FUNKSI HOSILASI MISOLLARDA QO'LLASH

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Annotatsiya: Ushbu maqolada biz funksi hosilasi misollarda qo'llashni ko'rib chiqdik.

Kalit so'zlar; funksiya hosilasi, bir tomonli hosila, cheksiz hosilalar, hosilani geometrik ma'nolari

1^0 Funksiya tarifini keltiraylik ; Aytaylik y=f(x) funksiya x_0 nuqtaning (x_0€R) biror atrofda berilgan bo'lsin. BU funksiya x_0 nuqtaning ortirmasi

$\Delta y=\Delta f(x_0)=f(x_0+\Delta x)-f(x_0)$ ning orttirmasi Δx ga nisbati $\Delta y/\Delta x = \Delta f(x_0)/\Delta x$ ($\Delta x\neq 0$) ni qaraymiz.

1-ta'rif. Agar ushbu $\lim_{\Delta x\to 0}(f(x_0+\Delta x)-f(x_0))/\Delta x$ limiti mavjud va chekli bo'lsa, u f(x) funksiyaning x_0 nuqtadagi hosilasi deyiladi va $d(f(x_0))/dx$ yoki $f'(x_0)$ kabi belgilanadi. Demak $f'(x_0)=\lim_{\Delta x\to 0}(f(x_0+\Delta x)-f(x_0))/\Delta x$ (1). Agar $x_0+\Delta x=x$ deyilsa unda $\Delta x=x-x_0$ va $\Delta x\to 0$ da $x\to x_0$ bo'lib (1) munosabat quyidagi $f'(x_0)=\lim_{x\to x_0}(f(x)-f(x_0)/(x-x_0)$ ko'rinishida keladi.

1-misol $Y=\log_a x$ tarifida foydalanib hisoblang.

$Y+\Delta y=\log_a x+\Delta x$,

$\Delta y=\log_a(x+\Delta x)-y$,

$\Delta y=\log_a(x+\Delta x)-\log_a x$

$\Delta y=\log_a\frac{x+\Delta x}{x}$ / : Δx

$$\frac{\Delta y}{\Delta x}=\frac{1}{\Delta x}\times\log_a(1+\frac{\Delta x}{x})=\frac{1}{x}\times\frac{x}{\Delta x}\times\log_a(1+\frac{\Delta x}{x})$$

$$\frac{\Delta y}{\Delta x}=\frac{1}{x}\times\log_a(1+\frac{\Delta x}{x})^{\frac{x}{\Delta x}} \quad (\frac{\Delta x}{x}=t)$$

$$\frac{\Delta y}{\Delta x}=\frac{1}{x}\times\log_a(1\_+t)^{\frac{1}{t}} \quad [(1+t)^{\frac{1}{t}}=e]$$

$$\frac{\Delta y}{\Delta x}=\frac{1}{x}\times\log_a e \quad [\log_a e=\frac{1}{\ln a}]$$

$$Y'=\lim_{\Delta x\to 0}(\frac{1}{x}\times\frac{1}{\ln a})=\frac{1}{x}\times\frac{1}{\ln a} \quad (0<a\neq 1, x>0)$$

2-misol $y=x^n$ ta'rifidan foydalanib hisoblang

$$y+\Delta y=(x+\Delta x)^n$$

$$\Delta y=(x+\Delta x)^n-x^n$$

$$\Delta y=x^n+n*x^{n-1}*\Delta x+\frac{n*(n-1)}{1*2}*x^{n-2}*(\Delta x)^2+\ldots\ldots+(\Delta x)^n-x^n$$

$\Delta y = n \times x^{n-1}\Delta x + \frac{n\times(n-1)}{1\times 2} \times x^{n-2} \times (\Delta x)^2 + \ldots + (\Delta x)^n \quad / :\Delta x$

$$\frac{\Delta y}{\Delta x} = n \times x^{n-1} + \frac{n \times (n-1)}{1 \times 2} \times x^{n-2} \times \Delta x + \ldots + (\Delta x)^{n-1}$$

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} [n \times x^{n-1} + \frac{n\times(n-1)}{1\times 2} \times x^{n-2} \times \Delta x + .. + (\Delta x)^{n-1}$

Y’ = n× x^{n-1}.

2^0 Bir tomonli hosilalar

2-ta’rif . Agar $\Delta x \to (+0)$ $[\Delta x \to (-0)]$ da $\frac{\Delta y}{\Delta x}$ nisbatining limiti

$\lim_{\Delta x \to +0} \frac{\Delta f(x_0)}{\Delta x} = \lim_{\Delta x \to +0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$

$[\lim_{\Delta x \to (-0)} \frac{\Delta f(x_0)}{\Delta x} = \lim_{\Delta x \to (-0)} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}]$

mavjud va chekli bo’lsa bu limit f(x) funksiyaning x_0 nuqtadagi o’ng (chap) hosilasi deb ataladi va uni f’$(x_0 + 0)$, $f'(x_0 - 0)$ kabi belgilanadi. Funksiyaning o’ng va chap hosilalari bir tomonli hosila deb ataladi .

3-misol y=$(\sin x)^{5e^x}$ x=0 nuqtadagi bir tomonli hosilasini toping .

$y + \Delta y = (\sin(x + \Delta x))^{5e^{x+\Delta x}}$

$\Delta y = (\sin(x + \Delta x))^{5e^{x+\Delta x}} - (\sin x)^{5e^x} \quad / :\Delta x$

$\frac{\Delta y}{\Delta x} = \frac{1}{\Delta x} \times (\sin(x + \Delta x))^{5e^{x+\Delta x}} - \frac{1}{\Delta x}(\sin x)^{5e^x}$ [x=0]

$\frac{\Delta y}{\Delta x} = \frac{1}{\Delta x} \times (\sin \Delta x)^{5e^{\Delta x}}$ [y’= $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$]

Y’= $\lim_{\Delta x \to +0} (\sin \Delta x)^{5e^{\Delta x}}$ =0 [y’= $\lim_{\Delta x \to (-0)} (\sin \Delta x)^{5e^{\Delta x}}$ =0] .

Demak berilgan funksiyaning x=o nuqtadagi o’ng hosilasi (f’(+0)=0) , chap hosilasi (f’(-0)=0) bo’ladi.

3^0 Cheksiz hosilalar .

Aytaylik y=f(x) funksiyasi x_0 nuqtaning (x_0€R) biror atrofida berilgan bo’lib , u x_0 nuqtada uzluksiz bo’lsin.

3 – ta’rif . Agar $\Delta x \to 0$ da $\frac{\Delta y}{\Delta x}$ nisbatining limiti $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$

$+\infty$ $yoki$ $(-\infty)$ bo’lsa , uni ham f(x) funksiyaning x_0 nuqtadagi hosilasi deyiladi . Bunday hosila cheksiz hosila deb ataladi .

Demak ; $\lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = +\infty \; yoki \; (-\infty)$

3- misol Y= $\frac{\cos 4x - (\cos 4x)^3}{4x^2}$ funksiya hosilasini toping .

Y’ = $\frac{\cos 4x(1-(\cos 4x)^2}{4x^2} = \lim_{x \to 0} \frac{\cos 4x(1-(\cos 4x)^2)}{4x^2}$

Y’ = $\lim_{x\to0}\frac{\cos 4x}{4x}\times\lim_{x\to0}\frac{1-(\cos 4x)^2}{x}$ $\quad$ [$1-(\cos 4x)^2=(\sin 4x)^2$]

4- misol . Ushbu y=$\sqrt[3]{\cos x}$ funksiyaning $x_0=\frac{\pi}{2}$ nuqtadagi hosilasini toping .

Y + $\Delta y=\sqrt[3]{\cos(x_0+\Delta x)}$ $\quad\leftrightarrow\quad \Delta y=\sqrt[3]{\cos(x_0+\Delta x)}-\sqrt[3]{\cos x_0}$

$\Delta y=\sqrt[3]{\cos(\frac{\pi}{2}+\Delta x)}-\sqrt[3]{\cos\frac{\pi}{2}}=\sqrt[3]{-\sin\Delta x}$ =-$\sqrt[3]{\sin\Delta x}$

$\frac{\Delta y}{\Delta x}=\frac{-\sqrt[3]{\sin\Delta x}}{\Delta x}\leftrightarrow\lim_{\Delta x\to0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to0}\frac{-\sqrt[3]{\sin\Delta x}}{\Delta x}$

$y'=-\lim_{\Delta x\to0}\sqrt[3]{\frac{\sin\Delta x}{\Delta x}}\times\frac{1}{\sqrt[3]{\Delta x}}$ =-∞

Y = x^{x^x} $\quad$ [$A=x^{x^x}$]

$(x^x\times\ln x)$’ =$(\ln A)'$ $\leftrightarrow$ (x^x)’ $\times\ln x+\frac{x^x}{x}=\frac{A'}{A}$ $\quad$ [A =x^{x^x}]

$(x^x)'+\ln x+\frac{x^x}{x}=\frac{A\prime}{x^x}$ $\quad$ [$(x^x)'$ =? $\quad B=x^x$]

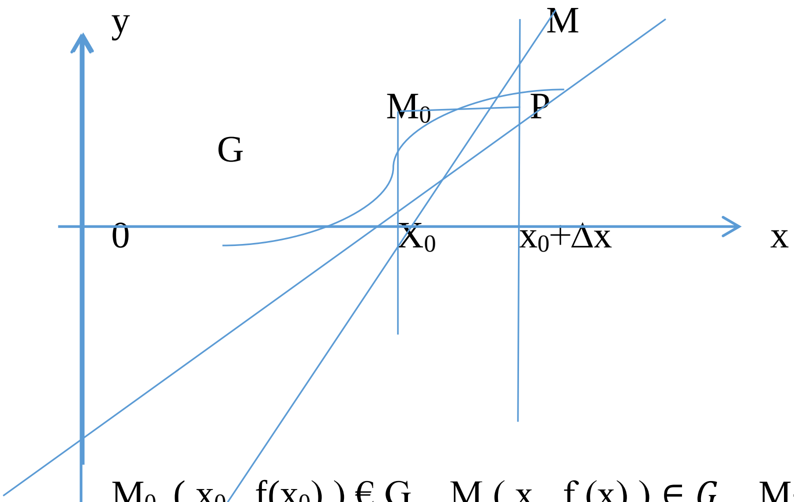
$x^x=B$ /× ln $\quad$ (X $*\ln x)'=(\ln B)'$

$\ln x+1=\frac{B\prime}{B}$ $\leftrightarrow$ [B = x^x] $\leftrightarrow$ $\ln x+1=\frac{B'}{x^x}$ $\leftrightarrow$ B’= $x^x(\ln x+1)$

$x^x\times\ln x(\ln x+1)+\frac{x^x}{x}=\frac{A\prime}{x^{x^x}}$; A’ = $x^{x^x}(x^x\times\ln x(\ln x+1)+\frac{x^x}{x})$

4^0 Hosilani geometrik ma’nolari ;Faraz qilaylik f(x) funksiya (a ;b) da berilgan bo’lib , x_0€ (a;b) nuqtada $f'(x_0)$ hosilaga ega bo’lsin . Bu f(x) funksiya grafigimizda tasvirlangan G egri chiziqni ifodalasin . Bu G chiziqda $M_0(x_0;y_0)$, M(x ;y) nuqtalarni olib

ular orqali o’tuvchi L kesuvchini qaraymiz.



M_0 (x_0, $f(x_0)$) € G , M (x , f (x)) ∈ G $\quad M\to M_0$ da L kesuvchi limit holati G chiziqqa M_0 nuqtada o’tkazilgan urinma deyiladi . Ravshanki , Φ burchak Δx ga bog’liq ; Φ = Φ(Δx) , f (x) funksiyaning grafigiga M_0 nuqtadan o’tkazilgan urinma mavjud bo’lishi uchun $\lim_{\Delta x\to0}\Phi(\Delta x)=\alpha$

ning mavjud bo’lishi lozim . Bunda α *urinmaning* Ox o’qining musbat yo’nalishi bilan tashkil etilgan burchagi M_0MP uchburchakda

$\tan\Phi(\Delta x)=\frac{MP}{M_0P}=\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$

Φ(Δx) =arctan$\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$

Bo’lishi kelib chiqadi . Funksiya uzluksizligidan foydalanib topamiz

$\lim_{\Delta x \to 0} \Phi(\Delta x) = \lim_{\Delta x \to 0} arc\tan \frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} =$ $=\arctan\left[\lim_{\Delta x \to 0} \frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}\right] = arc\tan f'(x_0)$

Demak $\Delta x \to 0$ da $\Phi(\Delta x)$ uning limiti mavjud va $\alpha = arc\tan f'(x)$

Kegingi tenglikdan $f'(x_0) = \tan\alpha$ bo'lishi kelib chiqadi . Demak funksiyaning x_0 nuqtadagi $f'(x_0)$ hosilasi urinmaning burchak koeffitsiyentini ifodalaydi.

Bunda urinma tengalamasi ; $Y = f(x_0) + f'(x_0)*(x-x_0)$, $y= f(x)$ funksiya grafigiga urinish nuqtasi $M_0(x_0 ; y_0)$ da otkazilgan normal tenglamasi

$y-y_0 = -\frac{1}{f'(x_0)} * (x - x_0)$ agar $f'(x) \neq 0$ bo'lsa .

6- misol $y= x^2$ chiziqning A (2 :4) va A' ($2+\Delta x$; $4+\Delta y$) nuqtalari orqali AA' kesuvchi o'tkazilgan . Agar ($\Delta x=1$) bu kesuvchi burchak koefffitsientini toping .

$Y=x^2$ Buning grafigi paroboladan iboratligini bilgan holda

$\Delta y = f(x + \Delta x)^2 - f(x)$; (X = 2 ; $\Delta x =1$); $\Delta y = (2+1)^2 - 4 = 5$

$\tan\alpha = \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{5}{1} = 5$

Foydalanilgan adabiyotlar;

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