HYBRID/HETEROGENEOUS V2X CONGESTION ALGORITHMS FOR URBAN TRANSPORTATION SYSTEMS

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Abstract: The mathematical modeling of traffic flows in controlled networks typically involves the use of various equations and variables to represent the behavior of vehicles and their interactions with the transportation infrastructure. One commonly used mathematical model for traffic flow in controlled networks is the Lighthill-Whitham-Richards (LWR) model, which is a macroscopic traffic flow model. The LWR model describes the relationship between traffic density, traffic velocity, and traffic flux (flow rate).

Here's the basic expression of the LWR model:

$\rho(x, t)$ be the traffic density at location x and time t (vehicles per unit length).

$v(\rho)$ be the velocity of traffic as a function of traffic density (typically decreases as density increases).

$q(\rho)$ be the traffic flux (flow rate) as a function of traffic density.

The LWR model can be expressed as a partial differential equation (PDE) for one-dimensional traffic flow as follows [1, 2, 3, 4]:

$$\partial\rho/\partial t + \partial q/\partial x = 0$$

This equation represents the conservation of vehicles in the network, stating that the rate of change of traffic density with respect to time ($\partial\rho/\partial t$) plus the rate of change of traffic flux with respect to location ($\partial q/\partial x$) is equal to zero. In other words, it ensures that the total number of vehicles in the network remains constant over time.

The relationship between traffic density, velocity, and flux can be further defined as [5, 6, 7, 8, 9]:

$$q(\rho) = \rho * v(\rho)$$

This equation describes how the traffic flux (flow rate) depends on the traffic density and velocity.

To introduce control measures, such as traffic signals or variable speed limits, into the mathematical model, additional terms or equations can be incorporated. These control measures can be modeled as functions of traffic density, time, or other relevant variables. For instance, you can introduce terms that represent the effect of traffic signals on the flow rate at intersections or the impact of variable speed limits on traffic velocity.

The LWR model forms the basis for many traffic flow models used in transportation engineering and traffic management. To create a more specific and detailed mathematical model for a controlled network, you would need to consider the particular control strategies and their impact on traffic density, velocity, and flow rates within the network. These models can become quite complex when considering multiple control measures and their interactions in a real-world transportation network [2, 3, 4].

The Lighthill-Whitham-Richards (LWR) model provides a macroscopic description of traffic flow in terms of traffic density, velocity, and flux. To study the time dependence of traffic flow and the effect of traffic lights in reducing traffic density, we need to introduce some additional elements to the basic LWR model.

LWR Model without Traffic Lights: The basic LWR model, without traffic lights, can be described by the following equation:

$$\partial\rho/\partial t + \partial q/\partial x = 0$$

In this equation, $\partial\rho/\partial t$ represents the rate of change of traffic density with respect to time, and $\partial q/\partial x$ represents the rate of change of traffic flux with respect to space (location) [9, 10].

Traffic Lights: To incorporate the effect of traffic lights, we need to modify the LWR model to account for variations in traffic flow caused by the switching of traffic lights. Traffic lights control the flow of traffic at intersections by regulating when vehicles from different directions can proceed. Let's introduce a control function, C(t), that represents the state of the traffic light system as a function of time.

Modified LWR Equation with Traffic Lights: With traffic lights, the LWR equation becomes:

$$\partial\rho/\partial t + \partial q/\partial x = -C(t)(q - q_{max})$$

C(t) represents the state of the traffic light control system, where C(t) = 0 when the light is red and C(t) = 1 when the light is green.

q represents the traffic flux (flow rate) as a function of traffic density, typically described by the fundamental diagram of traffic flow.

q_{max} is the maximum flow rate at the given location, which is determined by factors like road capacity and vehicle speed.

Effect of Traffic Lights: When traffic lights are green (C(t) = 1), the control term ($-C(t)(q - q_{max})$) has no effect on the LWR equation, and it operates like the standard LWR model. However, when the lights are red (C(t) = 0), the control term has a negative effect, reducing the traffic flux q by an amount proportional to the difference between the actual flow q and the maximum flow rate q max.

This reduction in traffic flow during red light phases leads to a temporary increase in traffic density at the intersection. As a result, traffic density may increase while the lights are red, and it will gradually decrease when the lights turn green again and traffic can flow freely.

This modified LWR model with traffic lights allows for the analysis of traffic dynamics at intersections and the time-dependent behavior of traffic flow under the influence of signalized control. It helps in understanding how traffic lights affect traffic density and how the switching of traffic lights influences the overall traffic flow patterns in a controlled network.

If the time of the red light is twice as long as the time of the green light at a traffic light-controlled intersection, and the maximum speed of cars is 5 kilometers per hour (km/h), we can analyze the effect of this signal timing on traffic flow using the modified LWR model mentioned earlier.

Let's assume:

Time of green light (T_green) = T seconds.

Time of red light (T_red) = 2T seconds.

Maximum vehicle speed (V_max) = 5 km/h.

The traffic light control function can be represented as follows:

C(t) = 1 during the green phase (0 <= t < T).

C(t) = 0 during the red phase (T <= t < 3T).

During the green phase, traffic can flow freely without any effect from the control term. However, during the red phase, the control term will reduce the traffic flow q since vehicles cannot pass through the intersection. This will lead to an accumulation of vehicles at the intersection and a temporary increase in traffic density.

Here's a simplified analysis of the traffic dynamics:

Green Phase (0 <= t < T):

Traffic density decreases as vehicles pass through the intersection.

Traffic flow is unrestricted, and vehicles move at their maximum speed (V_max = 5 km/h).

Red Phase (T <= t < 3T):

Traffic density increases because vehicles are halted at the intersection.

Traffic flow drops to zero during the entire red phase.

Transition between Phases (T seconds):

There will be a sharp increase in traffic density as vehicles queue up at the intersection when the light turns red.

When the light turns green, the queue of vehicles will gradually disperse as vehicles start moving again.

This alternating pattern of high traffic density during the red phase and lower traffic density during the green phase will continue as long as the signal timing remains unchanged.

Summary

It's important to note that this is a simplified analysis, and real-world traffic dynamics can be influenced by various factors, including driver behavior, the length of vehicle queues, and the capacity of the road segments leading to and from the intersection. In practice, traffic engineers use more sophisticated traffic flow models and consider factors like queue discharge rates and the capacity of the intersection to optimize signal timings and minimize congestion.

References

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