OLMOS PANJARADAGI DISKRET SHRYODINGER OPERATORINING SPEKTRI

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Annotatsiya. Ushbu maqolada olmos panjaradagi sistemada aniqlangan diskret Shryodinger operatori qaralgan. Olmos panjaradagi sistemada aniqlangan diskret Shryodinger operatori tavsivlangan hamda bu operatorning impuls tasviri olingan va muhim spektri oʻrganilgan. Ma'lumki, olmos panjaradagi ikki zarrachali Shryodinger operatorlarini o ʻrganish ochiq masala hisoblanadi. Mazkur maqolada oʻrganilayotgan operator olmos panjaradagi ikki zarrachali Shryodinger operatorini oʻrganishda muhim ahamiyat kasb etadi.

Kalit soʻzlar: diskret Shryodinger operatori, kompakt operator, muhim spektr, xos qiymat, xos funksiya.

SPECTRUM OF THE DISCRETE SCHROEDINGER OPERATOR ON A DIAMOND LATTICE

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Annotation. This paper considers the discrete Schrödinger operator defined in a system on a diamond lattice. The discrete Schrödinger operator defined in the diamond lattice system is described, and the pulse image of this operator is obtained and its essential spectrum is studied. It is known that the study of two-particle Schrödinger operators in a diamond lattice is an open problem. The operator studied in this article is of great importance in the study of the two-particle Schrödinger operator in the diamond lattice.

Keywords: Discrete Schrödinger operator, compact operator, critical spectrum, eigenvalue, eigenfunction.

$\mathbb{C}$– bir oʻlchamli kompleks sonlar fazosi boʻlsin. Ixtiyoriy $n \in N$ natural soni uchun $L_2[a,b]^n$ orqali $[a,b]^n$ da aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatli) funksiyalarning Hilbert fazosini belgilaymiz. $\mathbb{T}^d$ bilan d oʻlchamli tor, ya'ni

$$\mathbb{T}^d = \underbrace{\mathbb{T}^1 \times \mathbb{T}^1 \times \dots \times \mathbb{T}^1}_{dmarta}$$

ni belgilaymiz. d o‘lchamli tor $\mathbb{T}^d$ da aniqlangan, Haar ma’nosida o‘lchovga ega va

$$\int_{\mathbb{T}^d} |f(x)|^p dx < \infty$$

shartni qanoatlantiruvchi barcha $f: \mathbb{T}^d \to \mathbb{C}$ funksiyalarning chiziqli fazosini qaraymiz, bunda integralda o‘lchov Haar ma’nosida olinadi va p tayinlangan musbat son. Elementlarni qo‘shish va songa ko‘paytirish odatdagi funksiyalarni qo‘shish va songa ko‘paytirish kabi kiritiladi. Hosil bo‘lgan fazo $L_p(\mathbb{T}^d)$ kabi belgilanadi. Demak, $L_p(\mathbb{T}^d)$ fazoning elementlari $\mathbb{T}^d$ da aniqlangan va har bir o‘zgaruvchisi bo‘yicha 2π davrga ega bo‘lgan funksiyalardir [1-6].

Ushbu maqolada $L_2^{(2)}(\mathbb{T}^2)$ –Hilbert fazosida $H = Q + H_0$ ko‘rinishda aniqlangan Olmos panjaradagi diskret Shredinger operatori $H = H_0 + Q$ ning muhim spektrini o‘rganamiz.

Quyidagi toʻplamni kiritamiz:

$$A = \{v(n): \quad v(n) = n_1 v_1 + n_2 v_2 \quad n = (n_1\,; n_2), \ \ n \epsilon \mathbb{Z}^2\},$$

bu yerda

$$v_1 = e_3 - e_1 = (-1\,; 0; 1)\ , \quad v_2 = e_3 - e_1 = (0\,; -1\,; 1)\,.$$

Ta’rif. *A to‘plamga ikki o‘lchamli olmos panjara deyiladi.*

Quyidagi toʻplamni kiritamiz:

$$\Omega = A \cup (p + A)\,, \quad p = \frac{1}{3}(v_2 - v_1) = \frac{1}{3}(\,-1\,;\ -1; 2).$$

$\ell_2(\Omega)$ - orqali Ω da kvadrati bilan jamlanuvchi $\hat{f}(n) = (\hat{f}_1(n), \hat{f}_2(n))$ funksiyalar juftligini belgilaymiz. Bu fazo Hilbert fazosi bo‘lib, skalyar ko‘paytma quyidagicha aniqlangan:

$$(\hat{f}, \hat{g}) = \sum_{v \epsilon A} 3\hat{f}_1(n)\hat{g}_1(n) + \sum_{v \epsilon (p+A)} 3\hat{f}_2(n)\hat{g}_2(n).$$

$L_2^{(2)}(\mathbb{T}^2) - \mathbb{T}^2$ da aniqlangan kvadrati bilan integrallanuvchi $f(x) = (f_1(x), f_2(x))$ funksiyalar juftligining Hilbert fazosi bo‘lsin. Bu yerda skalyar ko‘paytma quyidagicha aniqlangan

$$(f, g) = (f_1, g_1) + (f_2, g_2)$$

Bunda

$$(f_i, g_i) = \int_{\mathbb{T}^2} f_i(x)\overline{g_i(x)}\, dx, \quad i = 1{,}2.$$

Quyidagi

$$\mathcal{F} : \ell_2(\Omega) \to L_2^{(2)}(\mathbb{T}^2)$$

unitar operatorni kiritamiz:

$$\mathcal{F} = \begin{pmatrix} \mathcal{F} & 0 \\ 0 & \mathcal{F} \end{pmatrix},$$

$$(\mathcal{F}\hat{f})(x) = \frac{\sqrt{3}}{2\pi} \sum_{n \in \mathbb{Z}^2} e^{i(x,s)} \hat{f}(s).$$

Bu operator teskarisi $\mathcal{F}^{-1}: L_2^{(2)}(\mathbb{T}^2) \to \ell_2(\Omega)$ quydagicha aniqlanadi:

$$\mathcal{F}^{-1} = \begin{pmatrix} \mathcal{F}^{-1} & 0 \\ 0 & \mathcal{F}^{-1} \end{pmatrix},$$

$$(\mathcal{F}^{-1}f)(s) = \frac{\sqrt{3}}{2\pi}\int_{\mathbb{T}^2} e^{-i(s,x)}f(x)dx.$$

bu yerda

$$(s,x) = s_1x_1 + s_2x_2.$$

Olmos panjaradagi diskret Shredinger operatori $\widehat{H}$ ushbu $\ell_2(\Omega)$ fazoda chegaralangan o‘z-o‘ziga qo‘shma operator sifatida quyidagicha aniqlanadi:

$$\widehat{H} = -3(\Delta_2 + 1) + \widehat{Q}.$$

Bunda

$$\left(-3(\Delta_2+1)\hat{f}\right)(v) = \left(\left(V_1\widehat{f_2}\right)(n);\ \left(V_2\widehat{f_1}\right)(n)\right)$$

Bu yerda

$$\left(V_1\widehat{f_2}\right)(n) = \widehat{f_2}(n) + \widehat{f_2}(n-e_1) + \widehat{f_2}(n-e_2)$$

$$\left(V_2\widehat{f_1}\right)(n) = \widehat{f_1}(n) + \widehat{f_1}(n-e_1) + \widehat{f_1}(n-e_2)$$

$$e_1, e_2 \in \Omega,\ n = (n_1\,;n_2),\ \ e_1 = (1\,;0),\ \ e_2 = (0\,;1).$$

$\hat{Q}$- Ω da aniqlangan zarrachalarning o‘zaro ta’sir potensiali bo‘lib, ular quyidagi formulalar bilan aniqlanadi.

$$(\hat{Q}f)(n) = \begin{pmatrix} \hat{Q}_1(n) & 0 \\ 0 & \hat{Q}_2(n) \end{pmatrix}\begin{pmatrix} \hat{f}_1(n) \\ \hat{f}_2(n) \end{pmatrix} = \begin{pmatrix} \hat{Q}_1(n)\hat{f}_1(n) \\ \hat{Q}_2(n)\hat{f}_2(n) \end{pmatrix}$$

bunda

$$\sum_{n\in A_2}\left|\hat{Q}_1(n)\right| < \infty\ ,\quad \sum_{n\in(p+A_2)}\left|\hat{Q}_2(n)\right| < \infty.$$

$\widehat{H}$ operatorni koordinata ko‘rinishidan impuls tasvirga o‘tish F almashtirishilari yordamida amalga oshiriladi

$$H = \mathcal{F}\widehat{H}\mathcal{F}^{-1} = \mathcal{F}\left(-3(\Delta_2+1)\right)\mathcal{F}^{-1} + \mathcal{F}\hat{Q}\mathcal{F}^{-1}.$$

H operator olmos panjaradagi diskret Shredinger operatorining impuls tasviri bo‘lib, u quydagicha aniqlanadi $H = H_0 + Q$, H_0va Q matritsa operatorlari bo‘lib, $L_2^{(2)}(\mathbb{T}^2)$ da quyidagicha aniqlanadi

$$(H_0f)(x) = \begin{pmatrix} 0 & E(x) \\ \overline{E(x)} & 0 \end{pmatrix}\begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} = \begin{pmatrix} E(x)f_2(x) \\ \overline{E(x)}f_1(x) \end{pmatrix},$$

$$(Qf)(x) = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}\begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} = \begin{pmatrix} (Q_1f_1)(x) \\ (Q_2f_2)(x) \end{pmatrix},$$

Bunda, E(x) −2 o‘zgaruvchili kompleks qiymatli funksiya

$E(x) = \frac{1}{3}\left(1 + e^{ix_1} + e^{ix_2}\right)$, $Q_i - L_2(\mathbb{T}^2)$ da aniqlangan integral operator

$$(Q_if_i)(x) = \int_{\mathbb{T}^2} Q_i(x-t)f_i(t)dt. \qquad i = 1, 2,$$

$Q_i(\cdot) - \mathbb{T}^2$da aniqlangan haqiqiy qiymatli biror uzluksiz, juft funksiya.

Quyidagi teoremani qaraymiz.

Teorema. $\sigma(H_0) = [-1\,;1]$.

Isbot. Bizga ma'lumki H_0 matritsaviy operatorining spektri quyidagi formula bilan aniqlanadi

$$\sigma(H_0)=\bigcup_{x\in\mathbb{T}^2}\sigma(H_0(x)). \qquad (2)$$

Bunda $H_0(x)-$ har bir fiksirlangan $x\in\mathbb{T}^2$ da ikki oʻlchamli matritsa, ya'ni

$$H_0(x)=\begin{pmatrix}0 & E(x)\\ \overline{E(x)} & 0\end{pmatrix},$$

$$E(x)=\frac{1}{3}\left(1+e^{ix_1}+e^{ix_2}\right).$$

Shuning uchun $H_0(x)$ ning spektri xos qiymatlaridan iborat boʻladi, ya'ni har bir tayinlangan $x\in\mathbb{T}^2$ larda $det|H_0-\lambda I|=0$ tenglamaning ildizlaridan iboratdir. Ushbu tenglamani tuzamiz:

$$(H_0-\lambda I)(x)=\begin{pmatrix}0 & E(x)\\ \overline{E(x)} & 0\end{pmatrix}-\begin{pmatrix}\lambda & 0\\ 0 & \lambda\end{pmatrix}=\begin{pmatrix}-\lambda & E(x)\\ \overline{E(x)} & -\lambda\end{pmatrix}.$$

$$det|H_0-\lambda I|=\begin{vmatrix}-\lambda & E(x)\\ \overline{E(x)} & -\lambda\end{vmatrix}=0,$$

$$\lambda^2-E(x)\cdot\overline{E(x)}=0,$$

$$\lambda_{1,2}=\pm|E(x)|,\quad x\in\mathbb{T}^2.$$

Bu yerda

$$|E(x)|^2=E(x)\overline{E(x)}=\frac{1}{3}\left(1+e^{ix_1}+e^{ix_2}\right)\frac{1}{3}\left(1+e^{-ix_1}+e^{-ix_2}\right)$$

$$=\frac{1}{9}\left(3+e^{ix_1}+e^{-ix_1}+e^{ix_2}+e^{-ix_2}++e^{i(x_1-x_2)}+e^{-i(x_1-x_2)}\right)=$$

$$=\frac{1}{9}(3+2\cos x_1+2\cos x_2+2\cos(x_1-x_2)).$$

Shunday qilib,

$$\sigma(H_0(x))=\{-|E(x)|\,;\,|E(x)|\}.$$

Demak, (2) ga koʻra

$$\sigma(H_0)=\bigcup_{x\in\mathbb{T}^2}\sigma(H_0(x))=\bigcup_{x\in\mathbb{T}^2}\{-|E(x)|\,;\,|E(x)|\}$$

$$=-Ran\{|E(x)|\}\cup Ran\{|E(x)|\}.$$

Endi

$$\max_{x\in\mathbb{T}^2}|E(x)|=\max_{x\in\mathbb{T}^2}\frac{1}{9}(3+2\cos x_1+2\cos x_2+2\cos(x_1-x_2))=1$$

va

$$\min_{x\in\mathbb{T}^2}|E(x)|=0$$

ekanligidan, ushbu

$$-Ran\{|E(x)|\}=[-1;0]$$

va

$$Ran\{|E(x)|\}=[0;1]$$

tengliklarni hosil qilamiz. Shunday qilib,

Demak $\sigma(H_0)=[-1\,;1]$.

Teorema isbotlandi [7-11].

Lemma. *$Q:L_2^{(2)}(\mathbb{T}^2)\to L_2^{(2)}(\mathbb{T}^2)$ kompakt operator.*

Isbot. $Q: L_2^{(2)}(\mathbb{T}^2) \to L_2^{(2)}(\mathbb{T}^2)$ operatorni ko‘rinishi quydagicha edi:

$$(Qf)(x) = \begin{pmatrix} (Q_1 f_1)(x) \\ (Q_2 f_2)(x) \end{pmatrix}$$

Bunda,

$$(Q_i f_i)(x) = \int_{\mathbb{T}^2} Q_i(x-t) f_i(t) dt. \qquad i = 1, 2.$$

$Q_i(\cdot)$ −ikkala o`zgaruvchi bo‘yicha ham $\mathbb{T}^2$ da aniqlangan biror uzluksiz funksiya. Biz Q operatorni kompaktligini ko‘rsatishimiz uchun har bir $i \in \{1, 2\}$ da $Q_i : L_2(\mathbb{T}^2) \to L_2(\mathbb{T}^2)$ operatorni kompakt ekanligini ko‘rsatamiz. Yuqoridagi lemmadan ma’lumki,

$$(Q_i f_i)(x) = \int_{\mathbb{T}^2} Q_i(x-t) f_i(t) dt$$

operator kompakt bo‘lishi uchun

$$\int_{\mathbb{T}^2} \int_{\mathbb{T}^2} |Q_i(x-t)|^2 dt dx < \infty$$

bo‘lishi zarur va yetarli edi. Shartga ko‘ra $Q_i(\cdot) - \mathbb{T}^2$ da aniqlangan biror uzluksiz funksiya.

Bundan

$$\int_{\mathbb{T}^2} \int_{\mathbb{T}^2} |Q_i(x-t)|^2 dt dx$$

integral mavjud va chekli. Demak Q_i kompakt , ya`ni Q kompakt operator.

Quyidagi teorema H operatorning muhim spektri va H_0 operatorning spektri o‘rtasidagi bog‘lanishni ifodalaydi.

Teorema. $\sigma_{ess}(H) = \sigma(H_0) = [-1\,; 1]$.

Isbot. Muhim spektr turg’unligi haqidagi Veyl teoremasiga ko‘ra,

$$H = Q + H_0$$

operatorning muhim spektri Q kompakt qo`zg‘alishda o‘zgarmaydi va H_0 operator spektri bilan ustma-ust tushadi. Q kompakt operator. Bundan esa $\sigma_{ess}(H) = \sigma(H_0) = [-1\,; 1]$ ekanligi kelib chiqadi. Teorema isbotlandi.

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