SINGULYAR INTEGRAL TENGLAMALAR VA UNI QO'LLANILISHI HAQIDA

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Annotatsiya. Ushbu maqolada buzilish chizig'iga ega aralash tipdagi tenglamalarning amaliy masalalarga tatbiqi o'rganilgan. Uni giperbolik tipdagi tenglamalarni yechishda qo'llanilishi bo'yicha olib borilgan ilmiy maqola tahlil qilingan. O'quvchilarga tushunarli bo'lishi uchun ilmiy maqola kengaytirib yoritilgan.

Kalit so'zlar: singulyar integral tenglama, Karleman tipli, Karleman va nokarleman siljishli tenglamalar, Gel'der sharti, Fredgol'm tenglamasi, kompleks tekisligi, integro-funksional tenglama.

ON SINGULAR INTEGRAL EQUATIONS
AND ITS APPLICATIONS

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Annotation. In this article, the application of mixed-type equations with distortion lines to practical problems is studied. A scientific article on its use in solving hyperbolic equations is analyzed. The scientific article has been expanded to make it understandable to readers.

Keywords: singular integral Karleman type, Karleman and non-Carlemann displacement equations, Gelder condition, Fredholm equation, complex plane, integro-functional equation.

$\Gamma(\beta_0 = 1, \alpha_0 < 0)$ masalasining qo'yilishi hamda yechimi mavjudligi va yagonaligi o'rganilgan. $P(\alpha_0, \beta_0) \in C_0 E_0$, bo'lsin, ya'ni $\alpha_0 = 0,\ -m/2 < \beta_0 < 1,\ \alpha = \beta$.

Ushbu holda $\Gamma(\beta_0 < 1, \alpha_0 \le 0)$ masalasida

$$v(x) = \int_{-1}^{x} \frac{K_1(x,t)v(t)dt}{|t-\varphi(x)|^l} + F_1(x)$$

bo'lib, integral tenglama quyidagi ko'rinishga ega bo'ladi:

$$v(x) = \int_{-1}^{x} \frac{H(x,t)v(t)dt}{t-\varphi(x)} + F(x),\ \ x \in I, \tag{1}$$

bunda

$$H(x,t)=\begin{cases}-K_1(x,t) & agar\ \ -1\le t\le\varphi(x),\\ K_1(x,t) & agar\ \ \varphi(x)\le t\le x,\end{cases}$$

$$\varphi(x)=\varphi_1(x),\ F(x)=F_1(x).$$

$\Gamma(\beta_0<1,\alpha_0\le 0)$ masalani R_1 sinfda izlaymiz. Faraz qilamiz, $\tau(x),\ \rho(x),\ \mu(x)\in C^1(\bar{I})$ va

$$\mu(x)=(1-x^2)\mu^*(x), \tag{2}$$

bo'lsin, $\mu^*(x)\in C^1(\bar{I})$.

$\varphi(x)<x$ bo'lganligi sababli (1) nokarleman $\varphi(x)$ siljishli singulyar integral tenglama bo'ladi. Qayd qilamiz, $\varphi(x)$ siljish k tartibli Karleman tipli siljish deyiladi, agar $k\ge 2:\underbrace{\varphi(\varphi...(\varphi(x))...)}_{k\ marta}=x$ bo'lsa. Masalan, $\varphi(x)=(x-c)/(xc-1)$ ikkinchi tartibli Karleman tipidagi siljish bo'ladi.

Karleman va nokarleman siljishli tenglamalar uchun Neter nazariya yetarlicha chuqur rivojlangan. (1) tenglamada $\varphi(x)$ siljish integrallash oralig'ini o'ziga akslantiradi va yagona $x=-1$ qo'zg'almas nuqtaga ega.

Teorema. 1) $\varphi(x)$- funksiya $\bar{I}$ da monoton o'suvchi va uzluksiz differensiallanuvchi va $\varphi'(x)<q_0<1,\ \varphi(-1)=-1$ va $\varphi(x)<x$, $\forall x\in\bar{I}\setminus\{-1\}$ bo'lsin;

2) $H(x,t)$ funksiya quyidagi ko'rinishga ega bo'lsin

$$H(x,t)=(1-x^2)H^*(x,t), \tag{3}$$

bunda $H^*(x,t)$ ikkala o'zgaruvchi (x,t) bo'yicha ham Gelder shartni qanoatlantiradi;

3) $F(x)$ funksiya I da Gelder shartini qanoatlantiradi, $x=1$ da uzluksiz, $x=-1$ da integrallanadigan $\lambda<1-\beta$ tartili maxsuslikka ega.

U holda $-(-y)^m u_{xx}+u_{yy}=0$ tenglama $v(x)$ noma'lum funksiyaga nisbatan ikkinchi tur Fredgolm tenglamasiga keltiriladi.

Isbot. $0<\gamma^{m/2}(x)\cdot\gamma'(x)\le 1$ ga asosan $\varphi(x)=2\delta(x)-x$ funksiya $\bar{I}$ da monoton o'suvchi va

$$\max_{x\in\bar{I}}\varphi(x)=\varphi(1)=2\delta(1)-1=2x_1-1.$$

Demak, (1) singulyar integral tenglamani yadrosi faqat $t\in[-1,2x_1-1]$ da maxsuslikka ega, $t\in(2x_1-1,1]$ da esa uzluksiz. Yuqoridagilarni inobatga olib, (1) tenglamani o’zgartiramiz

$$v(x)=H(x,\varphi(x))\int_{-1}^{p}\frac{v(t)dt}{t-\varphi(x)}+F(x), \qquad (4)$$

bunda $p=2x_1-1$,

$$F_0(x)=\int_{p}^{x}\frac{H(x,t)v(t)dt}{t-\varphi(x)}+\int_{-1}^{p}\frac{H(x,t)-H(x,\varphi(x))}{t-\varphi(x)}v(t)dt+F(x), \qquad (5)$$

$$H(x,\varphi(x))=\frac{\mu(x)T^{(\beta-1)/\beta}(x,\varphi(x))}{\Gamma(\beta)\Gamma(1-\beta)}\left(\frac{1+x}{x-\varphi(x)}\right)^{\beta}. \qquad (6)$$

(4) tenglamani

$$\int_{-1}^{p}\frac{v(t)dt}{t-\varphi(x)}=f_0(x),\ \ x\in I, \qquad (7)$$

ko’rinishda yozib olamiz, bunda

$$f_0(x)=\frac{v(x)-F_0(x)}{H(x,\varphi(x))},\ \ x\in I. \qquad (8)$$

(7) da $t=\varphi(s)$, $s=-1$, $t=1$; $s=1$, $t=p$, o’zgaruvchilarni almashtirish qilib

$$\int_{-1}^{1}\frac{v(\varphi(s))\varphi'(s)ds}{\varphi(s)-\varphi(x)}=f_0(x) \qquad (9)$$

ga ega bo’lamiz.

$$D(x,s)=\frac{\varphi'(s)}{\varphi(s)-\varphi(x)}-\frac{1}{s-x} \qquad (10)$$

funksiyani kiritib olamiz.

$\varphi(x)\in C^2(\bar{I})$ bo’lgani uchun $D(x,s)\in C(\bar{I}\times\bar{I})$ ekanligini aniqlash qiyin emas. (10) ga asosan (9) tenglamani

$$\int_{-1}^{1}\frac{v(\varphi(t))dt}{t-x}=f(x), \qquad (11)$$

ko’rinishga keltirib olamiz, bunda

$$f(x)=f_0(x)-\int_{-1}^{1}v(\varphi(t))D(x,t)dt. \qquad (12)$$

(4) integral tenglama uchun teskarilanish formulasini topamiz. Buning uchun z kompleks tekisligida $\bar{I}=[-1,1]$ kesmadan tashqarida

$$\Phi(z)=\frac{1}{2\pi i}\int_{-1}^{1}\frac{v(\varphi(t))dt}{t-z},\ \ \Phi(\infty)=0$$

analitik funksiyani qaraymiz.

Bu funksiya uchun

$$\Phi^{+}(x)=\frac{1}{2}v(\varphi(x))+\Phi_0(x),$$

$$\Phi^{-}(x)=-\frac{1}{2}v(\varphi(x))+\Phi_0(x),\ \ x\in\overline{I} \tag{13}$$

Soxotskogo-Plemelya formulasi o'rinli, bunda

$$\Phi_0(x)=\frac{1}{2\pi i}\int_{-1}^{1}\frac{v(\varphi(t))dt}{t-x}.$$

(13) ga asosan (11) tenglama

$$\Phi^{+}(x)=-\Phi^{-}(x)-i\frac{f(x)}{\pi},\ \ x\in\overline{I}. \tag{14}$$

ko'rinishga ega bo'ladi.

$$G(x)=\begin{cases}-1 \ \ agar \ \ z\in\overline{I},\\ 1 \ \ agar \ \ z\overline{\in}\overline{I};\end{cases}$$

$$f_1(x)=\begin{cases}-i\dfrac{f(x)}{\pi} \ \ agar \ x\in\overline{I},\\ 0 \ \ \ \ agar \ \ x\overline{\in}\overline{I}.\end{cases}$$

funksiyalarni kiritamiz. Shunday qilib, (14) tenglamani

$$\Phi^{+}(x)=G(x)\Phi^{-}(x)+f_1(x),\ \ x\in(-\infty,+\infty). \tag{15}$$

ko'rinishda yozish mumkin.

Shunday qilib, izlanayotgan masala $[-1,1]$ kesmada (14) shartni qanoatlantiradigan cheksizlikda yo'qolib ketadigan va $[-1,1]$ da analitik bo'lgan $\Phi(z)$ funksiyani izlashga keltirildi.

Ushbu masalaning yechimi

$$\Phi(z)=\frac{X(z)}{2\pi i}\int_{-1}^{1}\frac{f_1(t)dt}{X^{+}(t)(t-z)}=-\frac{X(z)}{2\pi^2}\int_{-1}^{1}\frac{f(t)dt}{X^{+}(t)(t-z)}, \tag{16}$$

ko'rinishda bo'ladi, bunda $X(z)=\sqrt{(z-1)/(z+1)}$.

(16) formuladan oson topishimiz mumkin, ya'ni

$$\Phi^{+}(x)-\Phi^{-}(x)=-\frac{X^{+}(x)}{\pi^2}\int_{-1}^{1}\frac{f(t)dt}{X^{+}(t)(t-x)},$$

bo'ladi, lekin (13) ga asosan

$$\Phi^{+}(x)-\Phi^{-}(x)=v(\varphi(x))$$

ga ega bo'lamiz. Bundan, oxirgi ikki tengliklarning o'ng tomonlarini tenglashtirib

$$v(\varphi(x)) = -\frac{X^+(x)}{\pi^2}\int_{-1}^{1}\frac{f(t)dt}{X^+(t)(t-x)} \tag{17}$$

topamiz.

(17) da x ni $\varphi(x)$ ga almashtirib

$$v(\varphi(\varphi(x))) = -\frac{X^+(x)}{\pi^2}\int_{-1}^{1}\frac{f(t)dt}{X^+(t)(t-\varphi(x))} \tag{18}$$

bo'lishini aniqlaymiz.

(12), (8) larga asosan (18) munosabatni

$$v(\varphi(\varphi(x))) = -\frac{1}{\pi^2 H(\varphi(x),\varphi(\varphi(x)))}\int_{-1}^{p}\frac{v(t)dt}{t-\varphi(x)} -$$

$$-\frac{X^+(\varphi(x))}{\pi^2}\int_{-1}^{p}\frac{v(t)dt}{t-\varphi(x)}\left\{\frac{1}{X^+(t)H(t,\varphi(t))}-\frac{1}{X^+(\varphi(x))H(\varphi(x),\varphi(\varphi(x)))}\right\}dt -$$

$$-\frac{X^+(\varphi(x))}{\pi^2}\int_{p}^{1}\frac{v(t)dt}{X^+(t)H(t,\varphi(t))(t-\varphi(x))}+\frac{X^+(\varphi(x))}{\pi^2}\int_{-1}^{1}\frac{F_0(t)dt}{X^+(t)H(t,\varphi(t))(t-\varphi(x))}+$$

$$+\frac{X^+(\varphi(x))}{\pi^2}\int_{1}^{p}v(t)\left(\varphi^*(t)\right)_t' dt\int_{-1}^{1}\frac{D(\varphi^*(t),s)ds}{X^+(s)(s-\varphi(x))}, \tag{19}$$

ko'rinishga keltiramiz, bunda $\varphi^*(x)$ - funksiya $\varphi(x)$ ga nisbatan teskari funksiya.

Endi (8) ni inobatga olib, (19) munosabatni birinchi qo'shiluvchisini

$$v(x) = a(x)v(\psi(x)) + \int_{1}^{p}H_0(x,t)v(t)dt + \int_{p}^{1}M_0(x,t)v(t)dt + \int_{-1}^{p}L_0(x,t)v(t)dt +$$

$$+F_0(x) - \int_{-1}^{1}M_0(x,t)F_0(t)dt, \tag{20}$$

o'zgartirib olamiz, bunda $\psi(x) = \varphi(\varphi(x))$,

$$a(x) = -\pi^2 H(x,\varphi(x))H(\varphi(x),\psi(x)); \tag{21}$$

$$b(x) = -X^+(\varphi(x))H(\varphi(x),\psi(x));$$

$$H_0(x,t) = \frac{b(x)}{t-\varphi(x)}\left(\frac{1}{X^+(t)H(t,\varphi(t))}-\frac{1}{X^+(\varphi(x))H(\varphi(x),\psi(x))}\right);$$

$$L_0(x,t) = -b(x)\left(\varphi^*(t)\right)_t'\int_{-1}^{1}\frac{D(\varphi^*(t),s)ds}{X^+(s)(s-\varphi(x))};$$

$$M_0(x,t) = -\frac{b(x)}{X^+(t)H(t,\varphi(t))(t-\varphi(x))}.$$

ko'rinishga keltiramiz. (5) ga asosan

$$F_0(x) - \int_{-1}^{1} M_0(x,t)F_0(t)dt = \int_{-1}^{p} G_1(x,t)v(t)dt + \int_{p}^{1} N_1(x,t)v(t)dt - \int_{x}^{p} \frac{H(x,t)v(t)dt}{t-\varphi(x)} + f(x), \quad (22)$$

bo'ladi, bunda

$$G_1(x,t) = \frac{H(x,t)-H(x,\varphi(x))}{t-\varphi(x)} - \int_{-1}^{1} \frac{M_0(x,s)\left(H(s,t)-H(s,\varphi(s))\right)ds}{t-\varphi(s)} - \int_{-1}^{t} \frac{M_0(x,s)H(s,t)ds}{t-\varphi(s)},$$

$$N_1(x,t) = -\int_{t}^{1} \frac{M_0(x,s)H(s,t)ds}{t-\varphi(s)}, \quad f(x) = F(x) - \int_{-1}^{1} M_0(x,t)F(t)dt$$

(22) ni inobatga olib, (20) dan $v_1(x)$ ga nisbatan quyidagi integro-funksional tenglamani olamiz:

$$v_1(x) = a_1(x)v_1(\psi(x)) + \int_{-1}^{1} C_1(x,t)v_1(t)dt - \int_{x}^{1} \frac{H_1(x,t)v_1(t)dt}{t-\varphi(x)} + f_1(x), \quad x \in I, \quad (23)$$

bunda

$$v_1(x) = (1+x)^{\lambda} v(x), \quad a_1(x) = \left(\frac{1+x}{1+\psi(x)}\right)^{\lambda} a(x), \quad f_1(x) = (1+x)^{\lambda} f(x); \quad (24)$$

$$C_1(x,t) = \left(\frac{1+x}{1+t}\right)^{\lambda} C(x,t), \quad H_1(x,t) = \left(\frac{1+x}{1+t}\right)^{\lambda} H(x,t); \quad (25)$$

$$C(x,t) = \begin{cases} H_0(x,t) + L_0(x,t) + G_1(x,t), & -1 \le t < p, \\ M_0(x,t) + N_1(x,t) + \dfrac{H(x,t)}{t-\varphi(x)}, & p < t \le 1. \end{cases} \quad (26)$$

(23) tenglamani yechimini I da Gelder shartini qanoatlantiruvchi va chetki nuqtalarida esa uzluksiz sinfdan qidiramiz.

(23) tenglamaga iteratsiyalar usulini qo'llab, n-iteratsiya uchun

$$v_1(x) = A_n(x)v_1(\psi_n(x)) + \sum_{k=0}^{n-1} A_k(x)\int_{-1}^{1} C_1(\psi_k(x),t)v_1(t)dt -$$

$$-\sum_{k=0}^{n-1} A_k(x) \int_{\psi_k(x)}^{1} \frac{H_1(\psi_k(x),t)v_1(t)dt}{t-\varphi(\psi_k(x))} + \sum_{k=0}^{n-1} A_k(x) f_1(\psi_k(x)) \quad (27)$$

ga ega bo'lamiz, bunda

$$A_k(x) = a_1(x)a_1(\psi_1(x))\dots a_1(\psi_{k-1}(x)), \quad A_0(x) = 1, \quad (28)$$

$$\psi_n(x) = \psi(\psi_{n-1}(x)), \quad \psi_0(x) = x, \quad k, n \in N. \quad (29)$$

$\psi(x) < x$, $\psi(-1) = -1$ bo'lgani uchun $\psi_n(x) = \psi(\psi_{n-1}(x)) < \psi_{n-1}(x)$.

Shunday qilib, $\{\psi_n(x)\}$ funksional ketma-ketlik monoton kamayuvchi va quyidan tekis chegaralangan, ya'ni $\psi_n(x) \geq -1$. $\lim_{n\to\infty} \psi_n(x) = \psi^0(x)$ bo'lsin. $\psi_n(x) = \psi(\psi_{n-1}(x))$ tenglikka $n \to \infty$ da limitga o'tib $\psi^0(x) = \psi(\psi^0(x))$ bo'lishini topamiz, $\psi^0(x) = -1$, chunki $\psi(x)$ yagona qo'zg'almas nuqtaga ega $x = -1$.

Shunday qilib,

$$\lim_{n\to\infty} \psi_n(x) = -1, \quad \forall x \in \bar{I}. \tag{30}$$

(3), (21) va (24) lardan osongina $a_1(-1) = 0$ ekanligini topamiz. Binobarin, (30) inobatga olib, $\forall\, const = p \in (0,1)$ lar uchun shunday $k_0(p_0) \in N$ mavjud bo'ladiki, $n > k_0$ bo'lganda

$$|a_1(\psi_n(x))| < p_0, \ \forall\, x \in I \tag{31}$$

bo'ladi.

(28) ga asosan

$$|A_n(x)| \leq a_0^{k_0} p_0^{n-k_0}, \tag{32}$$

bo'ladi, bunda $a_0 = \max_{x\in\bar{I}} a_1(x)$. Endi, (27) ga $n \to \infty$ da limitga o'tamiz va (32) ga asosan

$$v_1(x) = \int_{-1}^{1} L(x,t) v_1(t) dt + F(x), \ x \in I, \tag{33}$$

ekanligini topamiz, bu yerda

$$L_1(x,t) = K(x,t) - H(x,t), \tag{34}$$

$$K(x,t) = \sum_{k=0}^{\infty} A_k(x) C_1(\psi_k(x), t), \tag{35}$$

$$H = \begin{cases} \sum_{k=0}^{\infty} S_k(x,t), & x < t \leq 1; \\ \sum_{k=1}^{\infty} S_k(x,t), & \psi_1(x) < t \leq x; \\ \sum_{k=2}^{\infty} S_k(x,t), & \psi_2(x) < t \leq \psi_1(x); \\ \dots \ \dots \ \dots \ \dots \ \dots; \\ \sum_{k=n}^{\infty} S_k(x,t), & \psi_n(x) < t \leq \psi_{n-1}(x); \end{cases} \tag{36}$$

$$S_k(x,t) = A_k(x) H_1(\psi_k(x), t) / \left(t - \varphi(\psi_k(x))\right).$$

$$F(x)=\sum_{k=0}^{\infty}A_k(x)\,f_1(\psi_k(x)). \quad (37)$$

Barcha (35) – (37) funksional qatorlar (32) ga asosan $C^*\sum_{k=0}^{\infty}p_0^k$ qator bilan moslanadi va tekis yaqinlashadi, bunda C^* – o'zgarmas son.

$$F(x)=\sum_{k=0}^{\infty}A_k(x)\,f_1(\psi_k(x))\in H_\mu, \quad (38)$$

bo'lishini ko'rsatamiz. Buning uchun (38) funksional qatorni H_μ fazoda yaqinlashuvchi ekanligini isbotlaymiz.

Xususiy yig'indini qaraymiz: $F_n(x)=\sum_{k=0}^{n-1}A_k(x)\,f_1(\psi_k(x))$, bunda

$$\left\|F_n(x)\right\|_{H_\mu}=\max_{x\in\bar{I}}\left|F_n(x)\right|+\sup_{x_1,x_2\in\bar{I}}\frac{\left|F_n(x_2)-F_n(x_1)\right|}{\left|x_2-x_1\right|^\mu}.$$

$$\sup\frac{\left|F_n(x_2)-F_n(x_1)\right|}{\left|x_2-x_1\right|^\mu}=\sup\frac{\left|\sum_{k=0}^{n-1}A_k(x_2)\,f_1(\psi_k(x_2))-A_k(x_1)\,f_1(\psi_k(x_1))\right|}{\left|x_2-x_1\right|^\mu}.$$

Bundan

$$A_k(x_2)f_1(\psi_k(x_2))-A_k(x_1)f_1(\psi_k(x_1))=A_k(x_2)f_1(\psi_k(x_2))-A_k(x_1)f_1(\psi_k(x_2))+A_k(x_1)f_1(\psi_k(x_2))-$$
$$-A_k(x_1)f_1(\psi_k(x_1))=f_1(\psi_k(x_2))\left[A_k(x_2)-A_k(x_1)\right]+A_k(x_1)\left[f_1(\psi_k(x_2))-f_1(\psi_k(x_1))\right]$$

ayirmani aniqlaymiz.

$A_k(x_2)-A_k(x_1)$ ayirmani (qulaylik uchun (28) ga $a_1(x)=a(x)$ deb belgilaymiz):

$$A_k(x_2)-A_k(x_1)=\left[a(x_2)-a(x_1)\right]a(\psi_1(x_2))a(\psi_2(x_2))a(\psi_3(x_2))\ldots a(\psi_{k-1}(x_2))+a(x_1)\times$$
$$\times\left[a(\psi_1(x_2))-a(\psi_1(x_1))\right]\cdot a(\psi_2(x_2))a(\psi_3(x_2))\ldots a(\psi_{k-1}(x_2))+a(x_1)a(\psi_1(x_1))\left[a(\psi_2(x_2))-\right.$$
$$\left.-a(\psi_2(x_1))\right]\cdot a(\psi_3(x_2))\ldots a(\psi_{k-1}(x_2))+a(x_1)a(\psi_1(x_1))\cdot a(\psi_2(x_1))\cdot\left[a(\psi_3(x_2))-a(\psi_3(x_1))\right]\times$$
$$\times a(\psi_4(x_2))\ldots a(\psi_{k-1}(x_2))+\ldots+a(x_1)a(\psi_1(x_1))a(\psi_2(x_1))a(\psi_3(x_1))\times\ldots\times$$
$$\times a(\psi_{k-2}(x_1))\left[a(\psi_{k-1}(x_2))-a(\psi_{k-1}(x_1))\right]$$

ko'rinishda yozib olamiz.

$a_1(x)=a(x)$ va $\psi_k(x)$ uzluksiz differensiallanuvchi. Ko'rish qiyin emaski,

$$a(x_1)a(\psi_1(x_1))a(\psi_2(x_1))\ldots a(\psi_{m-1}(x_1))\left[a(\psi_m(x_2))-a(\psi_m(x_1))\right]\times$$
$$\times a(\psi_{m+2}(x_2))\ldots a(\psi_{k-1}(x_2))\le a_0^{k_0}p_0^{k-k_0}a(\psi_m(x_2))-a(\psi_m(x_1))\le$$
$$\le a_0^{k_0}p_0^{k-k_0}c^*\left[\psi_m(x_2)-\psi_m(x_1)\right]$$

bo'ladi. Bunda c^* - Lipshits konstantasi. $\psi(x) \le q_1 < 1$ ga ko'ra q_1 - o'zgarmas son, demak,

$$|\psi_m(x_2)-\psi_m(x_1)| = |\psi(\psi_{m-1}(x_2))-\psi(\psi_{m-1}(x_1))| < q_1|\psi_{m-1}(x_2)-\psi_{m-1}(x_1)| <$$
$$< q_1^2|\psi_{m-2}(x_2)-\psi_{m-2}(x_1)| < \ldots < q_1^m|x_2-x_1|.$$

Shunday qilib,

$$a(x_1)a(\psi_1(x_1))a(\psi_2(x_1))\ldots a(\psi_{m-1}(x_1))\left[a(\psi_m(x_2))-a(\psi_m(x_1))\right]\times$$
$$\times a(\psi_{m+1}(x_2))\ldots a(\psi_{k-1}(x_2)) \le a_0^{k_0} p_0^{k-k_0} q_1^m |x_2-x_1|.$$

U holda

$$|A_k(x_2)-A_k(x_1)| \le a_0^{k_0} p_0^{k-k_0} c^*\left(1+q_1+q_1^2+q_1^3+\ldots+q_1^{k-1}\right)|x_2-x_1| = a_0^{k_0} p_0^{k-k_0} c^* \frac{1-q_1^k}{1-q_1}|x_2-x_1|$$

bo'ladi. Binobarin,

$$\sum_{k=0}^{n-1}\frac{|A_k(x_2)f_1(\psi_k(x_1))-A_k(x_1)f_1(\psi_k(x_1))|}{|x_2-x_1|^\mu} \le \sum_{k=0}^{n-1}\frac{|f_1(\psi_k(x_2))|\cdot|A_k(x_2)-A_k(x_1)|}{|x_2-x_1|^\mu}+$$
$$+\sum_{k=0}^{n-1}\frac{|A_k(x_1)|\cdot|f_1(\psi_k(x_2))-f_1(\psi_k(x_1))|}{|x_2-x_1|^\mu} \le \left(\frac{a_0}{p_0}\right)^{k_0} M_0C^*\sum_{k=0}^{n-1}p_0^k\frac{1-q_1^k}{1-q_1}|x_2-x_1|^{1-\mu}+$$
$$+C_0^*\sum_{k=0}^{n-1}a_0^{k_0}p_0^{k-k_0}q_1^k|x_2-x_1|^{1-\mu} = \left[\left(\frac{a_0}{p_0}\right)^{k_0} M_0C^*\sum_{k=0}^{n-1}p_0^k\frac{1-q_1^k}{1-q_1}+\left(\frac{a_0}{p_0}\right)^{k_0} C_0^*\sum_{k=0}^{n-1}(p_0q_1)^k\right]\cdot|x_2-x_1|^{1-\mu},$$

bunda $M_0=\max\limits_{x\in\bar{I}}|f_1(x)|$, C^*, C_0^* - o'zgarmas sonlar.

Demak, $F(x)=\sum\limits_{k=0}^{\infty}A_k(x)f_1(\psi_k(x))$ qator H_μ da yaqinlashuvchi va bundan $F(x)\in H$ ekanligi kelib chiqadi.

$L(x,t)\in H$ ekanligi ham shunga o'xshash isbotlanadi.

Yuqorida tahlil qilingan ushbu masala bir qator o'zbek va xorijlik olimlar tomonidan o'rganilgan. Ushbu maqolada muallif tomonidan hisoblash ishlarini tahlil qilishga doir ma'lumotlar yoritilgan. Xususan ularni keltirib chiqarish juda murakkabligi uchun maqolada hisoblashlar kengaytirilgan va izohlar berilgan holda o'quvchilarfa tushuntirishga harakat qilingan. Kelgusida ilmiy bilan shug'ullanish maqsadida shu yo'nalish bo'yicha izlanmoqchi bo'lgan o'quvchilarga maqolani o'rganishlari osonlashtirildi. Sohaga doir ilmiy izlanishlar [1-15] larda ham chuqur o'rganilgan.

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