^{1-\delta}}\right] \tag{37}$$

(37)-ga birinchi va ikkinchi integrallarda $t=c+(x-c)\sigma$ va $t=1-(1-x)\sigma$ integral o'zgaruvchilarini almashtirish orqali

$$I_1(x) = \lim_{\delta\to 0}\left[-\frac{(x-c)^{\alpha+\delta}}{(1-c)^{2\alpha}}\int_0^1 \sigma^{\alpha}(1-\sigma)^{\delta-1}\left(1-\frac{(x-c)}{(1-c)}\sigma\right)^{-2\alpha} d\sigma \right.$$

$$\left. + \frac{(x-c)^{\alpha}}{(1-c)^{2\alpha-\delta}}\int_0^1 \sigma^{-2\alpha}(1-\sigma)^{\delta-1}\left(1-\frac{(1-x)}{(1-c)}\sigma\right)^{\alpha} d\sigma\right]$$

Bu yerda integral ko'rinishda gigipergeometrik funksiyani ishlatish orqali

$$I_1(x) = \lim_{\delta\to 0}\left[-\frac{\Gamma(1+\alpha)\Gamma(\delta)}{\Gamma(1+\alpha+\delta)}\frac{(x-c)^{\alpha+\delta}}{(1-c)^{2\alpha}}F\left(1+\alpha, 2\alpha, 1+\alpha+\delta; \frac{x-c}{1-c}\right) + \frac{\Gamma(1-2\alpha)\Gamma(\delta)}{\Gamma(1-2\alpha+\delta)}\frac{(1-c)^{\alpha}}{(1-x)^{2\alpha-\delta}}F\left(1-2\alpha, -\alpha, 1-2\alpha+\delta; \frac{1-x}{1-c}\right)\right] \quad (38)$$

-ni hosil qilib olamiz.

(26)-dagi avtotransformatsiya formulasini gipergeometrik funksiyaga va (38) tenglikning o'ng qismiga qo'llagan holda

$$I_1(x) = \lim_{\delta\to 0}\left[-\frac{\Gamma(1+\alpha)\Gamma(\delta)}{\Gamma(1+\alpha+\delta)}\frac{(x-c)^{\alpha+\delta}(1-x)^{\delta-2\alpha}}{(1-c)^{\delta}}F\left(\delta, 1-\alpha+\delta, 1+\alpha+\delta; \frac{x-c}{1-c}\right) + \frac{\Gamma(1-2\alpha)\Gamma(\delta)}{\Gamma(1-2\alpha+\delta)}\frac{(x-c)^{\alpha+\delta}}{(1-c)^{\delta}}(1-x)^{\delta-2\alpha}F\left(\delta, 1-\alpha+\delta, 1-2\alpha+\delta; \frac{1-x}{1-c}\right)\right] \quad (39)$$

-ga ega bo'lamiz.

Endi esa (39)-dagi gipergeometrik funksiyaning birinchi qo'shiluvchisiga Bolts formulasini qo'llab

$$I_1(x) = \lim_{\delta\to 0}\left[-\frac{(x-c)^{\delta-\alpha}(1-x)^{\delta-2\alpha}}{(1-c)^{\delta}}F\left(\delta, 1-\alpha+\delta, 1-2\alpha+\delta; \frac{1-x}{1-c}\right) - \left(\frac{\Gamma(2\alpha-\delta)}{\Gamma(2\alpha)} - \frac{\Gamma(1-2\alpha)}{\Gamma(1-2\alpha+\delta)}\right)\Gamma(\delta) - \frac{\Gamma(1+\alpha)\Gamma(\delta-2\alpha)}{\Gamma(1-\alpha+\delta)}\frac{(x-c)^{\alpha+\delta}}{(1-c)^{2\alpha}}F\left(1+\alpha, 2\alpha, 2\alpha-\delta+1; \frac{1-x}{1-c}\right)\right] \quad (40)$$

ko'rinishiga olib kelamiz.

(40)-da limit $\delta \to 0$-ga o'tganda

$$\lim_{\delta\to 0}\left(\frac{\Gamma(2\alpha-\delta)}{\Gamma(2\alpha)} - \frac{\Gamma(1-2\alpha)}{\Gamma(1-2\alpha+\delta)}\right)\frac{\Gamma(1+\delta)}{\delta} = \pi\, ctg 2\alpha\pi$$

-ni hisobga olgan holda (34) formulani olamiz.

2. (35) va (36) formulalar isboti uchun $I_2(s)$ va $I_3(s)$ integral ko'rinishdagi $t = 1-(1-c)\sigma$ va $t = c+(1-c)\sigma$ integral o'zgaruvchilarni almashtirgan holda, keyin esa integral ko'rinishdagi gipergeometrik funksiya ishlatamiz.

Endi esa (34), (35), (36)-dagi $I_1(x)$, $I_2(s)$, $I_3(x)$-larni (33) qo'ygan holda

$$A(x,s) = \frac{1}{x+ks-\rho}\left[-\frac{(x-c)}{(1-x)^{2\alpha}}\pi\, ctg\, 2\alpha\pi - \frac{\Gamma(1+\alpha)\Gamma(-2\alpha)}{(1-c)^{\alpha}\,\Gamma(1-\alpha)}F\left(\alpha, 1, 1+2\alpha; \frac{1-x}{1-c}\right) - \frac{(1-c)^{1-\alpha}\,\Gamma(1-2\alpha)\Gamma(1+\alpha)}{\Gamma(2-\alpha)(1+ks-\rho)}F\left(1-2\alpha, 1, 2-\alpha; \frac{1-c}{1+ks-\rho}\right)\right] - \frac{1}{1+kxs-\rho x}(\alpha I_3(x) - I_2(s)). \quad (41)$$

-ga ega bo'lamiz.

Endi esa (41)-ni (32) hisobga olgan holda (29)-ga qo'yamiz va

$$\tau(x)=\frac{\lambda k^{1-4\alpha}}{2}\int_c^1\left(\frac{x-c}{t-c}\right)^{4\alpha}\frac{\tau(t)dt}{(t-c)(k+(x-c)(t-c))}+R_2(\tau)+F_2(x),\quad x\in(c,1). \tag{42}$$

hosil qilib olamiz, bu yerda

$$R_2[\tau]=R_1[\tau]+\frac{\lambda\sin 2\alpha\,\pi\Gamma(1+\alpha)}{2\pi(1-c)^{\alpha}}(1-x)^{2\alpha}(x-c)^{3\alpha}\cdot\int_c^1\frac{\tau(t)q'(t)}{(c-q(t))^{4\alpha}(x-q(t))}$$

$$\left\{\frac{\Gamma(-2\alpha)}{\Gamma(1-\alpha)}F\left(\alpha,1,1+2\alpha;\frac{1-x}{1-c}\right)+\frac{(1-c)\Gamma(1-2\alpha)}{\Gamma(2-\alpha)(1+kt-\rho)}F\left(1-2\alpha,1,2-\alpha;\frac{1-c}{1+kt-\rho}\right)\right\}dt+$$

$$+\frac{\lambda\sin 2\alpha\pi}{2\pi}\int_c^1\frac{\tau(t)q'(t)}{(c-q(t))^{4\alpha}}\frac{xI_3(x)-I_2(t)}{1+kxt-\rho x}dt.$$

(42) tenglamada $t=c+(1-c)e^{-s}$, $x=c+(1-c)e^{-y}$ o'zgartiruvchilarni o'zgartirib va

$$\rho(y)=\tau[c+(1-c)e^{-y}]e^{4\alpha-\frac{1}{2}y}$$

-ni kiritish orqali (42) tenglamani

$$\rho(y)=\frac{\lambda k^{1-4\alpha}}{2}\int_0^{\infty}K(y-s)\rho(s)ds=R_3[\rho]+F_3(y) \tag{43}$$

ko'rinishiga keltirib olamiz, bu yerda

$$K(x)\equiv\frac{1}{kex+e^{x}},\quad F_3(y)=e^{\left(4\alpha-\frac{1}{2}\right)y}F_2[c+(1-c)e^{-y}],\quad R_3(\rho)=e^{\left(4\alpha-\frac{1}{2}\right)y}R_2[\tau]$$

doimiy operator.

(43) tenglama Viner-Xopf-ning integral tenglamasi hisoblanib, ushbu tenglama yordamida, Furyening xarakter ko'rinishiga almashtirib Koshining yadro integrali tenglamasi Rimanning chetki masalasiga keltiriladi va shu orqali kvadraturalar ishlanadi.

Fredgolm teoremasi integral tenglamalari to'plami faqat bitta holda, qachonki ushbu tenglamalar indekslari nolga teng bo'lganda o'rinli bo'ladi.

(43) tenglama indeksi

$$1-\frac{\lambda k^{1-4\alpha}}{2}K^{\wedge}(x), \tag{44}$$

-dagi ifoda indeksi hisoblanadi, teskari belgi bilan esa

$$K^{\wedge}(x)=\int_{-\infty}^{\infty}\frac{e^{ixt}dt}{ke^{t/2}+e^{-t/2}}\ . \tag{45}$$

Endi esa (44) ifodaning indeksi hisoblaymiz, $\mathrm{Re}\left(1-\lambda k^{1-\alpha}\cos\alpha\pi K^{\wedge}(x)\right)>0$ o'rinli bo'lganda

$$\mathrm{Re}\frac{\lambda k^{1-4\alpha}}{2}K^{\wedge}(x)=\frac{\lambda k^{1-4\alpha}}{2}\cdot\frac{\pi\cos(x\ln k)}{\sqrt{k}\ ch\pi x}\le\frac{\lambda k^{1-4\alpha}}{2}\cdot\frac{\pi}{\sqrt{k}}\le\cdot\frac{\lambda\pi k^{2\beta-\frac{1}{2}}}{2}<1$$

bo'ladi.

Bundan kelib chiqadiki, $Ind\left(1-\lambda k^{1-\alpha}\cos\alpha\pi K^{\wedge}(x)\right)=0$ ya'ni argument to'liq aylanada nolga teng, bu yerdan va GF masalasining yagona yechimga egaligidan (43) ning yechimi mavjudligi kelib chiqadi.

Ushbu masalalar o'zbek va xorijlik olimlar tomonidan o'rganilgan bo'lib, nashr qilingan maqolada juda qisqa ma'lumotlar berilgan. Ushbu maqolada deyarli barcha qiyin hisoblashlar kengaytirilgan va izohlar berilgan holda tishuntirishga harakat qilindi. Natijada kelgusida ilmiy qilish maqsadida shu soha bilan shug'ullanmoqchi bo'lgan o'quvchilarga maqolani o'rganish osonlashtirildi. Shu kabi masalalar [1-16] larda ham chuqur o'rganilgan.

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