IKKINCHI TARTIBLI DIFFERENSIAL TENGLAMALAR VA ULARNI KANONIK KO'RINISHGA KELTIRISH HAQIDA

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Annotatsiya. Ushbu maqolada ikkinchi tartibli differensial tenglamalar va ularni kanonik ko'rinishga keltirish haqida fikrlar bayon qilingan. Tor tebranishlarning bir jinsli bo'lmagan tenglamasi va gipergeometrik tenglamalar va ularning tasnifi keltirilgan. Shuningdek, bir nechta misollar yechib ko‘rsatilgan.

Kalit so‘zlar: xususiy hosila, differensial tenglama, giperbolik tipli tenglama, kanonik shakl, Koshi masalasi.

ON SECOND-ORDER DIFFERENTIAL EQUATIONS AND THEIR CANONICAL FORMATION

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Annotation. This article discusses second-order differential equations and their canonical representation. Inhomogeneous equation of narrow oscillations and hypergeometric equations and their classification are presented. A few examples are also shown.

Key words: particular derivative, differential equation, equation of hyperbolic type, canonical form, Cauchy problem.

Agar $u(x, y) -$ funksiya ikkita erkli o'zgaruvchili funksiya bo’lib, $a_{11}, a_{12}, a_{22}, a, b, c, f - x, y$ o’zgaruvchili funksiyalar berilganlar bo’lsa, u holda bunday tenglamalar xususiy hosilali differensial tenglamalar deyiladi.

$$a_{11}u_{xx} + 2a_{12}u_{xy} + a_{22}u_{yy} + au_x + bu_y + cu = f(x, y), \qquad (1)$$

ikkinchi tartibli chiziqli xususiy hosilali differensial tenglama deyiladi.

Agar $a_{11}, a_{12}, a_{22}, a, b, c, f -$ funksiyalar faqat x va y o'zgaruvchilarga emas, balki u va uning hosilalariga ham bog'liq bo'lsa, (1) tenglama kvazichiziqli deb ataladi.

$$a_{11}dy^2 - 2a_{12}dxdy + a_{22}dx^2 = 0 \qquad (2)$$

yuqoridagi (2) ko’rinishdagi tenglama xarakteristik tenglama deyiladi.

Agar biron bir D sohada

$$a^2{}_{12} - a_{11}a_{22} = 0 \qquad (3)$$

bo'lsa, bu sohadagi (1) tenglama giperbolik tipdagi tenglamalarga tegishli deymiz (yuqorida keltirilgan ta’rifga ko’ra). Bu holda xarakteristik tenglama ikkita tenglamaga ekvivalentdir:

$$a_{11}dy - (a_{12} + \sqrt{a^2{}_{12} - a_{11}a_{22}})\, dx = 0, \qquad (4)$$

$$a_{11}dy - (a_{12} - \sqrt{a^2{}_{12} - a_{11}a_{22}})\, dx = 0. \qquad (5)$$

$$\varphi(x,y) = C_1 \; va \; \psi(x,y) = C_2$$

bu tenglamalarning umumiy integrallari haqiqiydir va (1) tenglama xarakteristikasining ikki xil turkum.

O'zgaruvchilarning shakl almashinishi

$$\xi = \varphi(x,y), \qquad \eta = \psi(x,y) \qquad (6)$$

giperbolik tipdagi tenglamani kanonik shaklga keltiradi:

$$u_{\xi\eta} + a_1 u_\xi + b_1 u_\eta + c_1 u = d_1, \qquad (7)$$

bu yerda $a_1, b_1, c_1 d_1 - \xi$ va η funksiyalarning o'zgaruvchilari.

Endi bir nechta tipik misollarni yechib ko'rsatamiz.

Misol 1. Giperbolik tipdagi tenglamani qaraymiz (xarakteristik tenglamasi va yechimini toping):

$$a^2 u_{xx} - u_{yy} = 0.$$

Yechish. Bu tenglamaning xarakteristik tenglamasining ko'rinishi quyidagicha:

$$a^2 dy^2 - dx^2 = 0.$$

Bu tenglamaning umumiy integrallari tenglik bilan aniqlanadi:

$$x = \pm ay + const.$$

O'zgaruvchilarning almashtirsak

$$\xi = x + ay, \;\; \eta = x - ay,$$

ko'rib chiqilayotgan tenglamani kanonik shaklga keltiramiz:

$$u_{\xi\eta} = 0.$$

Tenglamaning umumiy yechimi

$$u(\xi,\eta) = f_1(\xi) + f_2(\eta),$$

$$u(x,y) = f_1(x + ay) + f_2(x - ay)$$

bo'ladi.

Misol 2. Quyidagi tenglamani tipini aniqlang:

$$u_{xx} + 2u_{xy} - 3u_{yy} + u_x + u_y = 0.$$

Yechish. Bu giperbolik tipdagi tenglamaga kiradi, chunki

$$a^2 - a_{11}a_{22} = 1 + 1 * 3 = 4 > 0.$$

Misol 3. Oldingi misol shartlarida tenglamani kanonik shaklga keltiring.

Yechish. Giperbolik tenglamaning xarakteristikalari tenglamasi ikkita tenglamaga bo'linadi:

$$dy - 3dx = 0,$$

$$dy + 3dx = 0.$$

Ushbu tenglamalarning umumiy integrallari:

$$3x - y = C_1, \qquad 3x + y = C_2.$$

Yangi o’zgaruvchi kiritamiz:

$$\xi = 3x - y, \qquad \eta = 3x + y.$$

U holda:

$$u_x = u_\xi \xi_x + u_\eta \eta_x = 3u_\xi + 3u_\eta, \qquad (\xi_x = 3, \quad \eta_x = 3),$$

$$u_y = u_y \xi_y + u_\eta \eta_y = -u_\xi + u_\eta, \qquad (\xi_y = -1, \quad \eta_y = 1),$$

$$u_{xx} = 3(u_\xi + u_\eta)_\xi \xi_x + 3(u_\xi + u_\eta)_\eta \eta_x = 9u_{\xi\xi} + 18u_{\xi\eta} + 9u_{\eta\eta} u_{xy} =$$

$$3(u_\xi + u_\eta)_\xi \xi_y + 3(u_\xi + u_\eta)_\eta \eta_y = -3u_{\xi\xi} + 3u_{\eta\eta},$$

$$u_{yy} = (-u_\xi + u_\eta)_\xi \xi_y + (-u_\xi + u_\eta)_\eta \eta_y = u_{\xi\xi} - 4u_{\xi\eta} + u_{\eta\eta}.$$

Yangi o'zgaruvchili tenglama giperbolik tipdagi kanonik tenglama shaklini oladi:

$$15\mathrm{u}_{\xi\eta} + u_\xi + 2u_\eta = 0.$$

Misol 4. Tenglama uchun Koshi masalasining yechimini toping:

$$y^2 u_{xy} + u_{yy} - \frac{2}{y} u_y = 0$$

yarim tekislikda $y > 0$ dastlabki shartni qanoatlantiradi:

$$u_{y=1} = 1 - x, \qquad u_{yy=1} = 3.$$

Yechish. Biz xarakteristik tenglamani tuzamiz:

$$-y^2 dxdy + dx^2 = 0.$$

Ularning umumiy integrallari

$$x = C_1, \qquad 3x - y^3 = C_2,$$

bo’ladi. Yangi o’zgaruvchilarni kiritamiz:

$$\xi = x, \quad \eta = 3x - y^3.$$

Tenglamaning umumiy ko’rinishi

$$u_{(\eta)} = 0$$

bo’ladi.

Uning umumiy yechimi quyidagi formula bilan beriladi:

$$u(x, y) = \varphi(\xi) + \psi(\eta) = \varphi(x) + \psi(3x - y^3),$$

bu yerda φ va ψ ixtiyoriy funksiyalardir.

Ushbu funksiyalarning shakli dastlabki shartlar bilan belgilanadi:

$$\varphi(x) + \psi(3x - 1) = 1 - x,$$

$$-3\psi'(3x - 1) = 3.$$

Oxirgi tenglamani integrallash orqali biz quyidagi tenglamaga kelamiz:

$$\psi(x) = -x - C.$$

U holda

$$\varphi(x) = 1 - x + 3x - 1 + C = 2x + C$$

masalaning yechimi quyidagi formula bilan aniqlanadi:

$$u(x,y) = 2x + C - 3x - (3x - y^3) - C \quad \Rightarrow \quad u(x,y) = y^3 - x.$$

Misol 5. Tenglamaning umumiy yechimini toping.

$$x^2u_{xx} - y^2u_{yy} - 2yu_y = 0$$

Yechish. $a^2 - a_{11}a_{22} = x^2y^2$ bo'lgani uchun (1) tenglama $x0y$ tekislikning koordinata o'qlarida yotmaydigan barcha nuqtalarida giperbolik tipga ega.

Xarakteristik tenglama:

$$x^2dy^2 - y^2dx^2 = 0 \quad \Rightarrow \quad dy = \pm\frac{y}{x}dx.$$

Tenglamani umumiy integrali:

$$xy = C_1, \qquad \frac{y}{x} = C_2.$$

Yangi o'zgaruvchi kiritamiz:

$$\xi = xy, \qquad \eta = \frac{y}{x},$$

$$\xi_x = y, \qquad \xi_y = x, \qquad \eta_x = -\frac{y}{x^2}, \qquad \eta_y = \frac{1}{x},$$

$$u_x = y\ u_\xi - \frac{y}{x^2}u_\eta, \qquad u_y = xu_\xi + \frac{1}{x}u_\eta,$$

$$u_{xx} = y^2u_{\xi\xi} - 2\frac{y^2}{x^2}u_{\xi\eta} + 2\frac{y}{x^3}u_\eta + \frac{y^2}{x^4}u_{\eta\eta},$$

$$u_{yy} = x^2u_{\xi\xi} + 2u_{\xi\eta} + \frac{1}{x^2}u_{\eta\eta},$$

$$2\eta u_{\xi\eta} + u_\xi = 0.$$

$z = u_\xi$ funksiyasini kiritamiz. Bu funksiya ajratiladigan o'zgaruvchilar bilan differentsial tenglamani qanoatlantiradi:

$$\frac{dz}{d\eta} = -\frac{z}{2\eta},$$

bunda ξ parametr vazifasini bajaradi.

$$ln_z + ln\sqrt{n} = ln\varphi(\xi) \quad \Longrightarrow \quad u_\xi = \frac{\varphi(\xi)}{\sqrt{\eta}},$$

bu yerda $\varphi(\xi)$ ixtiyoriy ikki marta differentsiallanuvchi funksiya.

Oxirgi tenglamani ξ dan ortiq integrallash orqali biz tenglamaning umumiy yechimini olamiz:

$$u = \frac{\varphi(\xi)}{\sqrt{\eta}} + \psi(\eta)$$

(integrallash konstantasi ixtiyoriy $\varphi(\eta)$ funksiyadir).

Shunday qilib,

$$u(x,y) = \sqrt{x/y}\,\varphi(xy) + \psi\left(\frac{y}{x}\right).$$

Misol 6. $x0y$ tekisligining birinchi va ikkinchi choraklarida tenglamaning umumiy yechimini toping:

$$2xu_{xx} + 2x^2yu_{yy} - u_x + x^2u_y = 0. \qquad (8)$$

Yechish. $a^2 - a_{11}a_{22} = -4x^3y$ bo'lgani uchun (8) tenglama $x0y$ tekislikning ikkinchi va to'rtinchi choragida giperbolik tipga ega; birinchi va uchinchi kvadrantlarda elliptik tipdagi tenglama bo'ladi. Koordinata o'qlaridagi parabolik turga ishora qiladi. Demak, xarakteristik tenglama quyidagi ko'rinishga ega:

$$dy^2 + xydx^2 = 0.$$

1. Agar $x > 0$ va $y > 0$ bo'lsa, u holda $dy = \pm i\sqrt{xy}dx$ tenglamaning umumiy integrali quyidagicha ko'rinishda bo'ladi:

$$\sqrt{x^3} + 3i\sqrt{y} = C_1, \quad \sqrt{x^3} - 3i\sqrt{y} = C_2.$$

Yangi o'zgaruvchi kiritsak,

$$\xi = \sqrt{x^3}, \qquad \eta = 3\sqrt{y},$$

$$\xi_x = \frac{3}{2}\sqrt{x}, \quad \xi_y = 0, \qquad \eta_x = 0, \quad \eta_y = \frac{3}{2\sqrt{y}},$$

$$u_x = \frac{3}{2}\sqrt{x}\,u_\xi, \quad u_y = \frac{3}{2\sqrt{y}}u_\eta,$$

$$u_{xx} = \frac{9}{4}xu_{\xi\xi} + \frac{3}{4\sqrt{x}}u_\xi, u_{yy} = \frac{9}{4y}u_{\eta\eta} - \frac{3}{4\sqrt{y^3}}u_\eta,$$

$$(u_{\xi\xi} + u_{\eta\eta}) = 0,$$

$$u(x,y) = \varphi\left(\sqrt{x^3} + 3i\sqrt{y}\right) + \psi\left(\sqrt{x^3} - 3i\sqrt{y}\right).$$

$x < 0$ va $y > 0, dy = \pm\sqrt{-xy}dx.$ Tenglamaning umumiy integrali quyidagicha ko'rinishda bo'ladi.

$$\sqrt{-x^3} + 3\sqrt{y} = C_1, \qquad \sqrt{-x^3} - 3\sqrt{y} = C_2.$$

Yangi o'zgaruvchi kiritsak,

$$\xi = \sqrt{-x^3} + 3\sqrt{y}, \qquad \eta = \sqrt{-x^3} - 3\sqrt{y}.$$

Kanonik ko'rinishdagi tenglamaga keltiramiz:

$$u_{\xi\eta} = 0.$$

Tenglamaning umumiy yechimining ko'rinishi quyidagicha:

$$u(x,y) = \varphi\left(\sqrt{-x^3} + 3\sqrt{y}\right) + \psi\left(\sqrt{-x^3} - 3\sqrt{y}\right).$$

Tor tebranishlarning bir jinsli bo'lmagan tenglamasi

Tor tebranishlarining bir jinsli bo'lmagan tenglamasi uchun Koshi masalasini ko'rib chiqamiz. Bizga $f(x,t) \in C^1 (R \times R_+)$ funksiya berilgan bo'lsin:

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2} + f(x,t), \quad x \in R, t \geq 0, \qquad (9)$$

$$u(x,0)=\varphi(x), \qquad \frac{\partial u}{\partial t}(x,0)=\psi(x), \quad x\in R. \qquad (10)$$

(9) va (10) masalalarni quyidagi ko'rinishda ifodalab tekshirish mumkin

$$u(x,t)=v(x,t)+w(x,t), \qquad (11)$$

bu yerda $v(x,t)$ – berilgan $\phi,\psi,f\ \equiv\ 0$ (erkin tebranishlar tenglamasi) bilan Koshi masalasining (9), (10) yechimi va $w(x,\ t)$ funksiyasi Koshi masalasini (9), (10) $\phi\ \equiv 0,\psi\ \equiv\ 0$ bo'lgandagi yechimi, ya'ni

$$\frac{\partial^2 w}{\partial t^2}=a^2\frac{\partial^2 w}{\partial x^2}+f(x,t),$$

$$w(x,0)=0, \quad \frac{\partial w}{\partial t}(x,0)=0. \qquad (12)$$

$w\ (x,t)$ funksiya Dalamber formulasi yordamida yoziladi va (12) masalaning yechimi formula yordamida topiladi:

$$w(x,t)=\frac{1}{2a}\int_0^t\left[\int_{x-a(t-r)}^{x+a(t+r)}f(\xi,\tau)d\xi\right]d\tau. \qquad (13)$$

Darhaqiqat, $w(x,0)\ \equiv\ 0$ shartni qanoatlantirilishi aniq. t va x ga nisbatan xususiy hosilalarni topamiz:

$$\frac{\partial w}{\partial t}=0+\frac{1}{2a}\int_0^t[af(x+a(t-\tau),\tau)+af(x-a(t-\tau),\tau]d\tau,$$

$$\frac{\partial^2 w}{\partial t^2}=f(x,t)+\frac{a}{2}\int_0^t\left[\frac{\partial f}{\partial x}(x+a(t-\tau),\tau)-\frac{\partial f}{\partial x}(x-a(t-\tau),\tau\right]d\tau,$$

$$\frac{\partial w}{\partial x}=\frac{1}{2a}\int_0^t[f(x+a(t-\tau),\tau)-f(x-a(t-\tau),\tau]d\tau,$$

$$\frac{\partial^2 w}{\partial x^2}=\frac{1}{2a}\int_0^t\left[\frac{\partial f}{\partial x}(x+a(t-\tau),\tau)-\frac{\partial f}{\partial x}(x-a(t-\tau),\tau\right]d\tau.$$

Bulardan kelib chiqib shuni xulosa qilamizki,

$$w(x,0)=0, \qquad \frac{\partial w}{\partial t}(x,0)=0, \qquad \frac{\partial^2 w}{\partial t^2}-a^2\frac{\partial^2 w}{\partial x^2}=f(x,t)$$

$w(x,t)$ – tenglama va boshlang'ich shartlarni qanoatlantiradi.

Ushbu yoʻnalishda oʻzbek va xorijlik olimlar tomonidan koʻplab ilmiy izlanishlar olib borilgan. Ular jumlasiga [1-15] maqolalarni kiritish mumkin.

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