ANIQ VA ANIQMAS INTEGRALLAR HAQIDA TUSHUNCHALAR

Muzaffarova Mohinur Umarovna
Buxoro davlat universiteti
Fizika-matematika fakulteti talabasi

Annotatsiya. Ushbu maqolada boshlang’ich funksiyani topish va aniq integral ta’rifini o’quvchilarga soddaroq tushuntirish yo’llari bayon qilingan. Aniqmas integralning xossalari va koordinatalar sistemasida egri chiziqlar kesishuvidan hosil bo’lgan shakllarning yuzasini topish yo’llari keltirilgan. Kasr tartibli integrallar haqida tushuncha va ayrim elementar funksiyalarning kasr tartibli integrlarlarini topish yo’llari sodda ravishda bayon qilingan.

Kalit so’zlar. Boshlang’ich funksiya, funksiya, grafik, chegara, yuza, bo’laklab integrallash usuli, kasr tartibli integrallar.

CONCEPTS OF DEFINITE AND INDEFINITE INTEGRALS

Muzaffarova Mohinur Umarovna
Bukhara State University
Student of the Faculty of Physics and Mathematics

Annotation. This article describes ways to find the initial function and explain the definition of the definite integral in a simpler way to the students. The properties of the indefinite integral and ways to find the surface of the shapes formed by the intersection of curves in the coordinate system are given. Concept of integrals of fractional order and ways of finding integrals of fractional order of some elementary functions are simply described.

Keywords. Initial integration method, function, graph, boundary, surface, fraction, integrals of fractional order.

Integral qadimgi Misrda ma’lum bo’lgan. Ammo, zamonaviy shaklda emas. O’shandan beri matematiklar ushbu mavzuga oid juda ko’p kitoblar yozdilar. Mazkur yo’nalishda asosiy nazariya Nyuton va Leybnits tomonidan yaratilgan.

Integrallarni noldan qanday tushunish mumkin? Ushbu mavzuni tushunish uchun matematik analiz fani to’g’risida asosiy bilimlar kerak bo’ladi. Integrallarni tushunish uchun limitlar va hosilalar to’g’risidagi tushunchalarni bilish shart.

Ta’rif. Agar $[a, b]$ oraliqda aniqlangan $f(x)$ funksiya uchun bu oraliqning barcha nuqtalarida $F'(x) = f(x)$ tenglik bajarilsa, $F(x)$ funksiya shu oraliqda $f(x)$ funksiyaga nisbatan boshlang`ich funksiya deb ataladi [1].

Ta’rif. Agar $F(x)$ funksiya biror oraliqda $f(x)$ funksiyaning boshlang’ich funksiyasi bo’lsa, u holda $F(x) + C$ (bu yerda C – ixtiyoriy $const$) funksiyalar to'plami shu kesmada $f(x)$ funksiyaning aniqmas integrali deyiladi va quyidagicha belgilanadi:

$$\int f(x)\,dx = F(x) + C$$

Aniqmas integralning xossalari:

1) Aniqmas integralning hosilasi integral ostidagi funksiyaga teng, ya'ni

$$\left(\int f(x)\ dx\right)' = f(x);$$

2) Aniqmas integralning differensiali integral belgisi ostidagi ifodaga teng, ya'ni

$$d\left(\int f\ (x)dx\right) = f\ (x)dx;$$

3) Biror funksiyaning hosilasidan olingan aniqmas integral shu funksiya bilan ihtiyoriy o'zgarmasning yig'indisiga teng, ya'ni

$$\int\ F'(x)dx = F(x) + C;$$

4) Biror funksiyaning differentsialidan olingan aniqmas integral shu funksiya bilan ihtiyoriy o'zgarmasning yig'indisiga teng, ya'ni

$$\int d\,F(x) =\ F(x) + C;$$

5) Chekli sondagi funksiyalarning algerbaik yig'indisidan olingan aniqmas integral shu funksiyalarning har biridan olingan aniqmas integrallarning algebraik yig'indisiga teng, ya'ni

$$\int(\,f_1(x) + f_2(x) + f_3(x))dx = \int f_1(x)\,dx + \int f_2(x)\,dx + \int f_3(x)\,dx.$$

Bu xossaning umumlashmasi bo'lgan quyidagi xossani keltirish mumkin:

- agar $f_1(x), f_2(x), \dots, f_n(x)$ funksiyalar $[a,b]$ oraliqda integrallansa, unda quyidagilar o'rinli

$$\int(a_1\,f_1(x) \pm a_2 f_2(x) \pm \cdots \pm a_n f_n(x))dx =$$

$$= a_1\int f_1(x)dx \pm a_2\int f_2(x)dx \pm a_n\int f_n(x)dx,$$

bunda $a_1, a_2, \dots, a_n$ –o'zgarmas sonlar.

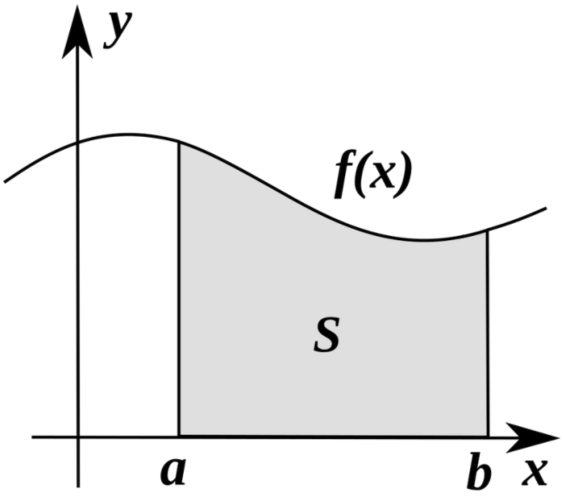
Integrallash jadvali

№	$f(x)$	$\int f(x)dx$
1	1	$x + c$
2	x^{α}	$\frac{x^{\alpha+1}}{\alpha+1} + c, \alpha \neq -1$
3	$\frac{1}{x}$	$ln\lvert x\rvert + c$

4	a^x	$\frac{a^x}{lna}+c$
5	$\cos x$	$\sin x+c$
6	$\sin x$	$-\cos x+c$
7	$\frac{1}{\cos^2 x}$	$tgx+c$
8	$\frac{1}{\sin^2 x}$	$-ctgx+c$
9	$\frac{1}{\sin x}$	$ln\left\|tg\frac{x}{2}\right\|+c$
10	$\frac{1}{\cos x}$	$ln\left\|tg\left(\frac{x}{2}+\frac{\pi}{4}\right)\right\|+c$
11	$\frac{1}{a^2+x^2}$	$\begin{cases}\frac{1}{a}arctg\frac{x}{a}+c\\ -\frac{1}{a}arctg\frac{x}{a}+c\end{cases}$
12	$\frac{1}{x^2-a^2}$	$\frac{1}{2a}ln\left\|\frac{x-a}{x+a}\right\|+c$
13	$\frac{1}{a^2-x^2}$	$\frac{1}{2a}ln\left\|\frac{a+x}{a-x}\right\|+c$
14	$\frac{1}{\sqrt{a^2-x^2}}$	$\begin{cases}arcsin\frac{x}{a}+c\\ -arccos\frac{x}{a}+c\end{cases}$
15	$\frac{1}{\sqrt{x^2\pm a^2}}$	$ln\left\|x+\sqrt{x^2\pm a^2}\right\|+c$
16	shx	$chx+c$
17	chx	$shx+c$
18	$\frac{1}{ch^2x}$	$thx+c$
19	$\frac{1}{sh^2x}$	$-cthx+c$

Aniq integral

Integral bilan ishlashda biz cheksiz kichik miqdorlarga ham duch kelamiz. Integral shaklning yuzasini, jism massasini, notekis harakat bilan bosib o’tilgan yo’lni va boshqa ko’p narsalarni hisoblashga yordam beradi. Shuni esda tutish kerakki, integral juda ko’p cheksiz kichik sonlar yig’indisidir.



Funksiya grafigi bilan chegaralangan shaklning yuzasini qanday topish mumkin?

Koordinata o'qlari va funksiya grafigi bilan chegaralangan egri chiziqli trapetsiyani cheksiz kichik qismlarga ajratamiz. Shunday qilib, berilgan shakl ingichka, kichik shakllarga bo'linadi. Shakllar yuzalarining yig'indisi trapetsiyaning yuzasini tashkil qiladi. Ammo esda tutish lozimki, bunday hisoblash taxminiy natija beradi. Biroq, bo'laklar qanchalik kichik bo'lsa, hisoblash shunchalik aniq bo'ladi. Bu esa quyidagicha yoziladigan aniq integral:

$$\int_a^b f(x)\,dx,$$

bu yerda a va b chegaralar.

Aniq integralni hisoblashda quyidagi Nyuton – Leybnits formulasidan foydalanamiz:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Aniq integral xossalari:

1. $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$
2. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
3. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx,$

bu yerda $a \le c \le b$.

Maqolada bo'laklab integrallash usuli va kasr funksiyalarni integrallash usullarini keltiramiz va ular yordamida misollarni yechib ko'rsatamiz.

Bo'laklab integrallash usuli. Ikki $U = U(x)$ va $V = V(x)$ funksiyalar (a, b) oraliqda uzluksiz $U'(x)$ va $V'(x)$ hosilalarga ega bo'lsin,

$$d[U(x)\cdot V(x)] = U(x)dV(x) + V(x)\cdot dU(x).$$

Bu tenglikdan

$$U(x)dV(x) = d[U(x)\cdot V(x)] - V(x)\cdot dU(x). \qquad (1)$$

Endi (1) tenglikni integrallab topamiz:

$$\int U(x)dV(x)\,dx = \int \big(d[U(x)\cdot V(x)] - V(x)\cdot dU(x)\big)\,dx =$$

$$= U(x)\cdot V(x) - \int V(x)dU(x)\,dx.$$

Shunday qilib quyidagi

$$\int U\,(x)dV(x) = U(x)V(x) - \int V(x)\,dU(x) \quad (2)$$

formulaga kelamiz. Bu (2) formula bo'laklab integrallash formulasi deyiladi.

Kasr funksiyalarni integrallash.Har qanday ratsional funksiyani ratsional kasr ko'rinishida, ya'ni ikki ko'phadning nisbati ko'rinishida tasvirlash mumkin:

$$\frac{A}{(x-a)^k} \qquad (k - butun\ musbat\ son\ k \geq 2)$$

$$\frac{Q(x)}{f(x)} = \frac{B_0x^m + B_1x^{m-1} + \cdots + B_m}{A_0x^n + A_1x^{n-1} + \cdots + A_n}$$

Muhokamaning umumiyligini cheklamasdan, bu ko'phadlar umumiy ildizga ega emas deb faraz qilamiz. Agar sur'atinng darajasi mahrajning darajasidan past bo'lsa, kasr to'gri kasr deb ataladi, aks holda noto'g'ri kasr deb ataladi.

Agar kasr no'to'g'ri bo'lsa, sur'atni maxrajga (ko'phadni ko'phadga bo'lish qoidasi boyicha) bo'lib, berilgan kasrni ko'phad bilan biror to'g'ri kasrning yig'indisi ko'rinishida tasvirlash mumkin:

$$\frac{Q(x)}{f(x)} = \mathrm{M(x)} + \frac{F(x)}{f(x)}$$

bu yerda M(x) - ko'phad, $\frac{F(x)}{f(x)}$-to'g'ri kasr.

1-misol. Noto'g'ri ratsional kasr berilgan bo'lsin:

$$\frac{x^4-3}{x^2+2x+1}$$

Sur'atni maxrajiga (ko'phadlarni bo'lish qoidasi boyicha) bo'lib, shuni hosil qilamiz:

$$\frac{x^4-3}{x^2+2x+1} = x^2 + 2x + 3 - \frac{4x-6}{x^2+2x+1}$$

Ko'phadlarni integrallash hech qanday qiyinchilik tug'dirmagani uchun, ratsional kasrlarni integrallashdagi asosiy qiyinchilik to'g'ri ratsional kasrlarni integrallashdan iboratdir:

Oddiy kasr kasrlarni integrallash quyidagi ko'rinishda amalga oshiriladi:

1) $\int \frac{A\,dx}{x-a} = A\ln|x-a| + C;$

2) $\int \frac{A\,dx}{(x-a)^n} = -\frac{A}{(n-1)(x-a)} + C, \qquad n \neq 1;$

3) $\int \frac{Mx+N\,dx}{x^2+px+q} = \frac{M}{2}\int \frac{2x+p}{x^2+px+q}dx + \left(N-\frac{Mp}{2}\right)\int \frac{dx}{x^2+px+q} =$

$= \frac{M}{2}\ln(x^2+px+q) + \left(N-\frac{Mp}{2}\right)\int \frac{dx}{(x+p/2)^2+q-p^2/4} =$

$= \frac{M}{2}\ln(x^2+px+q) + \frac{N-M\,p/2}{\sqrt{q-p^2/4}}arctg\frac{x+p/2}{\sqrt{q-p^2/4}} + C;$

4) $\int \frac{Mx+N\,dx}{(x^2+px+q)^n} = \frac{M}{2}\int \frac{2x+p}{(x^2+px+q)^n}dx + \left(N-\frac{Mp}{2}\right)\int \frac{dx}{(x^2+px+q)^n}$

$= \frac{M}{2}\frac{(x^2+px+q)^{1-n}}{1-n} + \left(N-\frac{Mp}{2}\right)\int \frac{dx}{((x+p/2)^2+q-p^2/4)^n}, \qquad n>1$

1-4 formulalardan, kasr funksiyalar integrali ratsional funksiyalar, logarifm, arctg orqali ifodalanishi kelib chiqadi. Demak, shu turdagi ratsional funksiyalar yuqorida keltirilgani kabi integrallanadi.

Shuning uchun, ratsional funksiyaning aniqlash sohasiga tegishli bo'lgan har qanday oraliqdagi aniqmas integrali ratsional funksiyalar, logarifmlar va arctg funksiyalarning kompozitsiyalarini algebraik yig'indisi sifatida ifodalanadigan elementar funksiyalardir.

2-misol. Noaniq integralni hisoblang:

$$\int \frac{dx}{\sqrt[3]{(2+x)(2-x)^5}}$$

Yechish.

Elementar almashtirishdan foydalanib, quyidagi ko'rinishga keltiramiz:

$$\int \sqrt[3]{\frac{2-x}{2+x}}\,\frac{dx}{(2-x)^2}.$$

Integral ostidagi funksiya o'zgaruvchilarga nisbatdan ratsionaldir.

$$x_1 = x, \qquad x_2 = \left(\frac{2-x}{2+x}\right)^{1/3}$$

Bu yerda $n = 1,\ p_1 = \frac{1}{3}, a = -1,\ b = 2,\ c = 1,\ d = 2.$

$$\frac{2-x}{2+x} = t^3,$$

Quyidagicha almashtiramiz

$$x = 2\frac{1-t^3}{1+t^3}, \qquad dx = -12\frac{t^2dt}{(1+t^3)^2}, \quad \frac{1}{2-x} = \frac{1+t^3}{4t^3}.$$

Demak,

$$\int \frac{dx}{\sqrt[3]{(2+x)(2-x)^5}} = \int \sqrt[3]{\frac{2-x}{2+x}} \frac{dx}{(2-x)^2} = -12\int \frac{(t^3+1)^2 t^3 dt}{16t^6(1+t^3)^2} = -\frac{3}{4\int \frac{dt}{t^3}}$$

$$= \frac{3}{8}\sqrt[3]{\left(\frac{2+x}{2-x}\right)^2} + C.$$

3-misol. Quyidagi aniq integralni hisoblang:

$$\int_0^{\pi} x^3 sinx\, dx$$

Yechish.

Funksiyaning integralini hisoblash uchun bo'laklab integrallash usulidan foydalanamiz.

$$U = x^3, \qquad dU = 3x^2\, dx,$$
$$dV = sinx\, dx, \qquad V = -cosx,$$

bu yerda

$$-x^3 cosx - \int (-\,cosx)3x^2 dx = -x^3 cosx + 3\int x^2 cosx\, dx$$

Integralni hisoblash uchun yana bo'laklab integrallash usulidan foydalanamiz.

$$U = x^2, \qquad dU = 2x\, dx$$
$$dV = cosx\, dx, \qquad V = sinx$$

bundan

$$-x^3 cosx + 3\left(x^2 sinx - \int 2xsinx\, dx\right) = -x^3 cosx + 3(x^2 sinx -$$
$$-2\int xsinx\, dx).$$

Integralni hisoblash uchun yana bo'laklab integrallash usulidan foydalanamiz.

$$U = x, \qquad dU = dx$$
$$dV = sinx\, dx, \qquad V = -cosx$$

$$-x^3 cosx + 3(x^2 sinx - 2\left(x(-cosx) - \int -cosx\;\; dx\right) = -x^3 cosx +$$
$$+3(x^2 sinx - 2\left(x(-cosx) + \int cosx\;\; dx\right) = -x^3 cosx + 3(x^2 sinx -$$
$$-2(x(-cosx) + \sin(x)) = -x^3 cosx + 3x^2 sinx + +6xcosx - 6sinx.$$

Topilgan boshlang'ich funksiyaning oraliqdagi qiymatini hisoblaymiz.

$$-\pi^3 cos\pi + 3\pi^2 sin\pi + 6\pi cos\pi - 6sin\pi -$$
$$-(0cos0 + 3 \times 0sin0 + 6 \times 0cos0 - 6sin0) = \pi^3 - 6\pi.$$

4-misol.

$$\int \frac{2x^2+41x-91}{(x-1)(x+3)(x-4)}dx$$

integralni hisoblang.

Yechish.

Berilgan integralni hisoblash uchun integral ostidagi kasrni turli maxrajli kasrlar yig'indisi ko'rinishiga keltiramiz.

$$\frac{2x^2+41x-91}{(x-1)(x+3)(x-4)}=\frac{A}{(x-1)}+\frac{B}{(x+3)}+\frac{C}{(x-4)}$$

$$\frac{A(x+3)(x-4)}{(x-1)(x+3)(x-4)}+\frac{B(x-1)(x-4)}{(x-1)(x+3)(x-4)}+\frac{C(x-1)(x+3)}{(x-1)(x+3)(x-4)}=$$

$$=\frac{A(x^2-x-12)}{(x-1)(x+3)(x-4)}+\frac{B(x^2-5x+4)}{(x-1)(x+3)(x-4)}+\frac{C(x^2+2x-3)}{(x-1)(x+3)(x-4)}=$$

$$=\frac{(A+B+C)x^2+(-A-5B+2C)x+(-12A+4B-3C)}{(x-1)(x+3)(x-4)}.$$

Ratsional kasrlar tengligi xossasiga ko'ra

$$2x^2+41x-91=(A+B+C)x^2+(-A-5B+2C)x+(-12A+4B-3C).$$

x ning bir xil darajalari oldidagi koeffitsientlari tengligidan kelib chiqib,

$$\begin{cases} A+B+C=2 \\ -A-5B+2C=41 \\ -12A+4B-3C=-91. \end{cases}$$

Bu yerda $A=4, B=-7, C=5$

$$\int\left(\frac{4}{(x-1)}+\frac{-7}{(x+3)}+\frac{5}{(x-4)}\right)dx=4\ln|x-1|-7\ln|x+3|+5\ln|x-4|$$

5-misol. Quyidagi aniq integralni hisoblang:

$$\int_{-1}^{1}(x^3-2x^2+x-1)dx$$

Yechish.

$$=\frac{x^4}{4}-\frac{2x^3}{3}+\frac{x^2}{2}-x\ \Big/_{-1}^{1}=$$

$$=\frac{1}{4}-\frac{2}{3}+\frac{1}{2}-1-\frac{1}{4}-\frac{2}{3}-\frac{1}{2}+1=-\frac{4}{3}=-1\frac{1}{3}$$

6-misol. Quyidagi aniq integralni hisoblang:

$$\int_{1}^{2}x^2\ln x dx.$$

Yechish.

Biz quyidagi misolni hisoblashda bo'laklab integrallash usulidan foydalanamiz.

$$\int U\,dV=UV-\int V\,dU$$

Bu yerda

$$U = lnx, \quad dU = \frac{1}{x}dx$$

$$dV = x^2 dx, \;\; V = \frac{x^3}{3}$$

$$\int_1^2 x^2 \ln x dx = \frac{x^3}{3} lnx/_1^2 - \int_1^2 \frac{x^3}{3} . \frac{1}{x} dx = \frac{8}{3} ln2 - \frac{1}{3} ln1 - \int_1^2 \frac{x^2}{3} dx =$$

$$= \frac{8}{3} ln2 - \frac{x^3}{9} /_1^2 = \frac{8}{3} ln2 - \frac{8}{9} + \frac{1}{9} = \frac{8}{3} ln2 - \frac{7}{9}$$

7-misol. Quyidagi aniq integrallarni hisoblang.

1) $\int_0^1 \frac{x^2 dx}{1+x^6}$

Yechish.

Berilgan aniq integralni hisoblash uchun avval aniqlanmagan integralni hisoblaymiz. Bu yerda $t = x^3$

$$\int \frac{1}{3(1+t^2)} dt = \frac{1}{3} \int \frac{dt}{1+t^2} = \frac{1}{3} arctg(t)$$

Endi t ni o'rniga x^3 ni keltirib qo'yamiz va berilgan oraliqlardagi qiymatini hisoblaymiz.

$$\frac{1}{3} arctg(x^3)|_0^1 = \frac{1}{3}\left(arctg(1) - arctg(0)\right) = \frac{1}{3}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{12}$$

2) $\int_3^4 \frac{x^2+3}{x-2} dx$

Yechish.

Berilgan aniq integralni hisoblash uchun avval aniqlanmagan integralni hisoblaymiz. Buning uchun kasrni oddiy kasrlar yig'indisi shakliga keltiramiz.

$$\int \frac{x^2+3}{x-2} dx = \int (x + 2 + \frac{7}{x-2}) dx = \int x\, dx + \int 2\, dx + \int \frac{7}{x-2} dx$$

$$= \frac{x^2}{2} + 2x + 7\ln(|x-2|)$$

Topilgan boshlang'ich funksiyaning oraliqdagi qiymatini hisoblaymiz.

$$\left(\frac{x^2}{2} + 2x + 7\ln(|x-2|)\right)|_3^4 = \left(\frac{4^2}{2} + 8 + 7\ln(|4-2|)\right) - \left(\frac{3^2}{2} + 6 + 7\ln(|3-2|)\right) = \frac{11}{2} 7ln2.$$

Kasr tartibli integral haqida tushuncha

Kasr tartibli integrallar yordamida matematik analiz qilish uch asrdan ko'proq tarixga egadir. Butun XIX asr va XX asrning birinchi yarmi matematik analizning mustaqil bo'limi sifatida natijalarni to'plash va kasr hisobini shakllantirish davriga aylangan. Bunda matematiklar va fiziklarning ilmiy ishlarining nashrlari paydo bo'lgan. Ular, Laplas, Furye, Riemann, Abel, Lyuvil, Grunvald, Xivisayd, Kuryant va boshqalarning ilmiy asarlaridir. Mashhur rus matematik olimi A.V. Letnikov kasr tartibli matematik analizining rivojlantirishga katta hissa qo‘shgan. 1868-1872-yillarda A.V. Letnikovning kasr tartibli hisoblash bo‘yicha birinchi ilmiy maqolalari chiqqan [2].

Olimlarning kasr tartibli hisoblashga bo‘lgan qiziqishlarining yangi to‘lqini 1974-yilda «Kasr tartibli hisoblash» (K.B. Oldham, J. Spanier) kitobi nashr etilgandan so‘ng paydo bo‘lgan. Ushbu kitobda kasr tartibli hisoblash nazariyasi tizimli ravishda keltirilgan. Shu vaqtdan boshlab turli xil jurnallarning tematik sohalari paydo bo‘la boshlagan, ular ilm-fan, texnika, tabiatshunoslikning turli sohalarida kasr tartibli hisoblashni qo‘llashga bag’ishlangan.

Hozirgi vaqtda kasr tartibli hisoblash nazariy jihatdan ham, amaliy jihatida ham tez rivojlanish bosqichidadir. Matematik analizning bu bo‘limi har xil (an’anaviy va fraktal) muhitdagi murakkab dinamik jarayonlarni matematik modellashtirish vositasiga aylangan, bu analiz, sintez, diagnostika va yangi boshqaruv tizimlarini yaratishning turli muammolarini hal qilishga imkon yaratib beradi.

Kasr tartibli integrallarni o‘rganishda $L_1(\Omega)$ sinfi muhim rol o‘ynaydi.

$\Omega = [a, b\,]$ bo‘lsin, bu erda $-\infty \le a \le b \le \infty$, ya’ni Ω chegaralangan kesma, yarim o‘q yoki butun o‘q bo‘lishi mumkin. Agar yarim va butun o‘qlar bo‘lsa, odatdagidek, biz $R^1 = [-\infty, \infty), R^1_+ = [0, \infty)$ va R^1 bilan bitta cheksiz to‘g'ri chiziq belgilanadi. Kelgusida Gyolderning sinflarini chegaralangan kesmada qaraymaiz.

Ω da aniqlangan $f(x)$ funksiya berilgan bo’lsin. $L_1(a, b\,)$ bilan $[a, b\,]$ di inegrallanuvchi va

$$\int_a^b |f(x)|dx < \infty$$

shartni qanoatlantiruvchi funksiyalar sinfi belgilanadi.

Ta’rif. $f(x) \in L_1(a, b\,)$ bo’lsin.

$$I_x^\alpha f(x) \overset{\text{def}}{=} \frac{1}{\Gamma(\alpha)} \int_a^x \frac{f(t)dt}{(x-t)^{1-\alpha}}, x > a,$$

bu yerda $\alpha > 0$, α tartibli kasr integral deyiladi.

7-misol. $f(x) = c = const$ ni α tartibli kasr integralini toping.

Yechish.

$$I_0^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x \frac{f(t)}{(x-t)^{1-\alpha}} dt = [t = xz, dt = xdz, x - t = x - xz$$

$$= x(1-z)] = \frac{1}{\Gamma(\alpha)} \int_0^x \frac{c\,x\,dz}{(x-xz)^{1-\alpha}} dt = \frac{c}{\Gamma(\alpha)} \int_0^x \frac{x\,dz}{x^{1-\alpha}(1-z)^{1-\alpha}} dt =$$

$$= \frac{c}{\Gamma(\alpha)} \int_0^x x^{\alpha} z^0 (1-z)^{1-\alpha} = \frac{cx^{\alpha}}{\Gamma(\alpha)} B(1,\alpha) = [\, B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)},$$

$$\Gamma(2) = 1,\ \ \Gamma(1) = 1] = \frac{cx^{\alpha}}{\Gamma(\alpha)} \frac{\Gamma(a)\Gamma(1)}{\Gamma(a+1)} = \frac{cx^{\alpha}}{\Gamma(a+1)} + C,$$

bu yerda

$$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}dx.$$

8-misol. $f(x) = x^{-1}$ ni α − tartibli kasr integralini toping.

$$I_0^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x \frac{f(t)}{(x-t)^{1-\alpha}} dt.$$

Yechish. Bizning holda $f(t) = t^{-1}$. Demak

$$\frac{1}{\Gamma(\alpha)} \int_0^x \frac{f(t)}{(x-t)^{1-\alpha}} dt = \frac{1}{\Gamma(\alpha)} \int_0^x \frac{t^{-1}}{(x-t)^{1-\alpha}} dt =$$

$$= \frac{1}{\Gamma(\alpha)} \ [t = xz, dt = xdz] =$$

$$= \frac{1}{\Gamma(\alpha)} \int_0^x \frac{x^{-1}z^{-1}x\,dz}{x^{1-\alpha}(1-z)^{1-\alpha}} = \frac{1}{\Gamma(\alpha)} x^{\alpha-1} \int_0^x z^{-1}(1-z)^{1-\alpha} dz =$$

$$= \frac{1}{\Gamma(\alpha)} x^{\alpha-1} B(0,\alpha) + C = \frac{x^{\alpha-1}}{\Gamma(\alpha)} \frac{\Gamma(0)\Gamma(\alpha)}{\Gamma(\alpha)} + C = \frac{x^{\alpha-1}}{\Gamma(\alpha)} + C.$$

Maqoladan asosiy koʻzlangan maqsad noaniq, aniq va kasr tartibli integrallarni talabalarga soddaroq tushuntirish hisoblanadi. Muallif tomonidan shu yoʻnalishda [3-5] maqolalar nashr qilingan.

FOYDALANILGAN ADABIYOTLAR:

1. Кудрявцев Л. Д., Кутасов А.Д., Чехлов В.И., Шабунин М.И. Сборник задач по математическому анализу. Том 2. 2003. 504 с.

2. Самко С.Т., Килбас А.А., Маричев О.И. Интегралы и производные дробного порядка и их приложения. – Минск: Наука и техника, 1987. – 688 с.

3. Muzaffarova, M. U. (2023). Nolga bo’lish mumkinmi savoli haqida. Science and Education, 4(4), 55–60.

4. Музаффарова М.У. (2023). Путешествие в мир простых и составных чисел. Scientific progress. 4:4, стр183-189.

5. Музаффарова Мохинур Умаровна. (2023). Признаки делимости чисел. Tadqiqotlar, 14(6), 112–117.